INTRODUCTION
The effects of coastal vegetation on wave attenuation have been well acknowledged and quantified. However, the studies on the effects of vegetation on wave setup and runup are still limited. Wave setup contributes to the elevated water level during a storm event. Flume tests (Wu et al. 2011) show that vegetation reduces the wave setup and neglecting vegetation will overestimate the mean water level (MWL). Dean and Bender (2006) theoretically examined the MWL change in vegetation. van Rooijen et al. (2018) numerically studied the effects of vegetation on wave setup using a storm impact model, XBead. The wave-averaged drag force ($\bar{f}_D$) in XBead was obtained through a wave shape model based on stream function wave theory. This wave shape model was calibrated with field data collected at barred beaches without vegetation, and thus may not be applicable to vegetated shorelines. The objective of this study is to model the effects of vegetation on the setup and runup of random waves using a cross-shore numerical model (CSHORE; Kobayashi et al. 1998). A new parametric model is proposed to compute $\bar{f}_D$ in CSHORE. Effects of vegetation on wave setup and runup are examined through numerical experiments.

METHODS
In CSHORE, short waves are governed by the wave action equation, whereas long waves are dependent on the cross-shore momentum equation as follows

$$\frac{\partial g}{\partial t} + \frac{\partial }{\partial x} \left( \alpha_3 \right) = \frac{\partial}{\partial x} \left( \alpha_2 \right) + \bar{f}_v + \bar{f}_{v,m}$$

where $h$ is the water depth, $\eta$ is the surface elevation, $x$ is the cross-shore distance, $S_{xx}$ is the radiation stress, $\tau_{bx}$ is the bottom shear stress, and $\bar{f}_{v,m}$ is the drag force due to the interactions between the mean flow (i.e. undertow, return flow) and vegetation. The overbar indicates phase-averaging. The current version of CSHORE computes $\bar{f}_v$ from the wave energy dissipation rate ($\bar{d}$) as below

$$\bar{f}_v = \left( 2n - 0.5 \right) D / C_g$$

based on an assumption that the MWL does not change in vegetation over a flat bottom, where $n = C_g / C$, $C_g$ is the group velocity and $C$ is the celerity. However, this assumption is not supported by laboratory or field measurements. $\bar{f}_v$ can also be computed theoretically from linear wave theory (LWT) (Dean and Bender 2006) as below

$$\bar{f}_v = \frac{\rho C_g b v N h_{rms}^2 \upsilon^2}{16 \sqrt{\eta} \sinh \upsilon h}$$

where $C_D$ is the drag coefficient, $b_v$ is the stem diameter, $N$ is the vegetation density, $H_{rms}$ is the root-mean-square wave height and $k_\upsilon$ is the wave number associated with the peak wave period ($T_p$). However, due to the symmetric horizontal velocity in LWT, $\bar{f}_v$ integrated along the submerged part of vegetation becomes zero, which is not true if using nonlinear wave theories. The drag force appears to be a function of the vegetation submergence ($h_v / h$), and thus a new formulation for $\bar{f}_v$ is proposed

$$\bar{f}_v = \frac{1}{2} \rho C_g b v N \upsilon c [u_c (\frac{h_v}{h})^{0.5}]$$

where $u_c$ is the velocity at the canopy from LWT. Stream function wave theory (SFWT) is applied to compute $\bar{f}_v$ and the index $m$ can be calculated from Eq. (4). A total of 1188 tests are numerically performed to determine $m$. $f_{v,m}$ is approximated using $\tau_{bx}$ as below

