CHAPTER 4

REFRACTION AND DIFFRACTION DIAGRAMS

James W. Dunham
Chief, Beach Erosion Unit
Los Angeles District, Corps of Engineers
Department of the Army
Los Angeles, California

SCOPE

The use of diagrams to indicate the effects of refraction and diffraction of ordinary wind waves and swell in offshore areas is by no means an innovation in coastal and harbor engineering. Refraction diagrams in particular have been used in various forms by engineers in the United States and in Europe for more than a decade. The principles and procedures for constructing refraction and diffraction diagrams have been developed by academic research and investigation. The purposes of this paper are (1) to review briefly these principles and procedures, and (2) to describe their practical application by the Corps of Engineers, Los Angeles District, Department of the Army.

REFRACTION DIAGRAMS

PRINCIPLES AND PROCEDURES OF DIAGRAM CONSTRUCTION

Applicable formulas. The basic principles underlying the construction of all types of refraction diagrams are expressed by Snell's law and by the formula for wave velocity in shallow water. Snell's law states that where the bottom contours are parallel the sine of the angle between the wave crest and the bottom contour is proportional to the velocity of wave propagation. This law generally is expressed by the formula:

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{C_1}{C_2} \tag{1}$$

where α_1, α_2 = angles between the wave crest and the bottom contour at two successive points along the orthogonal to the wave crests,

and C_1, C_2 = the corresponding velocities of wave propagation at the points where α_1 and α_2 are measured.

The formula for wave velocity is:

$$C^2 = \frac{g}{2\pi} L \tanh 2\pi \frac{d}{L}, \tag{2}$$

where d = depth,

C = wave velocity at depth, d,

L = wave length at depth, d,

g = acceleration of gravity.

Equation 2 contains three variables; d, C, and L. Because L = CT and T is a constant, equation 2 may be expressed in terms of the two variables, d, and C. The solution of the resulting equation is complex and time consuming. However, the University of California has reduced the equation to tabular form. The tables have been published by the Beach Erosion Board (Wiegel, 1948).

Currents of tributary streams. Because ocean currents and currents of tributary streams have a negligible effect on the velocity of wave propagation in the southern California area, their effect on refraction-diagram construction is not considered in detail in this paper. Where a refraction diagram is drawn across a line of current discontinuity (i.e., at the ocean mouth of a large river), the diagram must conform to applicable procedures. These procedures are described elsewhere (Johnson, 1947).

Wave-crest method. The first method of diagram construction is the wave-crest method. Successive positions of the wave crest are drawn by plotting the wave advance from point to point along the crest. The distance of wave advance is determined by the average depth and by the average velocity of propagation for the selected point on the wave crest over a given interval of advance, which is usually expressed as a multiple of the wave length. Graduated acetate scales, which show the actual forward displacement distance for any given depth, are used for this purpose. This type of scale is illustrated in available publications (Hydrographic Office, 1944; Johnson, O'Brien, and Isaacs, 1948). The scales shown were prepared for the dimensionless unit $\frac{d}{L_o}$. The value of $\frac{d}{L_o}$ must be determined separately for each plotted point because the value is based on the map scale and on the wave length in deep water. In the practical application of this procedure, the work may be greatly simplified by preparing a graduated acetate scale for each map scale used and for each wave period diagrammed. On each of these acetate scales the intervals of wave advance are given in some multiple of the wave length and are expressed in the same unit (i.e., feet, fathoms, or meters) as that of the bottom contours. After the full series of wave-crest positions from deep water to shore are plotted, orthogonals to these lines are drawn at any desired interval to determine wave convergence or divergence.

Crestless method. The second method of refraction-diagram construction omits the plotting of the successive wave-crest positions. Each orthogonal is plotted directly by determining its shoreward deflection as it crosses successive bottom contours. The determination of this deflection involves a relationship derived from Snell's law and expressed by the formula

$$\Delta\alpha = \frac{R}{J}\frac{\Delta L}{L_{av}} \sin\alpha$$

where $\Delta\alpha$ = the angular change in direction of the orthogonal,

R = the segment of the orthogonal over which the angular change occurs,

ΔL = the change in wave length as the wave crosses the R segment,

J = the distance between the bottom contours passing through each end of the R segment,

L_{av} = the average wave length as the wave crosses the R segment,

and α = the average angle between the wave crest and the bottom contours along the R segment.

This formula contains an approximation that renders its solution invalid for values of α greater than about 13°. By use of the relationship $\frac{R}{J} = \sec\alpha$ the above formula may be converted into

$$\Delta\alpha = \frac{\Delta L}{L_{av}} \tan\alpha$$

This converted formula is simpler than the original formula when α is less than about 80°. However, when α is more than about 80°, $\tan\alpha$ begins to approach infinity, and certain approximations are no longer valid. The first formula must then be used.

Before using the crestless method of diagram construction, a table showing the values of $\frac{\Delta L}{L_{av}}$ at various depths down to about half the deep-water wave length must be prepared. Two types of plotting aids especially designed for crestless orthogonal projection are shown in the publication of Johnson, O'Brien, and Isaacs (1948), which also gives explicit instructions for their use. One is essentially a protractor containing a simple nomograph for determining values of $\Delta\alpha$; the other, which is designed for use with a drafting arm, measures the angle α directly, indicates the value of $\Delta\alpha$ in the same step, and greatly expedites the plotting process.

Base hydrography. Regardless of the method used, a detailed, fairly large-scale map of the bottom topography is essential. Because some published charts of the Hydrographic Office are small in scale or lack sufficient sounding coverage for diagramming of the desired accuracy, bromide prints of the original field sheets on which the hydrographic surveys are plotted sometimes are used. These prints, which are obtained from the Hydrographic Office in Washington, D.C., contain a wealth of detail not shown on the published charts. The bromide prints, which generally show the areas contiguous to shore in fairly large scale, are especially useful in diagram construction. In the wave-crest method of refraction-diagram construction, drawings of bottom contours are not required; the average depths are determined by inspection of the soundings. In the crestless method, bottom contouring is required. The publication of Johnson, O'Brien, and Isaacs (1948) describes how the contours must be idealized, (i.e., modified to eliminate minor irregularities).

PRACTICAL APPLICATION BY THE LOS ANGELES DISTRICT

General. In the practical application of diagram construction by the Los Angeles District, the wave-crest method generally is used where (1) relatively flat banks and gentle slopes occur in the underwater terrain, and (2) the exact alinement of the bottom contours is difficult to determine. The crestless method generally is used where other conditions of bottom topography occur.

Inaccuracies may occur in both methods of diagram construction where exceptionally rough terrain occurs. Submerged reefs and scarps may cause a wave to break and re-form in several smaller waves instead of moving along sharp turns, as indicated on the diagram. These irregularities, especially such irregularities as projections of ledge rock near the surface, may also cause diffraction or flowing of wavy energy along the crest. This results in a wave pattern that does not conform to the pattern indicated by a diagram of pure refraction. Further investigation is required to determine those limits of bottom steepness beyond which the theoretical refraction pattern does not conform to the actual wave behavior.

