Surprises in Elementary Mathematics Reform: 
Toward a Model of Situated Professional Development

Helena P. Osana
Guillaume W. Jabbour
Anna Sampogna
Chantal Desrosiers
Mary Ann Chacko

Concordia University

1455 de Maisonneuve Blvd. Ouest
Education, LB-579
Montreal, Quebec H3G 1M8
Canada

Helena P. Osana is Assistant Professor in the Department of Education at Concordia University in the Early Childhood and Elementary Education program. Her research interests include mathematics education, statistical and scientific reasoning, and teacher cognitions.

Guillaume W. Jabbour is a first-year student in the teacher education program in the Department of Education at Concordia University. His interests lie in teachers’ expectations of student achievement.

Anna Sampogna is in her final year in the teacher education program in the Department of Education at Concordia University. Her research interests lie in children’s cognitive development and elementary mathematics curriculum and assessment.

Chantal Desrosiers is a third-year student in the Master’s of Teaching Mathematics program in the Department of Mathematics at Concordia University. Her research focuses on the effects of technology and communication on conceptual understanding in college-level calculus.

Mary Ann Chacko is a second-year Master’s student in the MA Child Studies program at Concordia University. Her research interests center around discourse in elementary mathematics instruction, particularly with respect to assisting students at risk of school failure.

Abstract

We engaged three elementary teachers in a reform-oriented professional development intervention in mathematics. The teachers work in an inner-city school with a student population exhibiting similar characteristics to students in border communities in the United States. Individual problem-solving sessions with four students in each of the teacher’s classrooms were videotaped, and over the course of eight weeks, we viewed the videoclips with the teachers in collaborative discussion sessions to observe and interpret their students’ solution strategies. Preliminary analyses of interview and classroom observation data indicate that our situated approach to professional development enhanced the teachers’ interest in learning about children’s thinking and helped them to recognize that mathematical understanding is constructed and not “received.” Moreover, our data indicated that our intervention gave them tools to discriminate among the various problem-solving strategies used by their students. Our journey toward a more relevant and personally meaningful approach to professional development for teachers of at-risk students is also reported.

One of the primary goals of the National Council of Teachers of Mathematics (NCTM, 2000), outlined in their Principles and Standards for School Mathematics document, is to engage all students in useful and meaningful mathematics through problem solving, argument, and communication. Although the original set of guidelines for the improvement of mathematics instruction was put forward by the NCTM in 1989,

\(^\text{1}\) All names are pseudonyms.
since that time little substantive improvement has been detected in children’s genuine understanding of mathematics or their ability to solve novel problems, particularly for children in border communities, many of whom are at risk for school failure (DeMonte, 2004; Secada et al., 1998; Stigler & Hiebert, 2004).

At the heart of the solution to this problem is professional development, at both the preservice and inservice levels. Mewborn (2003) outlined several guidelines for the design and implementation of effective teacher development. First, teachers should be “at the center” of their professional development, actively involved in their own learning (Schifter, 1998). In previous professional development initiatives, this has involved discussing teaching strategies, reviewing and critiquing instructional practices, and reflecting on personal and professional beliefs about teaching and learning (Borko & Putnam, 1996; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Lubinski & Jaber, 1997). Second, professional development should focus on the conceptual underpinnings of mathematics and how students come to understand them (Ball, 1996; Carpenter, Fennema, & Franke, 1996; Heaton, 2000; RAND Corporation, 2003; Swafford, Jones, & Thornton, 1997). Third, effective teaching in mathematics also involves listening to students to effectively discern the strengths and weakness in their mathematical thinking (Thompson & Thompson, 1994). Thus, professional development should also emphasize the nature of children’s intuitive conceptions and the way they develop with and without instruction (Fennema et al., 1996; Fennema & Nelson, 1997; Ginsburg, Klein, & Starkey, 1998). Finally, teachers should be able to try what they have learned in their professional development sessions with their own students, all within a context of support within their respective schools, districts, and partnerships, and subsequently reflect with their colleagues about the relative effectiveness of innovative instructional strategies (Learning First Alliance, 1998; Secada & Adajian, 1997).

