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Shear deformation of a non-linear solid undergoing deterioration of material properties

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Abstract

We study the degradation/enhancement of the response of a solid, through which a fluid is diffusing, whose material moduli depend on the concentration of the diffusant. A simplified model is used that couples the balance of linear momentum for the solid to a convection-reaction-diffusion equation for the fluid. At a fixed level of concentration, the solid responds as a Mooney-Rivlin material, the response being that of different Mooney-Rivlin material at different levels of concentration. The problem under consideration has relevance to many technological problems such as degradation in polymers due to diffusion of a fluid.

1. Introduction

There have been several models that have been proposed to describe the damage, or in other words deficient response of the body due to the deteriorating of material properties, in solids. Most such models are ad hoc, and others suffer from a surfeit of parameters (internal variable) to describe the model. A simple model that can possibly capture the salient features of the response in the case of deterioration of the properties of body due to the diffusion of a fluid could be the coupling of the balance of linear momentum of the solid with a convection-reaction –diffusion equation for the fluid, that assumes that the material parameters depend on the concentration of the different. Such a model would have relevance to the diffusion of moisture through solids, metals, that leads to corrosion or embrittlement due to the diffusion of hydrogen, the diffusion of moisture through concrete, etc.

Several studies concerning elastic solids wherein the material moduli are assumed to depend on the concentration have been carried out[1]. However, in most of these studies the variation of the concentration in the body is specified (assumed) a'priori, a convection-reaction diffusion equation is not solved to determine the concentration. Bouadi (Bouadi and Sun, 1989), (Bouadi and Sun, 1990) introduces a hygrothermal expansion coefficient in order to describe the effect of moisture. A very general model has been introduced by Weitsman (Weitsman , 1987) , (Weitsman, 1987) to describe the diffusion of fluid through polymer composites. However, as the material moduli depend on as many as thirty-two invariants, the model needs to be greatly simplified before it can be used to study specific initial-boundary value problems.

The response of viscoelastic bodies whose properties change with the concentration of a diffusant have also been studied. Rajagopal and Wineman (Rajagopal and Wineman)consider the possibility of the "internal clock" in linear viscoelastic bodies to change with the concentration of the diffusant and discuss its relevance to the response of biological materials. Mixture theory has been used to study the swelling of polymeric solids (see Shi, Rajagopal and Wineman (Shi, Rajagopal and Wineman, 1981)) but such an approach has its own inherent difficulties, that of prescribing boundary conditions (see Rajagopal, Wineman and Gandhi (Rajagopal, Wineman and Gandhi, 1987)).

Rajagopal, Srinivasa and Wineman (Rajagopal, Wineman and Srinivasa, 2007) studied simple shear and bending of a degrading beam of polymeric solid within a thermodynamic framework, the degradation being a consequence of the scission of a the polymeric network. This approach was later used by Soares, Moore and Rajagopal (Soares, Moore and Rajagopal) to analyse the deterioration in the material response of a solid that responds as an elastic solid in the absence of degradation.

[1] The bodies respond as elastic materials when the concentration of the diffusant is held constant. However, the material properties change with the concentration of the diffusant.

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All the above studies pertain to the degradation of the response characteristics of a solid. Similar degradation is possible in fluids. One could also study the degradation of fluids due to the injection of another fluid or a gas. Bridges and Rajagopal (Bridges and Rajagopal, 2006) have studied the response of a fluid whose properties change due to another fluid that is diffusing through it.

In this paper, we shall follow the procedure outlined in Muliana et al (Muliana, Shankar and Rajagopal) and Darbha and Rajagopal (Darbha and Rajagopal), and couple the balance of linear momentum with a convection- reaction-diffusion equation. We study shearing motions of a slab subject to a pulsating pressure gradient across which a fluid is diffusing. The slab responds like a Mooney-Rivlin solid when no degradation takes place. We study several simple problems to illustrate the consequences of degradation. We first study the problem of a fluid diffusing through a sheared slab. As a consequence of the diffusion the value of the material moduli those characterize the solid change. In this special problem, the equations of equilibrium for the solid and the convection-diffusion equation are uncoupled allowing one to solve the convection-diffusion equation for the concentration and then use this result in solving the equations of equilibrium for the solid. Next, we consider the diffusion of a fluid through a slab that is subject to a suddenly applied pressure gradient.

2. Degradation of the response of a sheared slab due to the diffusion of a fluid

We shall consider a body, which when the concentration does not change, behaves as a Mooney-Rivlin solid. As the concentration changes, the shear modulus of the solid changes, the Cauchy stress in such a body is given by

$$\mathbf{T} = -p\mathbf{1} + \mu_1(c)\mathbf{B} - \mu_2(c)\mathbf{B}^{-1} \tag{1}$$

where $-p\mathbf{1}$ is the reaction stress due to the constraint of incompressibility, $\mu_1(c)$ and $\mu_2(c)$ denote the material parameters that depend on the concentration c, and $\mathbf{B}$ is the left Cauchy-Green stretch tensor defined through

$$\mathbf{B} = \mathbf{F}\mathbf{F}^T, \tag{2}$$

where $\mathbf{F}$ denotes the deformation gradient.