An important limitation of the refraction principles is that they are applicable to oscillatory waves only. When the wave breaks or begins to spill, the refraction theory no longer applies. Consequently, in drawing a refraction diagram, the breaker depth for the height and period of the wave diagrammed should be determined; diagram construction should end at that limiting depth. The tide stage, which assumes considerable importance near the shore line, must be considered in diagram construction to determine the effects of waves on shore structures.

Advantages and disadvantages of wave-crest method. The advantages of the wave-crest method of diagram construction include the following: (1) the diagram can be drawn without contour lines over the base hydrography; (2) the completed diagram shows successive positions of the wave crest; and (3) the operators quickly attain skill in the application of the easily understood principles and procedures for the diagram's construction.

The disadvantages of the wave-crest method include the following: (1) a tendency to smooth out the wave-crest position on the diagram frequently results in the error of depicting a wave convergence instead of the actual crest severance with resultant crossing of the disrupted wave elements; (2) the selected interval of wave-crest advance is usually too large in areas of very shallow water and rugged bottom terrain; (3) the pattern of successive wave crests, which is the first step in the two-step process of developing orthogonals, is a rarely required refinement in diagram analysis; and (4) the practical use of the universal scale graduated in terms of the unit $\frac{d}{L_o}$ requires a separate acetate scale for each map scale used and for each wave period diagrammed.

Advantages and disadvantages of crestless method. The advantages of the crestless method of diagram construction include the following: (1) the orthogonals are developed in a single operation; (2) the points of crest severance and the areas of crossing wave trains are clearly indicated on the diagram; (3) the dimensionless protractors for diagram construction are used with equal facility for hydrography drawn to any scale and for all ranges of wave periods; (4) the limitation of $\Delta\alpha$ to a maximum of 13° provides a criterion that regulates the interval of advance and assures accurate operations; (5) the wave-crest position can be readily determined wherever required by drawing a curve perpendicular to the orthogonals; and (6) this method is the most efficient method when the purpose of the study is to determine the wave direction at a given point and a single orthogonal may suffice (i.e., the diagramming of a relatively broad area necessary for the development of a single orthogonal by the wave-crest method is not required).

The disadvantages of the crestless method include the following: (1) the depiction of successive wave-crest positions is eliminated; (2) an operator's faulty judgment in idealizing contours may result in errors in the diagram; and (3) the inexperienced operator must be closely supervised because the principles and procedures for the diagram's construction are somewhat difficult to understand. (A common error, especially where the orthogonal is nearly normal to the contours, is that of turning the orthogonal in the wrong direction.)

Diagrammatic comparison of the two methods. A diagrammatic comparison of the two methods was made by superimposing diagrams, one drawn by the wave-crest method and the other by the crestless method, over the same bottom contours (Fig. 1). The diagram in which the wave-crest method was used indicates incorrectly the development of an exceptionally strong wave convergence. The diagram in which the crestless method was used indicates correctly crest severance and a pattern of crossing wave trains.

Special techniques of diagram construction. Operators who are not familiar with the basic theories of the methods they are using may experience difficulty in constructing diagrams where irregular bottom contours occur. As previously stated, one disadvantage in using the wave-crest method is that errors may occur in the diagram if the selected interval of wave-crest advance is too large. These errors may be eliminated by transferring the diagram to larger-scale charts of the near shore area and by shortening the selected interval of wave-crest advance in that area. This type of transfer is shown in Fig. 4, where a small-scale drawing shows refraction over the broad continental shelf opposite Redondo Beach and a large-scale drawing shows refraction over the head of Redondo submarine canyon. The refraction over the head of Redondo submarine canyon causes convergence at the shore line between the Redondo Beach breakwater and Horseshoe Pier. This effect could not be shown to advantage on the small-scale drawing.

The inexperienced operator using the crestless method over irregular contours may experience difficulty in fitting the length of the R segment of the orthogonal to the contour pattern at each step. This must be done to prevent overshooting the limitations imposed by the approximations used in this method of diagram construction. For example, if the R length is too great, the orthogonal may advance into a region where the alinement of contours is considerably different from the contour alinement at the starting point. Thus, the R segment often must be shortened to such an extent that its distal end is between two of the plotted contours. As a result, a new contour line must be interpolated locally to pass through the desired point. The experienced operator who is able to judge the effects of the changing angles of incidence may be able to adjust his orthogonal alinement to the true position in a single long step instead of the several short steps that the inexperienced operator must take. To develop this technique, the inexperienced operator must first draw the orthogonals by short steps and repeat them with longer steps that are so adjusted as to obtain conformity. The ability to make shortcuts can be developed only through experience.

A technique used with considerable effectiveness in obtaining accurate results consists of requiring the operator at each step to draw a short dash at the distal end of the R segment and a small circle at the point of direction change. Constructing a diagram in this way enables the supervisor to easily check any part of the diagram.