Our research stems from a long-standing partnership between Concordia University and one English-language school board in Montreal, Canada. Our portion of the partnership entailed designing and implementing a professional development program for the first-, second-, and third-grade mathematics teachers at Vaughn Elementary School. Vaughn is a severely socioeconomically (SES) depressed school whose student body has similar characteristics to children in border communities in the United States: for many of them, English is not their first language, over 40% of the families of the children enrolled in the school report income below the poverty level, and a large proportion are at risk for school failure (Padrón, Waxman, & Rivera, 2002; Quebec Ministry of Education, 2003; Secada et al., 1998). Nevertheless, students at risk deserve, and indeed need, high quality instruction, which entails challenging them with relevant content, providing them the tools for future success, and holding them to high standards. This is particularly important in the early grades, for the factors that contribute to school failure and dropout begin early and are difficult to remEDIATE later (Frontera & Horowitz, 1995; Kennedy, Birman, & Demaline, 1986). Our work provides teachers in such settings the professional development that will assist them to provide top-quality education to those students who are typically left behind educationally and otherwise (Secada et al., 1998).

Our initial goals at the beginning of the school year were to provide professional development to a small group of teachers using the Cognitively Guided Instruction (CGI, Carpenter et al., 1996; Fennema et al., 1996) model, which incorporates many of the principles outlined by Mewborn (2003). CGI is predicated on the notion that elementary mathematics teaching should begin with the intuitive concepts and strategies children bring to the classroom. In CGI, teachers are introduced to basic forms of addition, subtraction, multiplication, and division problems and, through the viewing of videoclips of children solving problems, are invited to reflect on the relationship between word problem type and the development of children’s thinking. Teachers attempt instructional strategies that are built on their knowledge of children’s problem solving and return to subsequent workshop settings to discuss and reflect on the effectiveness of these strategies with their colleagues. CGI has been shown deepen teachers’ understandings of their students’ thinking thereby fostering meaningful problem solving through dialogue and collaborative learning in the classroom. As a result, students’ achievement on both alternative and standardized mathematics assessments has been shown to increase (e.g., Fennema et al., 1996).

We envisioned a similar structure for our professional development at Vaughn Elementary School, with an eye toward exploring how CGI could be adapted to meet the needs of the particular students at the school. Thus, we intended to meet with the teachers on a regular basis, provide them with information about “typical” children’s mathematical strategies (through CGI videoclips and readings), and to create reform-oriented instructional activities designed specifically for the students in their classrooms. What actually transpired, however, was significantly different from what we had envisioned. This article describes the direction we have now taken with respect to mathematics professional development at this particular school, outlines our revised research objectives, and presents preliminary results with respect to teacher growth.
Situating Professional Development

Three teachers agreed to be part of our professional development project: Karen, a first-grade teacher, Jackie, a second-grade teacher, and Valerie, who taught third-grade remedial mathematics to a class of fifth-grade students. Both Jackie and Valerie were veteran teachers; Jackie had 17-years’ experience at the elementary level, and Valerie had taught at both the elementary and secondary levels for a combined 30 years of teaching experience. Valerie had also acted as a special education consultant during her career. Karen was in her first year of teaching; after obtaining a college degree in Sociology, she attended an accelerated teacher education program that earned her a Bachelor’s degree in Kindergarten and Elementary Education.

Near the beginning of our CGI agenda, we realized that the workshops were having little, if any, impact on the three teachers. Although Jackie attempted a few CGI problems and small-group problem solving sessions in her classroom, it was clear from conversations with her that she was not convinced that her students were able to communicate about mathematical strategies nor that CGI would achieve the student results stipulated by the school district. She was searching for strategies that would have an immediate impact on the students in her class, both from a cognitive and affective perspective, and was skeptical that CGI would be fruitful in this regard. Valerie did not attempt to incorporate CGI into her current practices because of her perception that word problems are not intrinsically motivating and that the CGI approach lacked the structure necessary for the students in her class, all of whom had been coded with some form of learning disability or behavioral disorder. Karen joined the team late and had not attended any CGI workshops. She had attended two informal meetings with the team; her motivation for participating in the partnership between the school and the university was to receive new ideas and suggestions about teaching so that she would not end up “set” in the way she was taught in her teacher education program.

It became obvious to us that a different approach to professional development, at least with these teachers, was necessary. We hypothesized that the teachers were not terribly motivated to make the connection between “typical” development in mathematical thinking and the instructional strategies that they perceived were necessary for their specific students. We predicted that making the professional development more relevant and immediate would have a greater impact on the teachers’ thoughts and practices. We thus decided to videotape some of the students from their own classrooms solving mathematics problems. Our aim became to provide the teachers with specific information about the mathematical strategies used by the actual students in their respective classrooms and to use these videos as a vehicle for the creation and implementation of targeted curricula and instructional activities. We spent eight weeks viewing these videotapes with the teachers and discussing the unique characteristics of their students’ problem solving strategies. This approach provided opportunities for the teachers and researchers to work collaboratively on testing conjectures about the effectiveness of these activities within their own classrooms.