The material moduli μ_1 and μ_2 are defined through

$$\mu_1 = \mu\left(\frac{1}{2} + \beta\right), \quad \mu_2 = \mu\left(\frac{1}{2} - \beta\right), \tag{3}$$

where $\mu > 0$ and $-1/2 \le \beta \le 1/2$. When $\beta = \frac{1}{2}$, the above model reduces to the Neo-Hookean solid model.

We shall assume that the material moduli vary with the concentration in the following manner:

$$\mu_i(c) = \bar{\mu}_i - \hat{\mu}_i c, \quad i = 1, 2 \tag{4}$$

where $\bar{\mu}_i$ and $\hat{\mu}_i, \quad i = 1, 2$ are positive constants. The above assumption assumes that the material properties reduce with concentration. It is possible that bodies can be strengthened due to the infusion of a fluid, an example is the injection of medicine, and in this even the negative sign on the right hand side needs to be replaced by a positive sign.

We are interested in the unsteady shearing motions of a body B, where

$$B := \{(X, Y, Z) | -\infty \le X \le \infty, 0 \le Y \le H, -\infty \le Z \le \infty\}. \tag{5}$$

We are interested in two problems. The first wherein the boundaries $Y = 0$ and $Y = H$ are both held fixed and a suddenly applied pressure[2] gradient is applied along the Y *direction*, of the form

$$\frac{\partial p}{\partial X} = H(t), \tag{6}$$

where H(t) is the Heaviside function. The second corresponds to the bottom layer being held fixed while the top layer is sheared, the pressure being assumed to be constant in the slab.

[2] It is not correct to refer to p that appears in Eq. (1) as the pressure, if by pressure one means the mean normal stress. Note that $tr\mathbf{B}$ and $tr\mathbf{B}^{-1}$ are not necessarily zero and thus p is not the mean normal stress.

For both these problems under consideration we shall seek a shearing motion $(X, Y, Z) \rightarrow (x, y, z)$ of the form

$$
\begin{aligned}
x &= X + u(Y,t), \\
y &= Y. \\
z &= Z.
\end{aligned} \tag{7}_{1\text{-}3}
$$

For the above motion, a straight forward calculation leads to

$$
(\mathbf{F}) = \begin{pmatrix} 1 & \frac{\partial u}{\partial y} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}
$$

$$
(\mathbf{B}) = \begin{pmatrix} 1+\left(\frac{\partial u}{\partial y}\right)^2 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}
$$

$$
(\mathbf{B}^{-1}) = \begin{pmatrix} 1 & -\frac{\partial u}{\partial y} & 0 \\ -\frac{\partial u}{\partial y} & 1+\left(\frac{\partial u}{\partial y}\right)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{10}
$$

We shall assume that the concentration c has the form

$$
c = c(y,t). \tag{11}
$$

On entering Eq. (1) into the balance of linear momentum

$$
\rho \frac{d\mathbf{v}}{dt} = div\mathbf{T} + \rho\mathbf{b}, \tag{12}
$$

where ρ denotes the density, $\mathbf{v}$ the velocity and $\mathbf{b}$ the specific body force, and using Eq. (9), Eq. (10) and Eq. (11) we obtain

$$
\rho \frac{\partial^2 u}{\partial t^2} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left([\mu_1(c) + \mu_2(c)] \frac{\partial u}{\partial y} \right), \tag{13}
$$

$$
0 = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left(\mu_1(c) + \mu_2(c)\left(1+\frac{\partial u}{\partial y}\right)^2 \right), \tag{14}
$$

$$
0 = -\frac{\partial p}{\partial z}. \tag{15}
$$

On defining

$$p^* = p - \mu_1 + \mu_2(c)\left(1 + \left(\frac{\partial u}{\partial y}\right)^2\right), \tag{16}$$

we obtain

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{-\partial p^*}{\partial x} + \frac{\partial}{\partial y}\left([\mu_1(c) + \mu_2(c)]\frac{\partial u}{\partial y}\right), \tag{17}$$

$$0 = \frac{-\partial p^*}{\partial y}, \tag{18}$$

$$0 = -\frac{\partial p^*}{\partial z}. \tag{19}$$

It follows from Eq. (5), (18) and (19) that

$$\rho\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial y}\left([\mu_1(c) + \mu_2(c)]\frac{\partial u}{\partial y}\right) = H(t). \tag{20}$$

The concentration c is governed by the reaction diffusion equation

$$\frac{\partial c}{\partial t} + div(\rho c) = div\left(D\frac{\partial c}{\partial \mathbf{x}}\right), \tag{21}$$

where D is the diffusivity. In general the can depend on the deformation but we shall assume that it is a constant. In virtue of Eq. (7) and Eq. (11), we obtain

