

INTROCUTION

In this poster, a new third-order Lagrangian

asymptotic solution describing nonlinear water wave propagation on the surface of a uniform sloping bottom is presented. The model is formulated in the Lagrangian variables and we use a two parameter perturbation method to develop a new mathematical derivation. The analytical solution in Lagrangian form satisfies state of the normal pressure at the free surface. The condition of the conservation of mass flux is examined in detail for the first time. The two important properties in Lagrangian coordinates, Lagrangian wave frequency and Lagrangian mean level, are included in the third-order solution. The solution can also be used to estimate the mean return current for waves progressing over the sloping bottom. The Lagrangian solution untangle the description of the features of wave shoaling in the direction of wave propagation from deep to shallow water, as well as the process of successive deformation of a wave profile and water particle trajectories leading to wave breaking. So the analytical solution could be used to determine the breaker characteristics.



NAVE BREAKING

Because of the change of water depth, the wave shoals and is refracted in the propagation process from deep to shallow water. The celerity is reduced; hence, the particle velocity of the wave crest is faster than the celerity and the wave breaks. In order to describe the breaking wave mechanism, the determine of wave breaking is applied and the breaking criterion is U/C=1 where U is celerity and C is the horizontal velocity of particle at the wave crest. According to the determine of wave breaking two function could be obtained as follow :

FORMULATION OF THE PROBLEM

The fluid motion in the Lagrangian representation is described by tracing an individual fluid particle. For two-dimensional flow, a fluid particle is distinguished by the horizontal and vertical parameters (x_0, y_0) known as Lagrangian labels. Then fluid motion is described by a set of trajectories



$$u = x_{t} = c = f_{1,0t} + f_{1,1t} + f_{2,0t} + f_{2,0t}'' = 1$$
(1)
$$\frac{dy}{dS} = g_{1,0S} + g_{1,1S} + g_{2,0S} = 0$$
(2)

BREAKING CRITERIA

The limit wave steepness for breaking waves on a uniform water depth was first present in Miche(1951) based on the theoretical analysis. Goda(2004) expressed the breaking criterion graphically according to many experimental works and presented an approximate expression

for the curves. This empirical formula has been applied practically and widely in design of most maritime structures. Being united with Eq.(1) by Eq.(2) can determine the breaking point So our theoretical solution can discuss a succession of breaking indices. Compare our solution from Fig.3a-b with empirical formula of Goda(2004), we found that when the slope is 1/10, result of Goda(2004) and our theoretical results are consistent, and when slope is 1/3, the consistency of our results between theoretical solution and experimental data is better than the empirical formula of Goda(2004).



 $x(x_0, y_0, t)$ and $y(x_0, y_0, t)$, where x and y are in Cartesian coordinates. The dependent variables x and y indicate the position of any particle at time t and are function of the independent variables x_0 , y_0 and t. In a system of Lagrangian description, the governing equations and boundary conditions for two-dimensional irrotational free-surface flow are summarized as follow: $J = \frac{\partial(x, y)}{\partial(x_0, y_0)} = x_{x_0} y_{y_0} - x_{y_0} y_{x_0} = 1,$ $x_{x_0t} x_{y_0} - x_{y_0t} x_{x_0} + y_{x_0t} y_{y_0} - y_{y_0t} y_{x_0} = \frac{\partial(x_t, x)}{\partial(x_0, y_0)} + \frac{\partial(y_t, y)}{\partial(x_0, y_0)} = 0,$ $\begin{aligned} x_{x_{0}t}y_{y_{0}} - x_{y_{0}t}y_{x_{0}} + x_{x_{0}}y_{y_{0}t} - x_{y_{0}}y_{x_{0}t} &= \frac{\partial(x_{t}, y)}{\partial(x_{0}, y_{0})} + \frac{\partial(x, y_{t})}{\partial(x_{0}, y_{0})} = 0, \\ \frac{\partial\phi}{\partial x_{0}} &= x_{t}x_{x_{0}} + y_{t}y_{x_{0}}, \quad \frac{\partial\phi}{\partial y_{0}} = x_{t}x_{y_{0}} + y_{t}y_{y_{0}}, \quad \frac{P}{\rho} = -\frac{\partial\phi}{\partial t} - gy + \frac{1}{2}\left(x_{t}^{2} + y_{t}^{2}\right), \end{aligned}$

The third-order Lagrangian solutions which should be referred to Chen et al. (2006, 2009) can be solved as follow:

$$S = \int (k) dx_0 - \sigma t = \int k_{0,0} da_0 - \sigma t; \quad x(x_0, y_0, t) = x_0 + f_{1,0} + \alpha f_{1,1} + f_{2,0};$$

 $y(x_0, y_0, t) = y_0 + g_{1,0} + \alpha g_{1,1} + g_{2,0}; \ \phi = \phi_{1,0} + \alpha \phi_{1,1} + \phi_{2,0}; \ P = P_{1,0} + \alpha P_{1,1} + P_{2,0}.$

VAVE TRANSFORMATIONS

BREAKING WAVE HEIGHT

Some of empirical formulas presented the breaking wave height in terms of deepwater wave condition, such as in sunamura (1983) and in Rattanapiti-kon and shibayama(2000). Base on the results depicted in Fig.4 a-b, it showed that our theoretical results and the experimental data of breaking wave height are more consistent than other's empirical formula.



Both linear (up to the order of $\varepsilon \alpha$) and nonlinear solution (up to the order of $\varepsilon^2 \alpha$) are also implemented for comparison and the results are shown in Fig. 2. In general, the nonlinear wave profiles are higher than the linear solution for any wave steepness and bottom slope. Moreover, the breaking point predicted by the nonlinear solution occurs earlier than that by the linear solution. We remark here that in the comparisons made above, the nonlinear wave profiles prior to breaking point differ from the linear wave profiles in three ways: 1. Breaking takes place earlier in a deeper depth.

2. The wave height at the breaking point is much higher.

3. After breaking point, the tendency for the wave profile to plunge onto the sea bed is reduced.



Fig. 2 Comparison of the breaking wave height on three bottom slope.

CONCLUSIONS

The breaking wave Indices obtain from the present theory shows similar tendency to the experimental data and the theoretical curves and the experimental data are more consistent than other's empirical formula when the bottom slope steep. While the purpose of the present study is to establish a rational mathematical model, which satisfies the bottom boundary condition, its accuracy needs further improvement to include the effects of higher order wave steepness on a sloping bottom and the effect of bottom friction in shallow water.

REFERENCES

Fig. 2 Successive wave

profiles prior to breaking

plotted by linear.

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