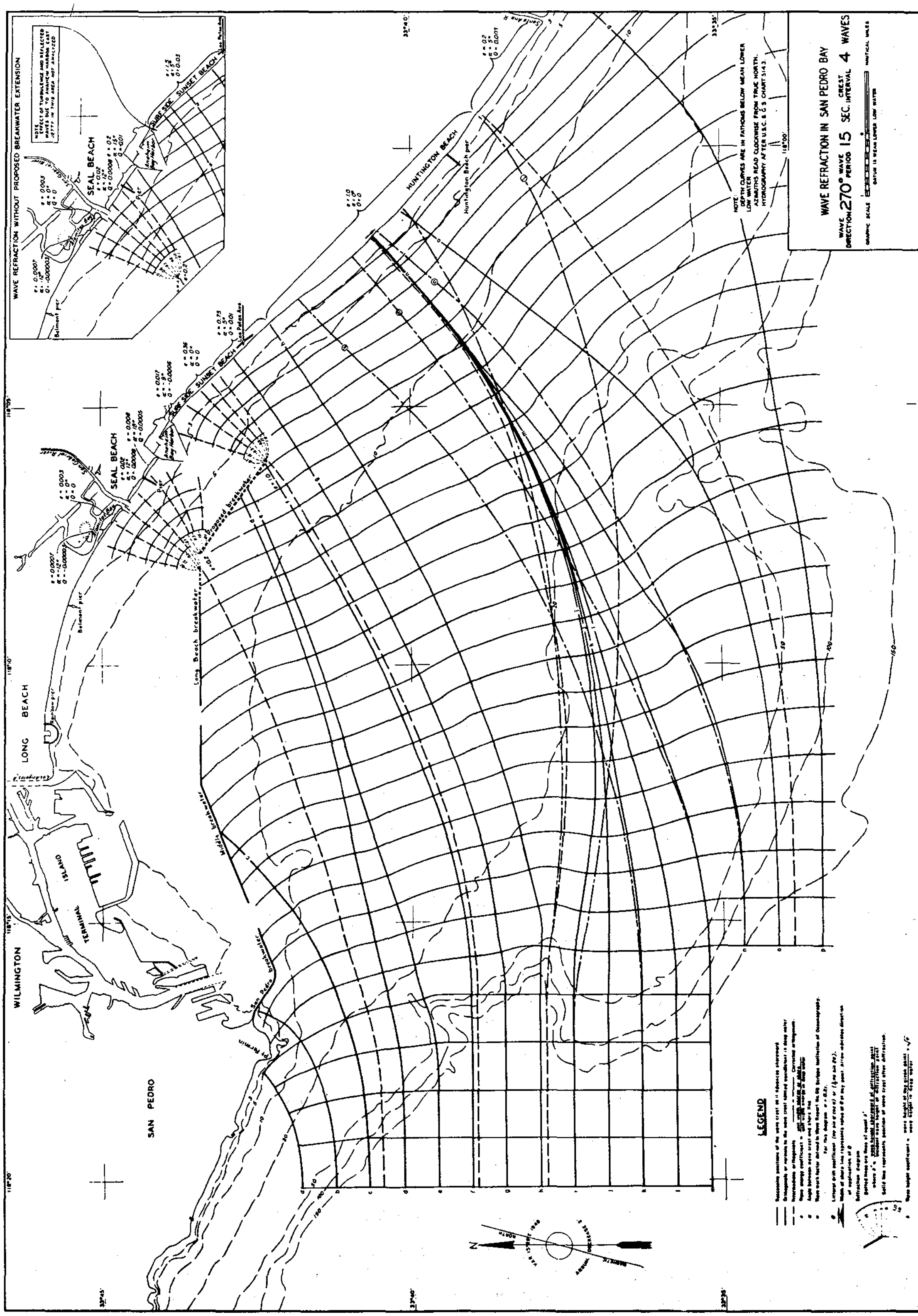

WAVE REFRACTION IN SAN PEDRO BAY
WAVE DIRECTION 270° WAVE PERIOD 15 SEC. CREST INTERVAL 4 WAVES
WAVE REFRACTION WITHOUT PROPOSED BREAKWATER EXTENSION
SEAL BEACH
SUNSET BEACH
HUNTINGTON BEACH
LONG BEACH
WILMINGTON
TERMINAL ISLAND
SAN PEDRO
Long Beach breakwater
Middle breakwater
San Pedro breakwater
LEGEND

Fig. 1

The training of a new operator should include explaining that the orthogonal itself is a curve that becomes tangent to his work lines at the beginning and end of each R segment. The work may be compared to that of the highway or railroad surveyor plotting a traverse in a series of tangents and then filling in the curves to meet BCs and ECs established through consideration of the terrain and minimum radii. The short dashes that mark the points of tangency and the small circles that mark the points of intersection point up the comparison.

Because of the approximations of the method, the greater the difference in alinement between the true orthogonal and the work line, the less accurate the work will be. Where the value of $\Delta\alpha$ is greater than 13°, the angle at which the orthogonal curve crosses the contour at the median point of the R segment may be considerably different from the angle at which the back tangent crosses that contour and the approximation that assumes these two angles to be identical is not valid. If the contours are sharply curved or irregular, the approximation limits may be violated unwittingly by crossing several differently skewed contours in a single step. The only sure method of obtaining accuracy under changing conditions is to shorten the interval of advance at each step by interpolating additional contours if necessary.

Another major difficulty encountered by the inexperienced operator is the change from the use of the simplified formula involving the tangent of α to the method involving the secant of the angle and the R/J ratio. The transition between these two methods occurs where α exceeds 80° and the orthogonal tends to follow along the contours. Here again, if the operator places a short dash across the orthogonal at each point of tangency and a small circle at each angle point, the transition is made without confusion. One should remember that when using the R/J method, the maximum curvature of the orthogonal is being indicated for the value of $\Delta L/L_{av}$ concerned. The orthogonal will soon begin to cross contours, plotted or interpolated, at an angle less than 80°; continuation of the R/J method at this point results in serious error. Where the contour spacing changes, the length of the R segment must be correspondingly changed or again the limits of approximation are violated. Operators of the Los Angeles District avoid these errors by "boxing in" the R-J segments, as shown in Fig. 2.

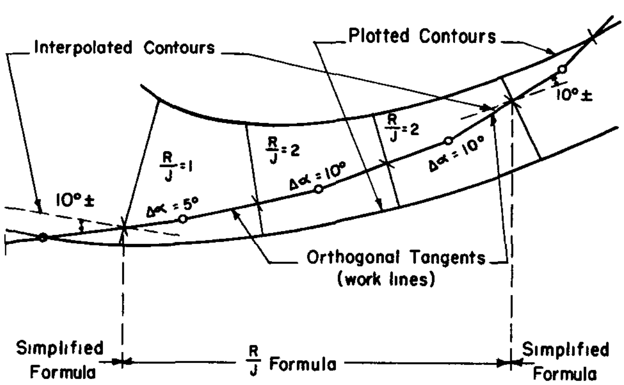


Fig. 2
Orthogonal plotted by R/J formula.

The inexperienced operator often expends time and effort unnecessarily by using too small an interval of advance although, because of the uniformity of the contours, he could use a larger interval without sacrificing accuracy. This is especially true where the orthogonals cross the contours approximately at right angles. The danger of error lies not so much in progressing with steps that are too long as in turning in the wrong direction. If the operator remembers that the orthogonal always turns toward the shallow side, he will avoid turning in the wrong direction, which is one of the most common sources of error in diagramming with the crestless method.

PRACTICAL APPLICATION OF DIAGRAM PROCEDURES TO COASTAL ENGINEERING

General. During the past decade, oceanographers and meteorologists have often collaborated in their studies to determine the relationship between storms at sea and subsequent wave action within a wide radius of the storm centers. By using the refraction-diagram procedure previously described, the effect of offshore wave conditions on nearshore areas has been fairly well determined (Munk and Traylor, 1947). Aerial photographs confirm the accuracy of the wave-forecasting and refraction-diagramming techniques. In nearly all cases, the conformity of the theoretical diagram to the actual photographed wave pattern was sufficiently close to dispel any doubts regarding the accuracy of refraction diagrams. The obvious conclusion was apparent: the refraction diagram combined with wave forecasts and hindcasts was a useful tool to the coastal engineer in many ways.

Problems of convergence. An important application of the refraction theory to coastal engineering practice is the determination of zones of wave convergence to be avoided in determining safe sites for proposed shore structures and to be considered in planning to protect existing structures. Studies of the beach-erosion problem at Redondo Beach indicated that existing structures could be protected by a proposed breakwater extension. This new structure, which would be offshore beyond the point of convergence, would intercept waves before they reached destructive heights of convergence.

The Standard Oil Company's oil-loading wharf at El Segundo, California is a structure that was built before application of the refraction theory to coastal engineering practice. As a result, it was unwittingly located in a convergence zone, and huge waves are not uncommon in the vicinity of the structure. A wave gage installed at the end of the wharf recorded waves in excess of 20 ft.; at the time the record was made, even higher waves were observed breaking beyond the wharf in water more than 35 ft. deep, although the concurrent wave action at other points along the coast was nominal. The average wave period was about 20 seconds at that time. Wave-refraction diagrams drawn for this exceptionally long-period swell disclosed a convergence caused by a broad offshore ridge lying between the Redondo and Santa Monica submarine canyons. Fig. 3 shows a diagram, drawn by the crestless method, of this convergence by 18-second waves from the west.