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2}. \tag{22}$$

3. Solution

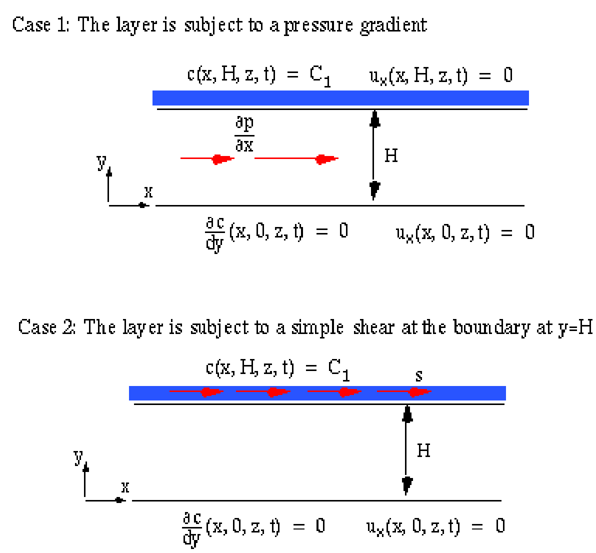

Figure 1: Prescribed loading and boundary conditions

We present solutions to the governing equations for the diffusion of fluid through a sheared slab (Figure1). The material moduli of the slab change due to the infusion of the fluid into the slab. We shall assume that the values of the material moduli decrease implying a decrease in the load carrying capacity of the slab, i. e., the slab degrades. It is assumed that the diffusion process and rate of mechanical loading are relatively slow so that the effect of inertia of the slab can be ignored.

The governing equations are summarized as follows (on dropping the asterisk for convenience):

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2} \tag{23}$$

$$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu_1(c(y,t))\frac{\partial u}{\partial y} + \mu_2(c(y,t))\frac{\partial u}{\partial y}\right] = 0.0 \tag{24}$$

$$-\frac{\partial p}{\partial y} = -\frac{\partial p}{\partial z} = 0.0 \tag{25}$$

We shall suppose that the concentration of the fluid satisfies the following initial and boundary conditions:

$$\begin{aligned} &c(y,0) = C_0 && \forall 0 \le y \le H \\ &c(H,t) = C_1 && \forall t \ge 0.0 \\ &\frac{\partial c}{\partial y}(0,t) = 0.0 && \forall t \ge 0.0 \end{aligned} \tag{26}$$

We seek a separation of variables solution in time and space for the concentration of fluid of the form

$$\begin{aligned} c(y,t) &= \sum_{m=1}^{\infty} A_m e^{-D\lambda_m t}\cos\lambda_m y + C_1 \\ A_m &= \frac{4(C_1 - C_0)}{(2m-1)\pi}(-1)^m \\ \lambda_m &= (2m-1)\frac{\pi}{2H} \end{aligned} \tag{27}$$

We study two cases of shear deformation. In case 1, we consider the diffusion of fluid through a slab subject to a pulsating pressure gradient (Figure1):

$$\frac{\partial p}{\partial x} = -\left[p + q\sin\left(\frac{2\pi t}{T}\right)\right]H(t) \tag{28}$$

The layer is fixed at y=0 and y=H. The displacement and displacement gradient at initial time are assumed zero. The following boundary and initial conditions for the displacement are prescribed to the slab in case 1:

$$\begin{aligned} &u(0,t) = u(H,t) = 0.0 && \forall t \ge 0.0 \\ &u(y,0) = \frac{\partial u}{\partial y}(y,0) = 0.0 && \forall 0 \le y \le H \end{aligned} \tag{29}$$

The governing equation for the deformation Eq. (24) is now given as:

$$\frac{\partial}{\partial y}\left[(\mu_1(c(y,t)) + \mu_2(c(y,t)))\frac{\partial u}{\partial y}\right] = -\left[p + q\sin\left(\frac{2\pi t}{T}\right)\right]H(t) \tag{30}$$

with $\mu = \mu_1 + \mu_2$, the above equation is now written as:

$$\frac{d\mu}{dc}\frac{\partial c}{\partial y}(y,t)\frac{\partial u}{\partial y} + \mu(c(y,t))\frac{\partial^2 u}{\partial y^2} = -\left[p + q\sin\left(\frac{2\pi t}{T}\right)\right]H(t) \tag{31}$$

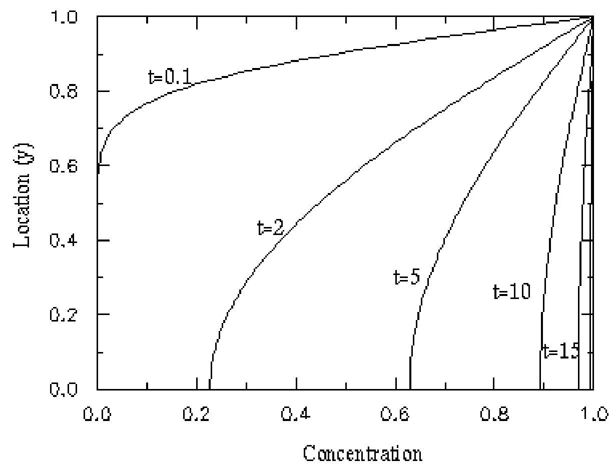

Figure 2: Profiles of the concentration of fluid c(y,t) when the diffusivity D=0.1