The crux of our research was to track the teachers’ reactions and thought processes as they participated in this more “situated” form of professional development in elementary mathematics. More precisely, we examined (a) what the teachers used as evidence of their students’ ability in mathematics in the context of problem solving and how the teachers refined their assessment strategies over the eight-week period, and (b) any changes in the teachers’ classroom practice as they acquired the tools with which to assess their students’ thinking at a more fine-grained level. Examining teacher change within the context of professional development allowed us to investigate the changes in teachers’ cognitions as they occurred at specific points in the intervention, thereby evaluating the effectiveness of our approach. Our results also promise to contribute to existing models of teachers’ cognitions in mathematics and provide insight into the ways effective teacher training can be made scalable for a large number of teachers who deal with our target population.

Research Activities

Measures and Instruments

We collected numerous types of data to evaluate change in the three participating teachers. First, before the video-viewing sessions began, we administered the Mathematics Beliefs Scale (MBS, Fennema, Carpenter, & Loe, 1990) and conducted individual audio-recorded pre-interviews with the teachers. The MBS yields a score out of 240 that measures pedagogical content beliefs in addition and subtraction. The scale contains three subscales that measure (a) teachers’ beliefs about how children learn mathematics, (b) the relationship between learning concepts and procedural skill, and (c) how mathematics should be taught. Higher total scores on the MBS indicate beliefs that children construct their own knowledge in mathematics and that their intuitive conceptions should form the basis of instruction that focuses on understanding and problem solving.

A portion of the pre-interviews required the teachers to sort 15 cards that each contained a description
of a hypothetical student solving a multi-digit mathematics problem and writing a representation of his or her solution. An example of one of these cards is presented in Figure 1. We carefully designed the cards so that they contained the elements of four dimensions that appear in the literature as important indicators of the level of sophistication of students’ mathematical problem solving (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Ginsburg, 1989; Hiebert, 1986; Kilpatrick, Martin, & Schifter, 2003; Kilpatrick, Swafford, & Findell, 2001; Smith, 2003): meaningfulness of strategy, representation of solution, execution of procedure, and conceptual understanding of the standard algorithm. As illustrated in Figure 2, a range of strategies were represented on the cards; in some cases the strategy showed conceptual understanding, in others it did not, and in two cases there was not enough evidence to make a determination about student understanding. In addition, we deliberately constructed several scenarios in which the student used a meaningful, and on some cards, sophisticated strategy, but made a calculation error and arrived at the wrong answer (e.g., Card 5).

The teachers were asked to sort the cards according to “how good you think the students are in math.” They were free to create as many categories as they wished and were asked to think out loud as they worked through the task. After the sorting, the teachers were asked to describe their categories and explain the placement of each card. The same cards and procedure were used on the post-interview after the intervention was completed, including a re-administration of the MBS.

Description of Intervention

The intervention consisted of four video-viewing sessions conducted individually with each teacher during an eight-week period. The sessions had two primary objectives: to assist the teachers in acquiring useful conceptual tools with which to observe and interpret their students’ problem solving and to brainstorm with the teachers about ways to tailor instruction so that specific weaknesses could be targeted. The Direct Modeling-Counting-Derived Fact/Invented Algorithm framework outlined by Carpenter et al. (1999) was used to guide the discussions. Carpenter et al. proposed that students who engage in Direct Modeling need to physically represent the actions and objects in the problem, often with blocks or tally marks on paper. Children who use Counting strategies often use objects (fingers are typical) to keep track of counts as they enact the solution. For addition and subtraction problems, they start the counting sequence not from one, but from a number presented in the problem. In this sense, Counting strategies are more abstract and efficient than Direct Modeling. Students who use either Derived Fact or Invented Algorithm strategies use number combinations that they already know to derive a solution to the problem; such strategies are often performed without external tools. This framework was described to the teachers on an as-needed basis and as such constituted “just-in-time” scaffolds as they grew in their understandings of children’s thinking.

Also during the eight-week period, we visited each of their classrooms to collect detailed time-sampling observations, which are currently being analyzed and will be the focus of a forthcoming paper. These data will allow us to examine any changes in the teachers’ instructional practice during the data collection period, presumably as a result of our collaborative video-viewing sessions.