Utilization of wave hindcasts. The designer of a marine structure is interested in the direction of approach of the strongest waves anticipated at the site of the structure. Observations at the shore line may indicate whether the largest waves come from upcoast or down coast of the normal to shore, but give little information concerning the wave regimen offshore. The determination of predominant wave characteristics beyond the breaker zone often is important in designing structures such as piers, jetties, breakwaters, and groins. This can be done by, (1) analyzing a comprehensive series of aerial photographs made during a representative time interval, or (2) constructing and analyzing refraction diagrams of some of the more recurrent deep-water storm waves that wave hindcasts, based on historical weather maps, indicate are common in the area. The refraction-diagram method generally is the least expensive and the most efficient method.

Wave-hindcasting, which is a specialized field of science in itself, is described by R. S. Arthur in Chapter 8. However, the method of utilizing the hindcasts in refraction-diffraction analysis should be described briefly. The results of hindcast analysis generally are presented in tables and graphs that indicate, (1) the duration of waves of various significant heights and directions, and (2) the work expended by those waves over a given time interval with a breakdown by significant wave periods. The data shown in these tables are generally obtained for a number of deep-water stations offshore. If an island screen occurs between the line of offshore stations and the coast (as is the case in the southern California area), refraction studies must be made to indicate the effect of the island screen on those waves of each different period and direction that affect the study.

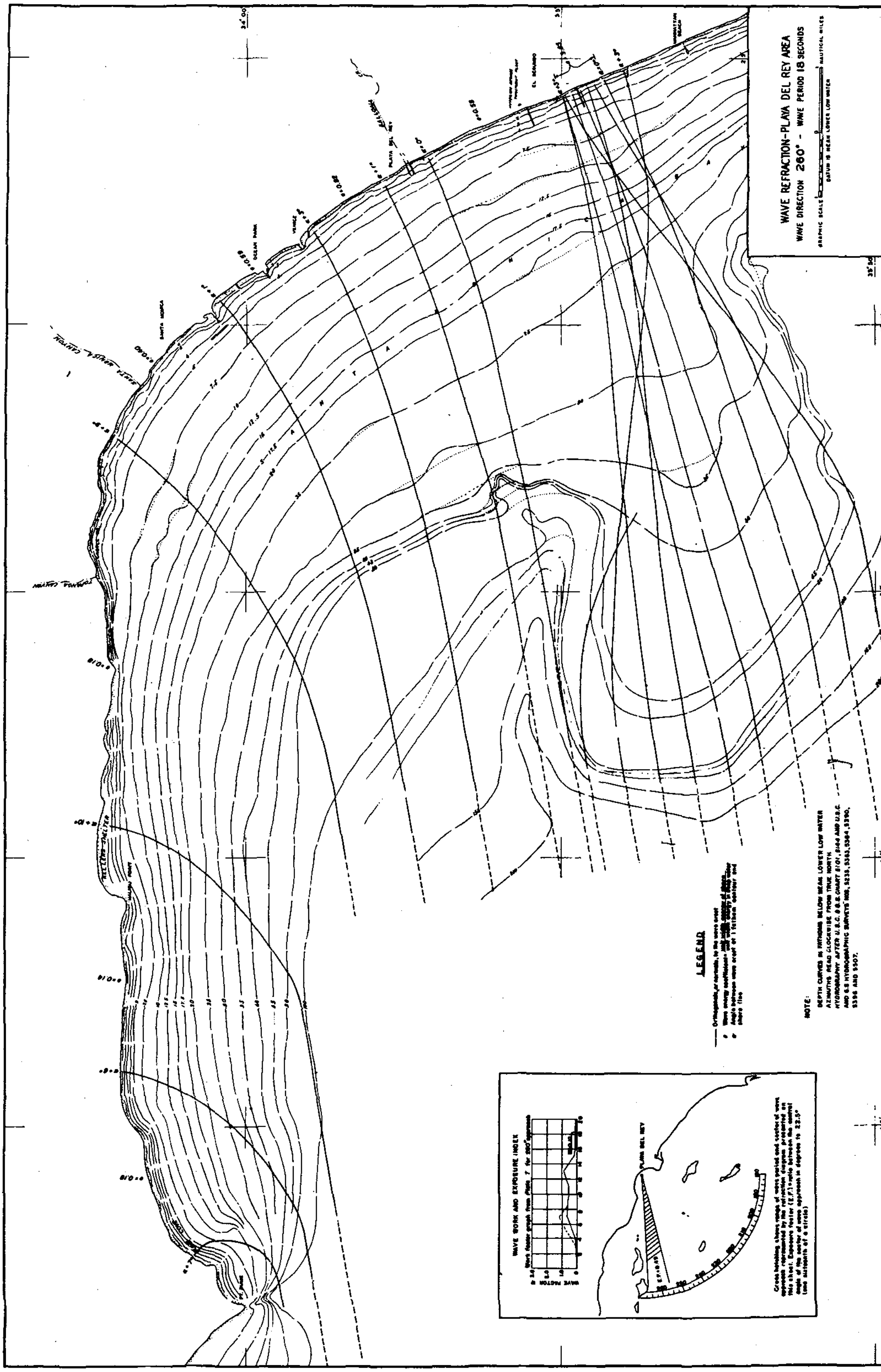

WAVE REFRACTION–PLAYA DEL REY AREA
WAVE DIRECTION 260° - WAVE PERIOD 18 SECONDS
LEGEND
NOTE:
PLAYA DEL REY
WAVE WORK AND EXPOSURE INDEX