We solve the above governing equation numerically by spatially discretizing the interval [0, H] into k equal parts; the size of each part is $\Delta y = \frac{H}{k}$. Let u_n denote $u\ (y_n)$ and $y_n = y_0 + n\Delta y$ where y_0=0.0 and n=1, 2, … k. The derivatives of *u* w.r.t. *y* are approximated as:

$$\begin{aligned} \frac{\partial u}{\partial y}(y_n) &\approx \frac{u_{n+1} - u_{n-1}}{2\Delta y} \\ \frac{\partial^2 u}{\partial y^2}(y_n) &\approx \frac{u_{n+1} - u_n + u_{n-1}}{\Delta y^2} \end{aligned} \tag{32}$$

In this way, we transform the differential equation into a difference equation:

$$\begin{aligned} &u_0 = 0.0 \\ &\alpha_1 u_{n-1} - \alpha_2 u_n + \alpha_3 u_{n+1} = f(t) \\ &u_k = 0.0 \end{aligned} \tag{33}$$

where $\alpha_1 = \frac{\mu(c(y_n,t))}{\Delta y^2} - \frac{1}{2\Delta y}\frac{d\mu}{dc}\frac{\partial c}{\partial y}(y_n,t)$, $\alpha_2 = \frac{2\mu(c(y_n,t))}{\Delta y^2}$, $\alpha_3 = \frac{\mu(c(y_n,t))}{\Delta y^2} + \frac{1}{2\Delta y}\frac{d\mu}{dc}\frac{\partial c}{\partial y}(y_n,t)$, and *f(t)* is the right hand side of Eq. (31). We seek solutions for u_n at every instant of time.

We examine the effects of the diffusion process and fluid concentration dependent moduli on the overall deformation of the slab. Let us consider a slab with H=1 and D=0.1 which results in a relatively fast diffusion process. The steady state is reached after about 25 units of time. Figure 2 illustrates the concentration of fluid, at many time instants, in the slab. Let us now consider the material moduli to be mildly dependent on the fluid concentration in that the slope with which it decreases is less than unity. This can be reflected by the following choices:

$$\begin{aligned} \mu_1 &= 1.5 - 0.75c(y,t) \\ \mu_2 &= 1. - 0.5c(y,t) \end{aligned} \tag{34}$$

Figure 3 illustrates a pulsating pressure gradient that corresponds to f(t)

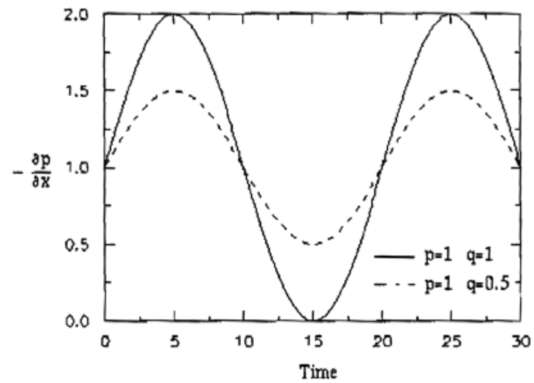

Figure 3: Pulsating pressure gradient

Figure 4 shows the displacement and shear stress in the slab during the transient diffusion process when the parameters p and q are equal to 1 and the period T is equal to 20 (Eq. (28)). The relatively fast diffusion process and mild concentration dependent material moduli result in nearly symmetrical displacement and anti-symmetrical shear stress except at early instants of time. This is in virtue of the diffusion being sufficiently fast the material degradation is complete in a very short time and symmetric displacement field that corresponds to the steady state solution, results very quickly. The time-dependent results of the concentration of fluid, displacement, and shear stress at three different locations in the slab (y=0.25, 0.5, and 0.75) are illustrated in Figure 5.

Next, we consider a slow diffusion process and we assume that the material moduli of the slab are more strongly dependent on the concentration of fluid than previously considered in that the material moduli decrease faster with the concentration. For this purpose, we take the diffusivity of the slab D=0.01 and the material moduli are given by:

$$\mu_1 = 1.5 - 1.35c(y,t)$$
$$\mu_2 = 1. - 0.85c(y,t) \tag{35}$$

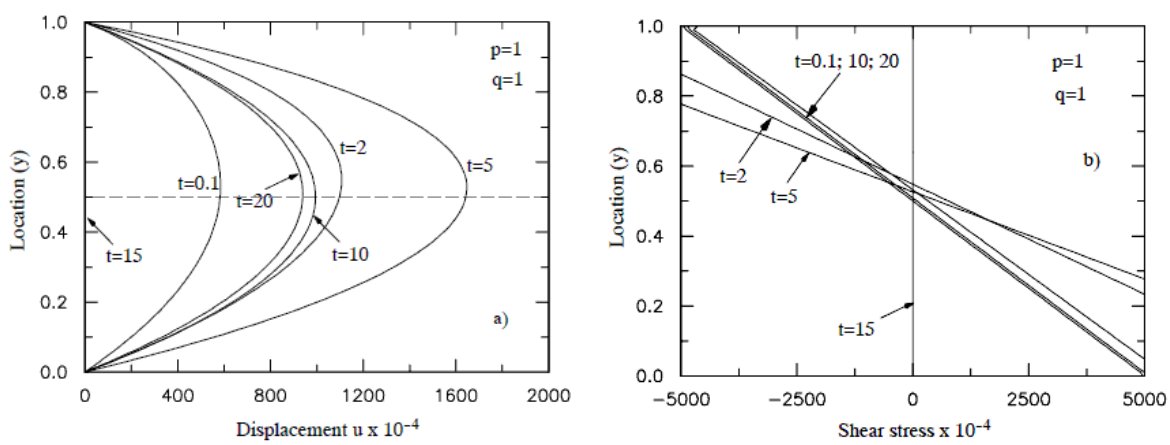

Figure 4: Responses due to a pulsating pressure gradient during transient analysis (D=0.1; $\mu_1 = 1.5 - 0.75c(y,t)$ and $\mu_2 = 1. - 0.5c(y,t)$): (a) displacement (b) shear stress