Teacher Change: Preliminary Results

In this section, we present preliminary results of teacher change using data from the MBS and the think-aloud protocols from the card sort task. In the description of the card sort data below, we use the label “Category 1” to describe the teachers’ “best at math” category, “Category 2” for their “second-highest” category, and so forth.

Mathematics Beliefs Scale Data

For both Valerie and Karen, the total scores on the MBS were higher after the intervention compared to before (Valerie’s score increased by 9 points and Karen’s by 29). Jackie’s total score decreased by 9 points. An analysis of the subscale scores, however, indicated that all three teachers believed more strongly after the intervention that skills should be taught in conjunction with conceptual understanding and problem solving. In addition, Valerie and Karen indicated a stronger awareness that children learn mathematics not by receiving knowledge from the teacher but through constructing strategies that are meaningful to them. Only Karen grew to appreciate the instructional implications of children’s mathematical development, at least as measured by the MBS; after the intervention, she indicated a stronger belief that instruction should facilitate children’s construction of meaningful strategies rather than be designed to “transmit” skills and standard procedures.

Card Sort Data

In our comparison of the teachers’ categorizations on the card sort before and after the intervention, we found evidence that the teachers changed in their interpretations of student thinking in mathematics. A change common to all three teachers was the use of more fine-grained criteria on the post card-sort. Karen, for example, used four more criteria to classify the cards on the post card-sort and our analysis
involved more than twice the number of codes than for the pre card-sort. New criteria on the post card-sort included the recognition of Direct Modeling as a developmentally primitive problem-solving strategy and an awareness that conceptual understanding can exist even in the presence of counting errors. In addition, Karen went from creating three categories of student problem solving on the pre card-sort to five on the post card-sort. Taken together, this revealed the development of more finely-tuned intellectual tools to assess student thinking.

Both Karen and Jackie learned to make finer distinctions among different counting strategies: instead of making vague references to students as “counters,” on the post card-sort, they were able to recognize the sophistication in the “counting on” strategy, which entails starting the counting sequence from a quantity given in the problem instead of starting it from 1. Finally, on the pre card-sort, Valerie was impressed with any invented procedure used by a student that was not taught by the teacher. This criterion was enough for her to place these students in a “high” category. On the post card-sort, she was able to see finer distinctions in such strategies: she saw the value in using numbers that were meaningful and easy to manage and could discern the reasons behind students’ manipulations of numbers.

We also uncovered evidence of idiosyncratic change. Jackie, for example, placed great emphasis on mental computation before the intervention; cards that involved manipulatives or counting (with the use of fingers) were placed in lower categories because of the perceived “non-intellectual” aspect of external tool use. On the post card-sort, she was better able to “see past” the concreteness of the strategy to evaluate mathematical understanding. Karen, on the other hand, did not attend to manipulative use on the pre card-sort, but used it (Direct Modeling, in particular) as an indication of lower mathematical ability on the post card-sort. On Card 15, for example, a student uses a Direct Modeling strategy to correctly solve 17 + 19 and demonstrates a correct representation of the standard algorithm. Karen placed this card in a “medium” category before the intervention but in the lowest of five categories on the post card-sort because of the student’s reliance on physical representations to solve the problem. Thus, the intervention appeared to have helped Karen to make clearer distinctions among strategies that vary on a developmental continuum.

Our analysis of the card sort data also revealed general patterns of interpretation that were observed before the intervention and appeared to have persisted in face of it. Several of these observations deserve attention here because they reveal interpretation skills that have, for the most part, not been reported previously in the teacher cognition literature. First, even though the teachers appeared to have appropriated a more refined framework with which to discriminate and describe the differences among scenarios, the difference between Direct Modeling and Counting strategies was not salient to the teachers, even after the intervention. Although Valerie and Jackie, for instance, were better able to discriminate both of these strategies from Invented Algorithms after the intervention, they continued to lump Direct Modeling and Counting Strategies together on the post card-sort.

Second, we found that teachers were rarely focused on students’ use of non-standard written representations (such as the use of arrows instead of conventional “+” and “=” symbols or idiosyncratic placement of numbers in the execution of a standard algorithm). Even though mentioned briefly by Karen and Valerie, not once did written representation surface as a determinant of students’ mathematical ability. Furthermore, we noticed that problem features, such as mathematical structure or problem wording, was also not a salient feature in the teachers’ interpretations. This may be due to the lack of attention paid to problem type in our professional development intervention; unlike CGI (Fennema et al., 1996), the relationship between mathematics problems and children’s thinking was not the central focus of our work with the teachers. Instead, our approach was to let the teachers guide the professional development activities by continually informing us about what they perceived to be immediately relevant to them and their specific students’ needs.