Fig. 3

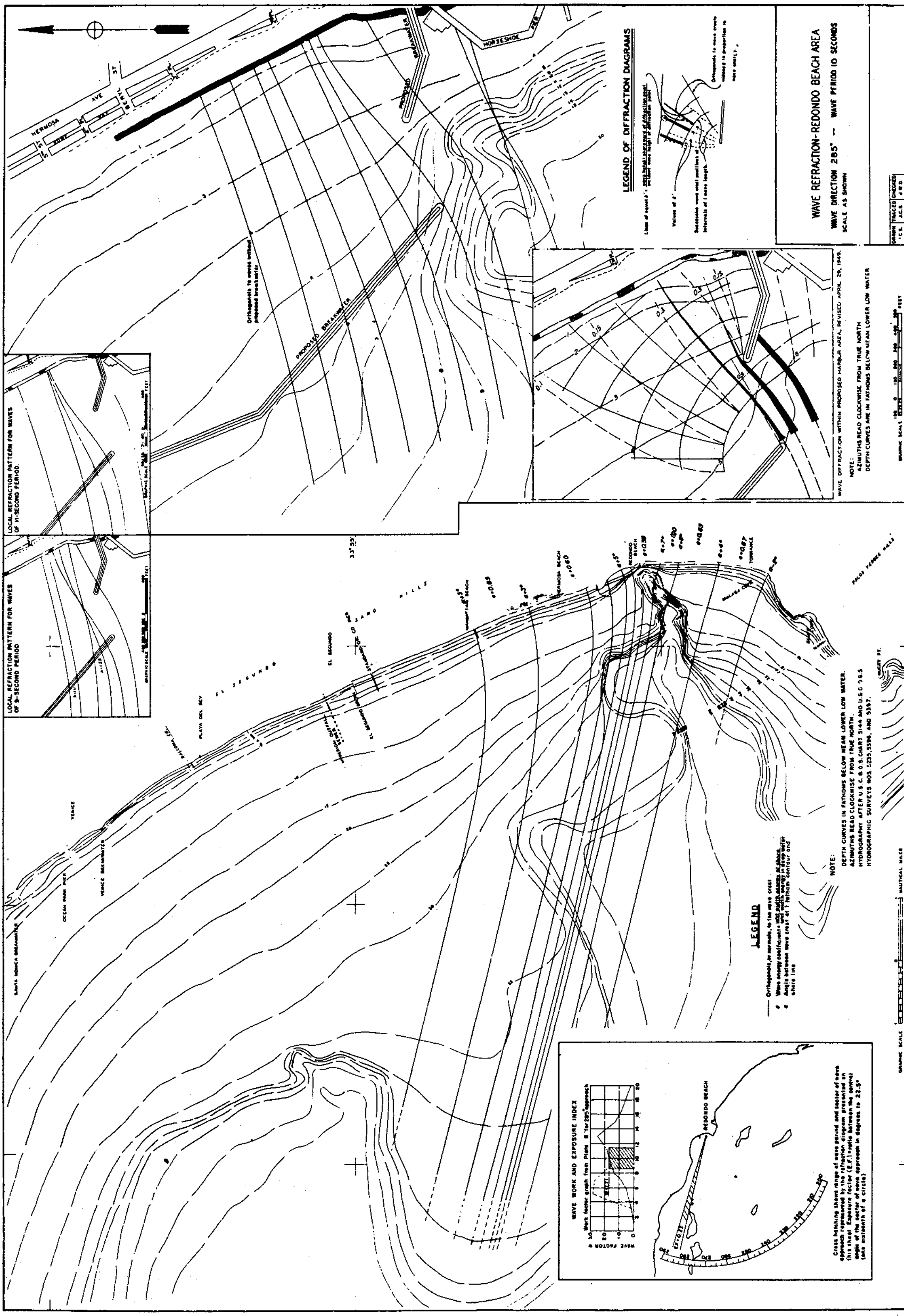


Fig. 4

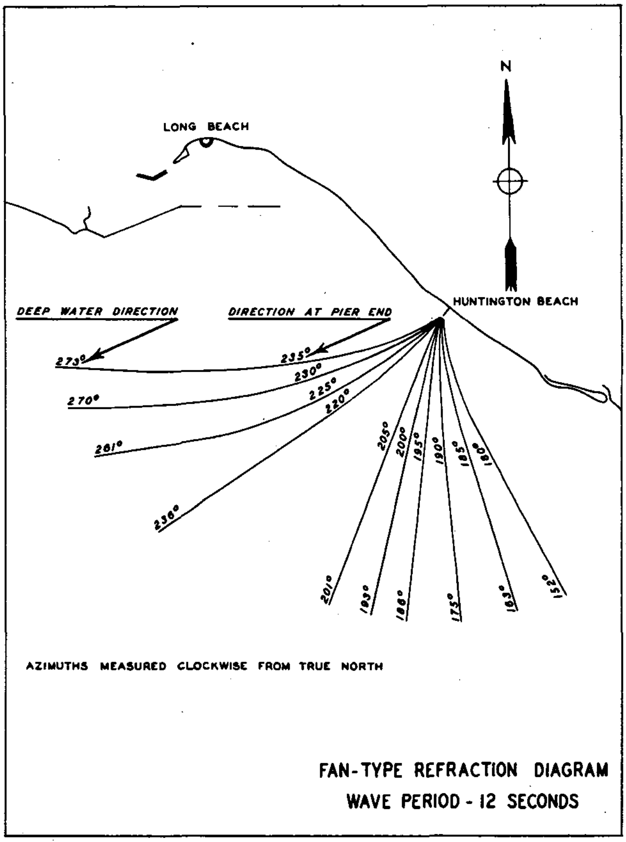
To complete the analysis, refraction diagrams are made to show the effect of the continental shelf on those wave segments that penetrate the screen.

Selection of wave types for study. Obviously, drawing a refraction diagram for every possible wave period and direction is impracticable. The practicable alternative is the study of a limited number of wave types, each of which may be considered representative of all waves within certain limits of direction and period. All waves within a sector of 10° to 20° at the offshore station and within a period range of 2 or 3 seconds generally can be represented on a single diagram. Where islands and mainland-coast headlands cut off waves from certain directions, the diagram coverage may be limited to the sectors of unobstructed approach. Although the width of these sectors may vary for different wave periods, for practical purposes the sector widths for the predominant wave period are considered representative of all periods.

Effect of island screens on waves. When a wave with crest length assumed to be infinite approaches an island, a central segment of the crest converges toward the island shore, which absorbs or reflects the wave energy. Adjacent intermediate segments on both sides of the central segment are refracted by the insular shelf into the island lee where they cross and continue in their diverse directions. Unrefracted segments along the outer sides of the intermediate segments continue shoreward in an unchanged direction. In analyzing the effect of the island screen, the effects of refraction on the intermediate segments of each wave crest are disregarded. The unrefracted sectors approaching a mainland station are assumed to include the intermediate segments even though they have been diverted away from the station. The reasoning behind this anomalous procedure is as follows: For every intermediate wave segment refracted away from the mainland station, part of a wave crest from another direction is probably diverted toward that station by refraction over the same insular shelf.

Presentation of analytic data. Insets on Figs. 3 and 4 show the offshore direction sector represented by the refraction diagrams shown on these figures. Wave-work diagrams for the 22-1/2° sectors covered by each refraction diagram are also given for the nearest offshore station of a wave hindcast study. These wave-work diagrams are also shown on Figs. 3 and 4. The exposure factor EF is the ratio of the sector width covered by the refraction diagram to the sector width represented by the work diagram. On Fig. 4, the period range represented by the refraction diagram is shown by cross-hatching on the work diagram. The angle between the wave crest and the contour at breaker depth is given on all diagrams for each orthogonal. The convergence or divergence of the orthogonals is indicated by the energy coefficient e, which is the ratio of the spacing between orthogonals in deep water to the spacing between orthogonals at the breaker line.

Pin-pointing the analysis. As a result of analyzing a complete series of refraction



FAN-TYPE REFRACTION DIAGRAM
WAVE PERIOD - 12 SECONDS

Fig. 5

diagrams for a given shore segment, information is obtained on (1) the probable direction of littoral drift along the shore, (2) the zones of convergence for each type of wave, and (3) the cumulative duration of each type of wave train over a period of time. This type of general study may suggest the advisability of a more detailed study of waves within a limited period range or direction sector where slight changes in period and direction produce major changes in the surf zone. The marked effects of one-second changes in wave periods are shown on Fig. 5. A detailed study of the effect of changes in period in the Santa Barbara Channel approach indicated that only waves of 11 seconds and less could reach the Redondo Beach area through that corridor. Longer-period waves were diverted toward shore or away from the Redondo Beach area by banks and shelves at the east end of the Santa Barbara Channel.

Application to groin design. Where a system of groins is required to stabilize a beach affected by a strong littoral drift, the alinement of the orthogonals of the predominant waves will indicate the proper orientation of each groin axis. The fillets of sand trapped by the groins generally assume a shore alinement perpendicular to the orthogonals (i.e., parallel to the breaker crest). By determining the probable limit of seaward advance of the shore line on the up-drift side of each groin, the length of each fillet along the shore may readily be predicted. As a result, the proper location for the root of the next groin on the up-drift side may be determined. By utilizing this method of groin-field design, each groin is alined properly according to the predominant approach direction and the groin spacing is adjusted to utilize to maximum advantage the sand-trapping effect of each structure.