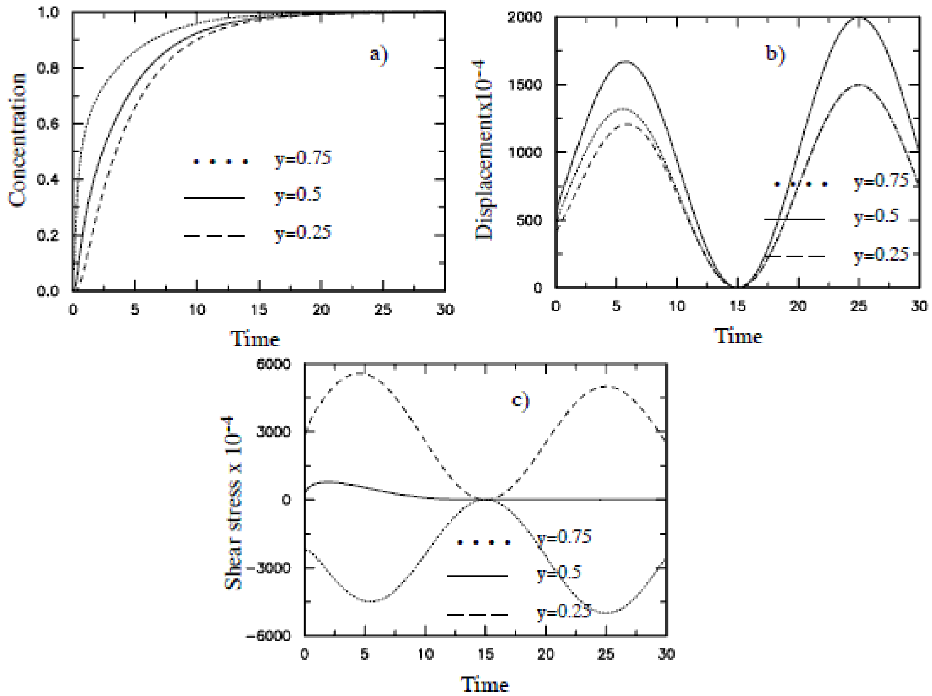

Figure 5: Time-dependent responses due to a pulsating pressure gradient during transient analysis at three locations (D=0.1; $\mu_1 = 1.5 - 0.75\,c(y,t)$ and $\mu_2 = 1. - 0.5c(y,t)$): (a) concentration, (b) displacement, (c) shear stress

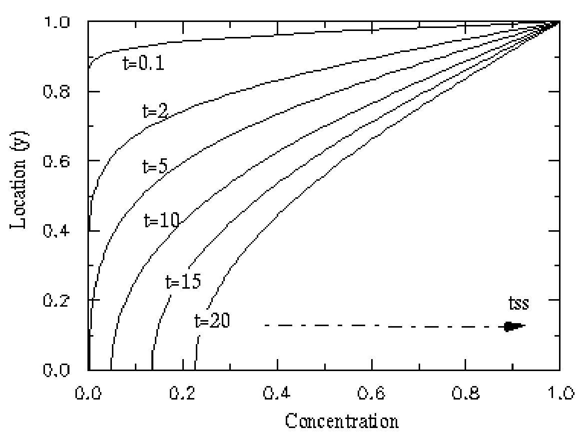

Figure 6: Profiles of the concentration of fluid c(y,t), D=0.01

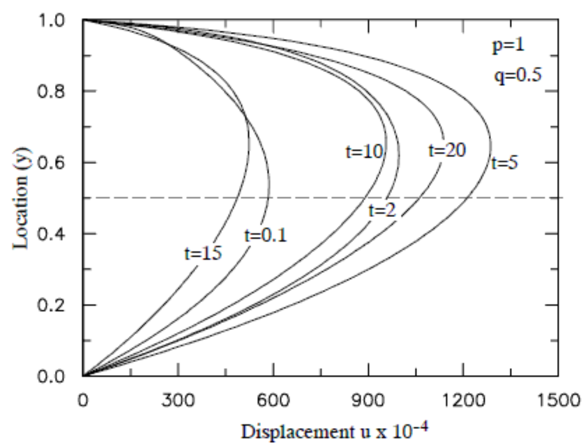

Figure 7: Displacement due to a pulsating pressure gradient (D=0.01; $\mu_1 = 1.5 - 1.35\,c(y,t)$ and $\mu_2 = 1. - 0.85c(y,t)$)

Figures 6 and 7 portray the profiles of the concentration of fluid and displacement of the slab, at each instant of time. It is seen that the displacement profiles at the reported times are unsymmetrical. This is a consequence of diffusion being slow whereby the concentration at the different locations are far from the steady state value with a clear variation in the concentration as a function of the y-co-ordinate. This leads to the asymmetry of the displacement field. The parameters p and q are equal to 1 and 0.5, respectively and the period T is equal to 20 (Eq. (28)).