Finally, the teachers were less attuned to the students’ final answers during the card sort task than we had anticipated. To illustrate, Card 13 depicts a student who uses an Invented Algorithm to arrive at a solution of 45 for the problem 39 + 16. The student takes 1 from the 16 to turn the 39 into a 40, thereby transforming the problem into 40 + 15. The student adds these two numbers incorrectly, however, and concludes by saying, “So 40 and 15 is 45. So the answer is 45...”. None of the teachers was concerned with the final answer being incorrect; they focused on the child’s creativity of solution and his ability to manipulate numbers (“make groups of ten”) while problem solving. In fact, Valerie placed this card in the second highest category on the pre card-sort and in the highest category on the post card-sort, during which she said, “This is an invented way of getting to a Base 10 and then adding... keeping numbers in your head to work out the problem.”
Discussion

Early in our work at Vaughn Elementary School, we noticed that the participating teachers were less than enthusiastic about committing to the “standard” CGI professional development agenda. Indeed, not all teachers are, what we call, “CGI-ready.” A move toward CGI necessitates a radical change in thinking and orientation on which not all teachers are ready to embark. It was thus necessary for us to halt our activities and shift direction, from both a practical and research perspective. We searched for an alternate approach that would make the professional development more personally relevant to the teachers and that might convince them that their students were indeed capable of generating a multitude of meaningful strategies and talking to their peers about them.

Our decision to individually interview several students from each of the teacher’s classrooms for the purposes of discussion and information sharing appeared to have benefited the teachers, and our preliminary analyses of the data have allowed us to arrive at a few tentative conclusions. First, the viewing of their own students enhanced their commitment to and interest in learning about the development of children’s thinking, even when the discussion moved away from the precise strategies seen in the videos. Without their own students as “anchors,” the teachers were less motivated to make the connection between children’s learning “in theory” and the specific goings-on in their respective classrooms. Furthermore, after the intervention, the teachers were more sensitive to the importance of conceptual understanding as a foundation for the development of procedural proficiency in mathematics. We speculate that this arose in part because of the growth they exhibited in interpreting their students’ problem solving strategies, although more research is needed to explore this relationship directly.

Second, our informal passes through the observation data have revealed changes in the teachers’ instructional practices. During the intervention, all three teachers began to use CGI problem types in their classrooms, Karen and Jackie on a regular basis. The personalized professional development we offered demonstrates that children’s problem solving is an excellent window into the mathematical minds of children. This stands in contrast to the teachers’ initial reactions to engaging their students in the solving of word problems—both Valerie and Jackie suggested that CGI problem types would not be enough to sustain interest in their mathematics classrooms.

Finally, Jackie became less tied to Interactions (e.g., Hope & Small, 1994), the highly-structured curriculum that the first- and second-grade teachers in the school had been using. Informal conversations with her have indicated that her understanding of teacher professional development is not tied to a pre-set curriculum or guide; changing teaching in mathematics involves a radical shift in knowledge, thinking, and educational philosophy.

It should not be surprising that a situated approach had a substantial impact on the teachers. Sociocultural perspectives on learning have proposed that growth and development are dependent on the contexts in which they are embedded (Brown, Collins, & Duguid, 1989; Lave, 1991; Lave & Wenger, 1991; Rogoff, 1990), and there is little reason to think that teacher growth is different. The teachers in our study learned about children’s thinking in contexts that were personally valuable and that offered appropriate tools (expert guidance, videoclips, opportunities for discussion and reflection) as they were needed. The issue of scalability becomes even trickier under these circumstances, however, particularly for professional development that is deliberately non-prescriptive and based inherently on enhancing teachers’ knowledge about such a complex phenomenon as children’s thinking.

In sum, there is no reason to believe that our professional development efforts would not be effective for teachers in border communities as well, for they too interact with at-risk children who demonstrate a variety of mathematical strategies and would benefit from challenging, conceptually-based mathematics instruction (Secada et al., 1998). All teachers, but particularly those who serve at-risk students, need support in creating meaningful relationships with children as they engage in interesting and challenging mathematical content, regardless of the community in which they reside. Learning about students and their thinking provides a pedagogically sound way to foster educational equity and to help students know that they matter.
References

*Phi Delta Kappan*, 77 (7), 500-509.