Fan-type diagram. The deep-water direction of approach of any wave may be determined, when its period and its direction at a point near shore are known, by plotting an orthogonal by the crestless method of diagram construction from that point seaward into deep water. A number of these orthogonals plotted for waves of the same period but of different directions at the near-shore point (origin) produces a fan-type diagram that has many uses. For example, at Huntington Beach an automatic wave recorder with a device intended to measure wave direction was established at the end of a pier. The deep-water direction of the recorded waves could readily be determined by a series of fan-type refraction diagrams covering the probable range of wave period and near-shore direction. Fig. 5 shows one of the diagrams of this series drawn for waves of 12-second period.

DIFFRACTION

PRINCIPLES AND PROCEDURES OF DIAGRAM CONSTRUCTION

General. The term diffraction, as used in this paper, may be defined as the phenomenon in which the propagation of water waves continues into a sheltered region formed by a breakwater or similar barrier that interrupts part of an otherwise regular wave train.

Putnam and Arthur (1948) state that Penney and Price (1944) showed that Sommerfeld's solution of the optical diffraction problem, described by Bateman (1944), is also a solution of the water-wave diffraction problem. The article by Putnam and Arthur (1948) describes the development of a simplified solution to the water-wave diffraction problem and gives data on the verification of both the complete and the simplified theoretical solutions by experiments with deep-water waves. These experiments, which were made for the water area affected by the tip of a single breakwater, did not attempt to verify the theoretical diffracted wave-crest pattern. More recently Blue and Johnson (1949) reported on experiments with diffraction at a breakwater gap. The diffracted wave pattern and comparative wave heights were investigated for both deep- and shallow-water waves.

Diffraction equation. Putnam and Arthur (1948) found that the expression for the surface elevation, Z_s, is

$$Z_s = (k_1 k_2 C/g)\, e^{-ik_2Ct} \cosh k_2 d \,.\, F(x,y),$$

where C = incident wave velocity,

d = depth of water (assumed uniform),

t = time,

y = horizontal distance in direction of incident wave travel,

x = horizontal distance perpendicular to y,

e = base of natural logarithms = 2.7183,

i = $\sqrt{-1}$,

g = acceleration of gravity,

and k_1, k_2 = constants.

The only factor changed by diffraction is $F(x,y)$. Thus, the modulus of $F(x,y)$ determines the wave height, and the argument of $F(x,y)$ determines the wave pattern. Putnam and Arthur (1948) presented graphs of the modulus and argument of $F(x,y)$ based on tabulated values of the Fresnel integrals.

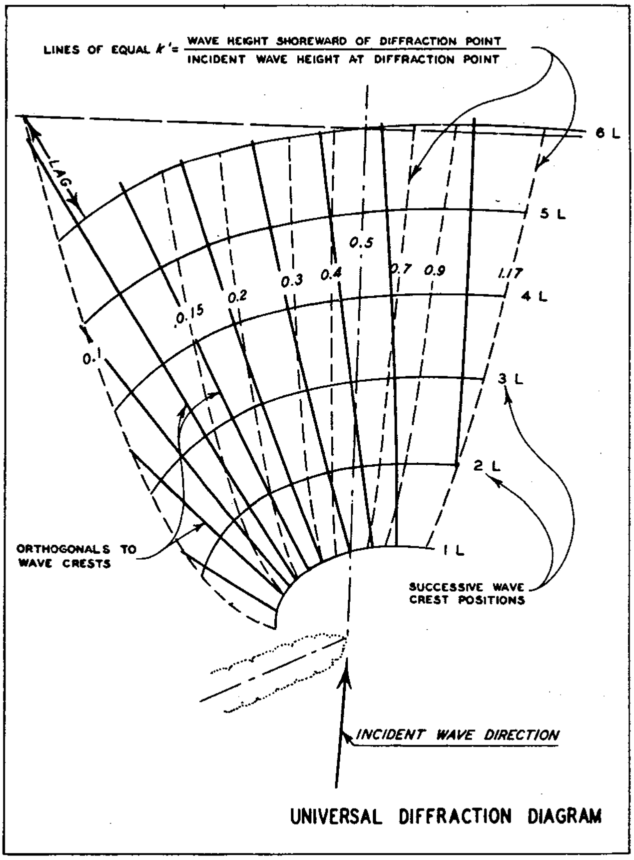


Fig. 6

The complete solution of the equation is somewhat complex because some of the terms affect the diffracted wave height and crest pattern only under unusual conditions or beyond the region in which the wave analyst is generally interested in diffraction effects. Consequently, Putnam and Arthur (1948) developed a simplified equation, which omitted those terms, as follows:

$$K' = F(x,y) = e^{-2\pi i\, y/L} \cdot f(u),$$

where
$$u^2 = (4/L\,(\sqrt{x^2 + y^2} - y),$$

K' = diffraction coefficient = $\dfrac{\text{diffracted wave height at point } (x,y)}{\text{incident wave height at the breakwater}}$

(the choice of K' to represent this function is dictated by its analogy to the K_d factor of refraction theory),

L = wave length at point of diffraction.

<u>Diffraction diagram.</u> In solving this equation for various values of x and y, lines of equal K' are determined that take the form of parabolas centering on the y axis and passing through the point of diffraction. The line for K' = 0.5 is the y axis itself or the orthogonal to the incident wave projected past the point of diffraction. The area on the sheltered side of the y axis is within the geometric shadow of the breakwater, and the area on the other side is outside the geometric shadow. Although this diagram applies only to sharp-cornered breakwater tips, the use of rounded tips similar to those found in rubble-mound construction did not produce large differences between experiment and theory. The angle between the diffraction axis and the breakwater axis theoretically has no effect on the dif-

fraction pattern. The position of the diffracted wave crest is established from values of crest lag, which are determined from the argument of f(u), and which are constant along any line of equal K'. Fig. 6 shows a diagram of wave diffraction carried to the wave-crest position six wave lengths beyond the point of diffraction. The diagram shows both the lines of equal K' and the successive crest positions at even wave-length intervals.

Experimental verification. The experiments conducted by Putnam and Arthur (1948) showed generally close agreement between the experimental and theoretical values of K'. This agreement, which generally was closer for the simplified solution than for the completed solution, was most markedly shown near the diffraction axis and within the geometric shadow. Outside the geometric shadow, experimental heights were less than the theoretical heights.

The results of experiments by Blue and Johnson (1949) generally corroborated those by Putnam and Arthur (1948) regarding the values of K', and disclosed fair conformity of experimental wave patterns to theoretical patterns for both deep- and shallow-water waves. The experimental wave patterns, which generally were ahead of their theoretical positions along the gap center line, showed a more sharply convex crest shape than the theoretical patterns. Irregularities in the experimental wave patterns evidently were related to wave steepness, the steeper the incident wave, the greater being the velocity increase along the diffraction axis and outside the geometric shadow and the more irregular being the diffraction pattern. The theoretical diffraction pattern generally is within the limits of accuracy required for investigations of harbor design, especially for long-period wind waves and swell. Within the geometric shadow the theoretical values of wave height determined from the calculated values of K' normally are within about 10 percent of the actual value. Because the theoretical heights of waves generally exceed the actual heights and the design of structures inside the harbor is based on the theoretical heights, maximum protection is provided.