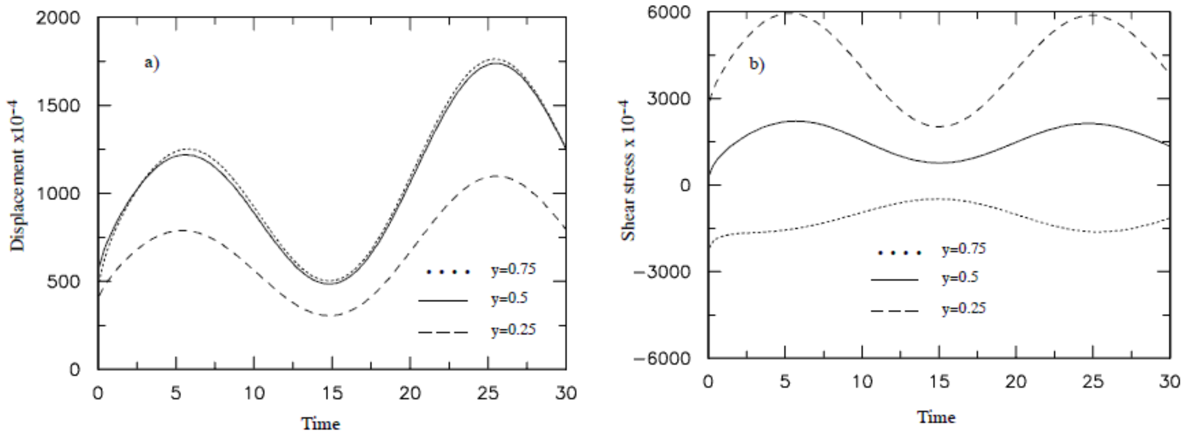

Figure 8: Response due to a pulsating pressure gradient during transient analysis at three locations (D=0.01; $\mu_1 = 1.5 - 1.35c(y,t)$ and $\mu_2 = 1. - 0.85c(y,t)$): (a) displacement and (b) shear stress

The time-dependent response for the displacement and shear stress at three different locations in the slab (y=0.25, 0.5, and 0.75) are illustrated in Figure 8. It is seen that the magnitude of the displacement at y=0.5 and y=0.75 are nearly identical and the maximum displacement is located at a point between y=0.5 and y=0.75. As time progresses to that when the steady state condition is reached, we find that the displacement profiles are nearly symmetrical. In Figure 9 we present the displacement corresponding to a pulsating pressure gradient in the absence of fluid concentration and when the entire slab is subject to a fixed amount of concentration of fluid (one may assume this as the steady state condition). In this case, the governing equation reduces to:

$$\mu(c)\frac{\partial^2 u}{\partial y^2} = -\left[p + q\sin\left(\frac{2\pi t}{T}\right)\right]H(t) \tag{36}$$

It can be seen that when the diffusion process is relatively fast w.r.t. the rate of pulsating pressure gradient and the material moduli are mildly dependent on the concentration of fluid, solving a steady-state condition of the diffusion of fluid and the deformation in the slab yields reasonably good approximate solutions. However, in the other extreme situation when we have a slow diffusion process and the material moduli strongly depend on the concentration of fluid, it is necessary to solve the transient diffusion of the fluid and the deformation in the slab simultaneously as transient effects are significant and take a long time to die down.

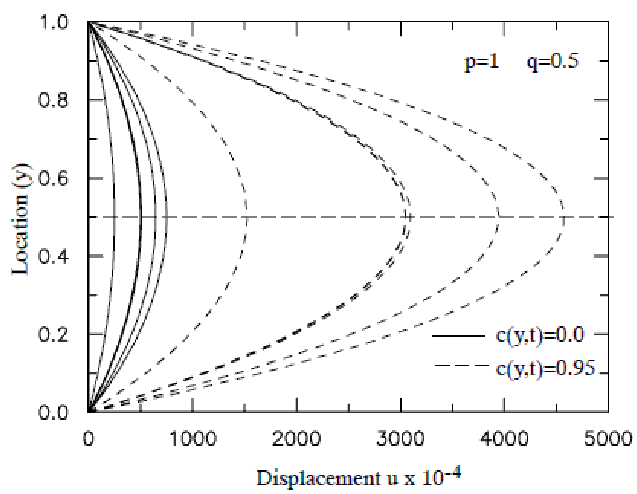

Figure 9: Displacement due to a pulsating pressure gradient under uniform fluid concentration.

Next, we subject the slab to a constant surface traction at y=H and a pulsating pressure gradient. The bottom surface of the slab is fixed. The prescribed boundary conditions are expressed as:

$$
\begin{aligned}
& u(0,t) = 0.0 \qquad \forall t \geq 0.0 \\
& T_{xy}(H,t) = p \\
& u(y,0) = \frac{\partial u}{\partial y}(y,0) = 0.0 \qquad \forall 0 \leq y \leq H
\end{aligned}
\tag{37}
$$

Figure 10 (a) and (b) illustrate displacement and shear stress during the transient diffusion of fluid. It is seen that the maximum displacement occurs at the top surface and its value increases with increasing time as the material moduli degrade with the concentration of fluid. The maximum shear stress is at the bottom surface. We also calculate the Frobenius norm of the Green-St. Venant strain (Figure 10c) given by:

$$
\begin{aligned}
& \overline{E} = \left(\left|E_{ij}\right|\left|E_{ij}\right|\right)^{1/2} \\
& E_{ij} = \frac{1}{2}\left[F^T F - I\right]
\end{aligned}
\tag{38}
$$

In the second case, we examine the response of a sheared slab due to a diffusion of fluid through the slab (Figure 1). The following boundary and initial conditions for the deformation are prescribed to the slab:

$$
\begin{aligned}
& u(0,t) = 0.0 \qquad \forall t \geq 0.0 \\
& T_{xy}(H,t) = \left(\mu_1(c(H,t)) + \mu_2(c(H,t))\right)\frac{\partial u}{\partial y}(H,t) = s \qquad \forall t \geq 0.0
\end{aligned}
\tag{39}
$$

The difference equation in Eq. (33) is now written as

$$
\begin{aligned}
& u_0 = 0.0 \\
& \alpha_1 u_{n-1} - \alpha_2 u_n + \alpha_3 u_{n+1} = 0.0 \\
& -u_{k-1} + u_k = s\Delta y
\end{aligned}
\tag{40}
$$

Figure 11 illustrates the displacement due to a simple shear (s=1) and the diffusion of the fluid that causes degradation of the slab, under the assumption that the diffusion process is fast, and that the material moduli are dependent on the concentration of fluid as given by Eq. (34). Figure 12 presents the displacement when the diffusion process is slow and the material moduli strongly depend on the fluid concentration as given by Eq. (35). Once again, we notice that in the second problem it is necessary to solve the transient diffusion of fluid and the deformation in the slab simultaneously in that steady state conditions are attained slowly.

The effect of degradation of the slab due to the infusion of the fluid is brought out best in Tables 1 and 2 for the two problems considered in the paper. We notice that in both the cases, the steady state solution, corresponding to the concentration being unity, for the average and maximum values of the strain and the displacement are much higher than when there is no fluid infused in the solid. We notice from Table 1 that the maximum value of the Frobenius norm for the strain could be as high as 1400 %, while the displacement could be as high as 800%, as in the non-degraded slab.

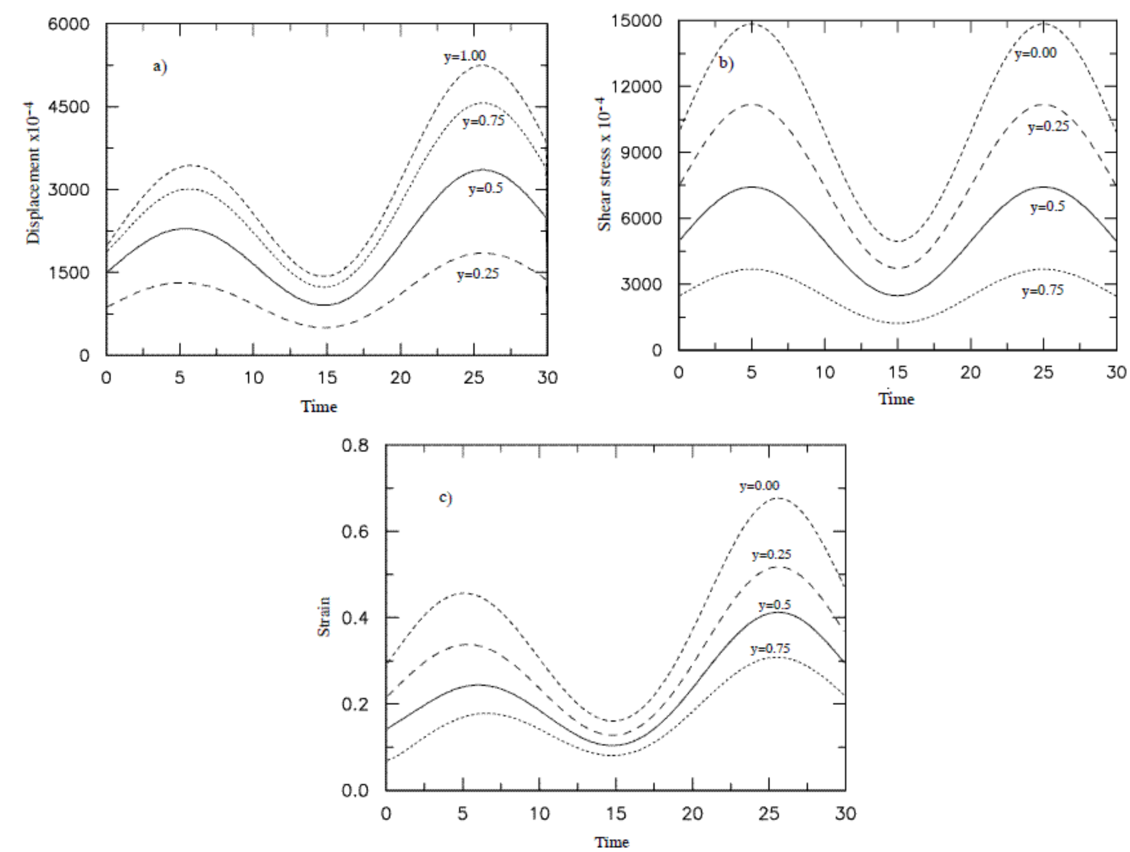

Figure 10: Response due to constant surface traction and pulsating pressure gradient under transient conditions: a) displacement, b) shear stress, c) norm of the strain