APPLICATION TO COASTAL ENGINEERING

Artificial harbors. An obvious application of diffraction-diagram construction to coastal engineering is the determination of wave heights and crest patterns within a proposed or existing breakwater-protected harbor for any given characteristics of an incident wave train. Because the degree of variation in diffraction patterns is the same in all directions for all changes in wave length, the same diagram may be used for any combination of map scale and wave period by enlarging or reducing the diagram to fit the incident wave length. A diffraction diagram of any harbor of uniform depth may be quickly drawn by (1) determining the incident wave length, (2) marking off a series of wave lengths along the prolongation of the orthogonal through the breakwater tip, (3) enlarging or reducing the diffraction diagram until the wave lengths of the diagram and the harbor map are the same, and (4) tracing the diagram (with its origin at the breakwater tip and its axis coinciding with the prolonged orthogonal) onto the harbor map.

Breakwater gaps. Some breakwaters have gaps at intervals along their axes to facilitate navigation and to prevent traffic congestion. A determination of the wave pattern and the distribution of wave energy within these harbors in the vicinity of each gap is useful. The findings of Blue and Johnson (1949) for waves with incident direction normal to the breakwater axis indicate that a fairly accurate approximation of the diffracted wave pattern and of the values of K' may be obtained by drawing two mirror-image diagrams with the axis of each diagram passing through one of the two breakwater tips. Considerable judgment must be used in connecting the wave-crest positions between the two axes because the agreement between experiment and theory is poorest in that area. The practice of the Los Angeles District is to round the crest positions between the diffraction axes so that they blend smoothly into the convex curves of the crest positions within the geometric shadows. As a result, the wave crests at the gap center line are somewhat ahead of their theoretical positions. Values of K_d between the two axes are then determined by the refraction-diagram procedure instead of by theoretical computations using the diffraction formula.

Breakwater gaps with oblique wave approach. Where the direction of incident wave propagation is oblique to a straight line connecting the two breakwater tips, the diagram-construction procedure is similar to that described in the preceding paragraph up to the point of connecting the two mirror-image diagrams. The incident wave crest reaches one breakwater tip and there begins to diffract before it reaches the other breakwater tip. Unless the wave crest reaches the second breakwater tip an exact multiple of the wave period later, the two diffraction diagrams will be out of phase. One of these diagrams must then be modified by interpolating crest positions that are in synchronization with the other diagram. These synchronized crest positions then can be connected with the crest positions of the other diagram as described in the preceding paragraph. A diagram of diffraction at a breakwater gap with oblique incident waves is shown in Fig. 1.

Converging breakwaters. Interior harbors often are designed with converging breakwaters for protection of the entrance to the inner harbor. The principle behind this type of entrance protection is that the diffraction of waves passing through the gap between the outer ends of the breakwaters reduces the wave height progressively from the gap to the harbor entrance. A pervious dike or a barrier beach generally is provided to absorb the wave energy on each side of the entrance axis between the landward ends of the converging jetties and the entrance to the interior harbor. Here again, the diffraction diagram may be used to predict the effectiveness of this type of construction in reducing wave heights. The application of diffraction and refraction diagramming principles to the problems of converging breakwaters is the same as the application of these principles to the problems of breakwater gaps.

REFRACTION-DIFFRACTION COMBINED

PRINCIPLES AND PROCEDURES OF DIAGRAM CONSTRUCTION

General. If the bottom contours near a breakwater are not normal to the wave direction, refraction as well as diffraction must be considered; after the first few wave lengths shoreward from the barrier, the effects of refraction may be more significant than those of diffraction. Thus, the breakwater problem usually combines the problems of (1) refraction over the continental shelf to the point of diffraction at the breakwater, (2) diffraction for some distance beyond the breakwater, and (3) refraction between that point and shore.

Abrupt-change diagram. In considering the effects of various offshore wave directions and periods near the breakwater, the direction and length of each wave to be considered must be determined by a refraction diagram for the area between deep water and the diffraction point. Shoreward from the diffraction point, diffraction and refraction occur concurrently. However, for practical purposes and in the absence of experimental data on the combined effects of these two phenomena, the Los Angeles District uses a pure diffraction diagram only for a short distance beyond the point of diffraction, and thence reverts abruptly to pure refraction theory by extending the orthogonals to shore by refraction principles. This may be done by (1) the wave-crest method -- to determine the successive positions of the wave crest beyond the last position revealed by the diffraction theory or (2) the crestless method (by starting each orthogonal normal to the most shoreward wave crest of the diffraction diagram) -- to determine the divergence of orthogonals without delineating the wave-crest positions. By use of the lines of equal K', refraction coefficients may be carried from the offshore area through the diffraction area and into shore.

Modified diffraction diagram. If the orthogonal drawn through the breakwater tip by refraction procedures curves sharply, greater accuracy can be obtained by using the orthogonal as the diffraction axis and by varying each successive wave-crest position of the diffraction diagram to conform to that axis. The diffraction diagram thus modified should not be carried more than two or three wave lengths beyond the diffraction point before changing to refraction procedures. Otherwise, the wave pattern would be distorted.