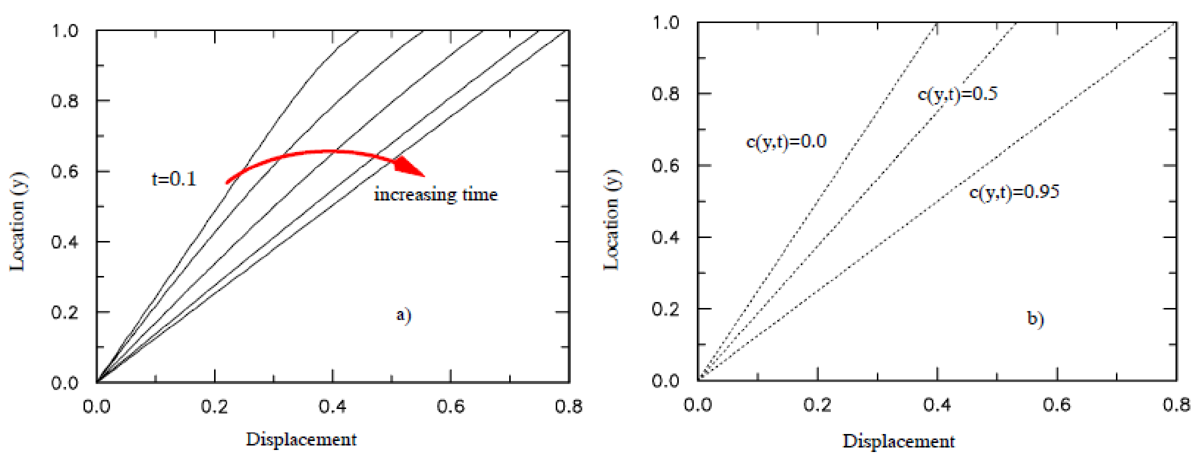

Figure 11: Displacement due to simple shear (D=0.1; $\mu_1 = 1.5 - 0.75c(y,t)$ and $\mu_2 = 1. - 0.5c(y,t)$): (a) transient diffusion analysis and (b) steady state diffusion analysis

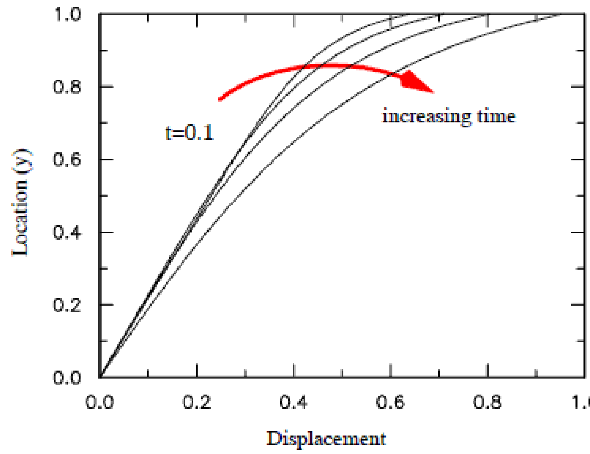

Figure 12: Displacement due to simple shear (D=0.01; $\mu_1 = 1.5 - 1.35c(y,t)$ and $\mu_2 = 1. - 0.85c(y,t)$)

Problem 1

$$u(0,t) = u(H,t) = 0.0 \qquad \forall t \geq 0.0$$

$$u(y,0) = \frac{\partial u}{\partial y}(y,0) = 0.0 \qquad \forall 0 \leq y \leq H$$

Table 1. Maximum and average field variables when $\frac{\partial p}{\partial x} = -1$

Concentration	$\bar{t}_{\max}$	$\bar{t}_{aver}$	$\bar{E}_{\max}$	$\bar{E}_{aver}$	$\bar{u}_{\max}$	$\bar{u}_{aver}$
0	0.495	0.255	0.143	0.072	0.05	0.033
1 (SS)	0.495	0.255	1.793	0.779	0.42	0.28

Problem 2

$$u(0,t) = 0.0 \qquad \forall t \geq 0.0$$

$$u(H,t) = s \qquad \forall t \geq 0.0$$

Table 2. Average field variables due to a simple shear (s=1)

Concentration	$\bar{t}_{aver}$	$\bar{E}_{aver}$	$\bar{u}_{aver}$
0	2.525	0.875	0.505
1 (SS)	0.303	0.875	0.505

For both problems, the material moduli degrade with concentration as shown below:

$$\mu_1 = 1.5 - 1.35c(y,t)$$

$$\mu_2 = 1. - 0.85c(y,t)$$

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