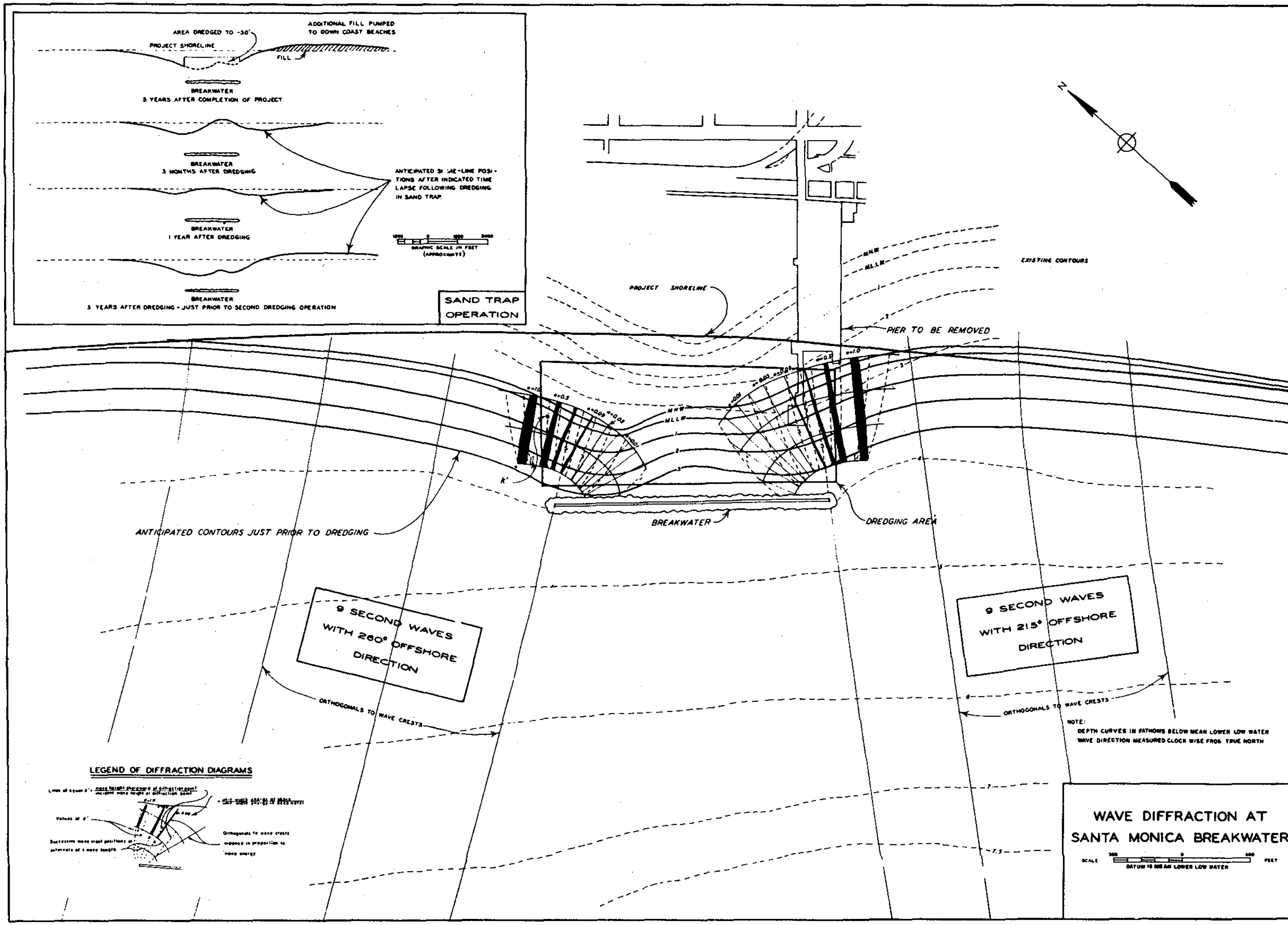

AREA DREDGED TO -30'
ADDITIONAL FILL PUMPED TO DOWN COAST BEACHES
PROJECT SHORELINE
FILL
BREAKWATER
5 YEARS AFTER COMPLETION OF PROJECT
3 MONTHS AFTER DREDGING
1 YEAR AFTER DREDGING
5 YEARS AFTER DREDGING - JUST PRIOR TO SECOND DREDGING OPERATION
SAND TRAP OPERATION
PROJECT SHORELINE
PIER TO BE REMOVED
EXISTING CONTOURS
MHW
MLLW
ANTICIPATED CONTOURS JUST PRIOR TO DREDGING
BREAKWATER
DREDGING AREA
9 SECOND WAVES WITH 260° OFFSHORE DIRECTION
9 SECOND WAVES WITH 215° OFFSHORE DIRECTION
ORTHOGONALS TO WAVE CRESTS
ORTHOGONALS TO WAVE CRESTS
NOTE:
DEPTH CURVES IN FATHOMS BELOW MEAN LOWER LOW WATER
WAVE DIRECTION MEASURED CLOCK WISE FROM TRUE NORTH
LEGEND OF DIFFRACTION DIAGRAMS
WAVE DIFFRACTION AT SANTA MONICA BREAKWATER
DATUM IS MEAN LOWER LOW WATER

Fig. 7

Weighted-line orthogonal diagram. In studying the wave characteristics near an offshore obstruction or in the lee of a breakwater, information often is required regarding the wave height and the energy per unit length of wave crest at all points in the area diagrammed. This may be done by widening the orthogonals in such a way that the line width is proportional to either of these two values. Orthogonals thus widened are called weighted-line orthogonals. By using the square root of the energy coefficient in weighting the orthogonals, the values of K_d and K' (i.e., wave heights) can be represented. Figs. 4 and 7 show diagrams of diffraction and refraction with orthogonals weighted in proportion to the wave energy per unit crest length from a point seaward of this point of diffraction to shore. A diagram of this type is particularly useful in showing the distribution of wave energy in such a manner that the layman can quickly grasp the implication of the diagram.

PRACTICAL APPLICATION BY THE LOS ANGELES DISTRICT

Sand-trap characteristics of breakwaters. Offshore breakwaters tend to trap littoral drift through the reduction of wave energy in their lee. This tendency is markedly shown at Santa Monica, California, where the breakwater, originally built to form a sheltered area for harbor use, has trapped several million cubic yards of littoral material since its construction in 1934. The beach in its lee has advanced seaward more than 800 feet, considerably reducing the effective harbor area. A proposal has been made to utilize the breakwater's sand-trapping tendencies by operating a dredge in its protected lee to distribute the trapped material along adjacent beaches as required. Fig. 7 shows how refraction-diffraction diagrams have been used to predict successive positions of the shore line and offshore contours as the fill continues to advance seaward year after year despite dredging operations. The rate of littoral drift had been determined previously by computing the accretion rate.

The weighted orthogonals indicate that the energy distribution in the breakwater lee drops off rapidly. A cursory examination of the diagram would indicate that the large quantities of sand that have reached the sheltered area of the breakwater lee could not have been transported by the negligible amount of wave energy indicated. However, much of this sand has been transported by storm waves that often in the southern California area have ten times the energy of normal waves. Under storm conditions waves along the diffraction parabola K' = 0.3 are as high as normal waves in the unprotected area, and adjacent parts of the wave crest are also proportionately higher. The wave energy, which assumes sizeable proportions, is directed toward the center of the breakwater shadow. The transporting capacities of the diffracted wave elements are fully utilized -- littoral drift is carried from outside both parts of the breakwater toward the center of the breakwater lee.

Upcoast and down coast from the sheltered area, the shore line tends to assume a position generally normal to the direction of wave approach. Although the exact position of the shore line cannot be predicted by theoretical analysis, a general consideration of the applied forces will indicate the probable shore alinement that would result from any sustained wave action of constant period and direction. The inset in Fig. 7 shows several anticipated positions of the shore line after the proposed dredging operation in the sand-trap area.

Littoral drift opposite an offshore breakwater tip. Fig. 1 shows a diagram that indicates the use of refraction and diffraction diagrams in studying the effects on the adjacent shore line of an existing breakwater and a proposed breakwater extension in San Pedro Bay. Sand movement along the beach is strongly affected by the offshore breakwater as well as by the shore-connected structures in this region. Studies that included the use of wave hindcasts and refraction-diffraction diagrams have been made of the anomalous behavior of the beach and surf in the affected area. As a result, corrective action is being planned.

Development of natural harbors. Natural harbors and sheltered anchorage areas protected by jutting headlands or reefs often present problems of both refraction and diffraction. Because natural obstacles to wave propagation are seldom so sharply defined or so abrupt as artificial barriers, the combined effects of refraction and diffraction result in a complicated pattern of wave behavior. The Los Angeles District has not yet had occasion to make a refraction-diffraction study of a natural harbor. Such a study could be made to compare the effectiveness of two or more harbor sites, each having partial natural protection, under various conditions of wave attack. The study probably would indicate methods of improving a natural shelter to form a safe harbor, possibly by extending a reef or headland artificially. The actual method of diagram construction used would be based on local conditions, but the principles and procedures presented in this discussion would provide a basis for a reliable analysis.

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