$$f_{v,m} = C_D, n b_v N \min \left( h, h_v \right) \frac{T_p}{\bar{f}_v}$$

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A different set of drag coefficients $C_{D,m}$ is applied in Eq. (5) to account for the uncertainty in the modeled mean flow velocity. The energy dissipation rate due to vegetation ($D_v$) was combined with the energy dissipation rate due to bottom friction ($D_f$), and $f_{v,m}$ was combined with the bottom shear stress ($\tau_{bx}$) in the original version of CSHORE. In contrast, Eqs. (1) and (5) are written in a way that $\tau_{bx}$ only accounts for the bottom shear stress and $f_{v,m}$ is separated from $\tau_{bx}$. Together with the proposed model of $\bar{f}_v$, $D_v$ is computed separately by following Chen and Zhao (2012), and $D_f$ is solely due to bottom friction. CSHORE assumes that the runup heights without overtopping follow the Rayleigh distribution, whereas the wave heights in saltmarsh follow a Weibull distribution (Jadhav et al. 2013). To model the vegetation effects on wave runup, the Weibull distribution is employed with the shape parameter calibrated using measurements.

RESULTS
It is found that the index $m$ can be parameterized with Ursell number ($Ur$), relative wave height ($h_v / h$), and vegetation submergence ($h_v / h$). Fig. 1a shows the variations of $m$ with $Ur$ for all test cases. With some algebraic operations, a single-valued relationship between $m$ and $Ur$ is found (red circles in Fig. 1b). The coefficients $\alpha_1$ and $\alpha_2$ in the single-valued function in Fig. 1b are determined as follows

$$\alpha_1 = \begin{cases} -0.1 \frac{h_v}{h} & 0.2 \leq \frac{h_v}{h} \leq 0.8 \\ 1.09 \frac{h_v}{h} & 0.2 < \frac{h_v}{h} < 0.8 \\ 0.35 \frac{h_v}{h} - 0.16 \end{cases}$$

$$\alpha_2 = \begin{cases} 0.35 \frac{h_v}{h} - 0.16 & 0.2 \leq \frac{h_v}{h} \leq 0.8 \\ 0.03 \frac{h_v}{h} + 0.65 & \frac{h_v}{h} > 0.8 \end{cases}$$

The index $m$ can be linearly interpolated from Fig. 1b. For random waves, $\bar{f}_v$ is expressed as

$$\bar{f}_v = \frac{1}{16 \sqrt{\eta} \sinh \upsilon h} \left( \frac{h}{h_v} \right)^m$$

where $\bar{m} = 2 \pi / T, \bar{P} = T_p / (1.35)$ is the mean wave period.

![Figure 1 - Variations of $m$ with $Ur$.](image-url)

The flume experiments conducted by Wu et al. (2011) are
used to examine the three models for $f_e$ (i.e. Eqs. (2)-(5)). For all cases, $N = 3150$ stems/m$^2$, $b_h = 3.175$ mm, $h_b = 0.2$ m. The wave characteristics and drag coefficients are listed in Table 1. $C_D$ and $C_D,m$ are calibrated for wave height decay and MWL, respectively. Fig. 2 shows the modeled and measured MWL for all seven flume tests. The wave height decay modeled by CSHORE fits well with the flume data. CSHORE with $f_e$ and $f_{em}$ computed using the first two methods (i.e. Eqs. (2), (3) and (5)) tends to overpredict the wave setup, whereas CSHORE with the proposed model of $f_e$ (i.e. Eqs. (4) and (5)) captures the MWL change accurately. The modeled $H_{rms}$ agrees well with the measured $H_{rms}$ (Fig. 3).

Table 1. Wave and vegetation conditions of 7 test cases

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<th>Case ID</th>
<th>$H_w$ (cm)</th>
<th>$T_p$ (s)</th>
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In order to examine the effects of vegetation on the wave runup, a series of numerical experiments are conducted to simulate the wave runup on a dike with the slope of 1:4, fronted by a sloping beach with the slope of 1:100, with and without vegetation. The wave conditions and beach and dike dimensions are from the flume experiments by van Gent (1999). A vegetation patch with the same properties as those in Wu et al. (2011) is placed on the sloping beach as shown in Fig. 4a. Fig. 4b shows the modeled wave runup ($R_{2\%}$) with and without vegetation. It is observed that the vegetation patch reduces the wave runup by at least 20% and in some scenarios cuts the wave runup by half. The reduction rate varies with the vegetation properties and wave conditions. Note that the reduction in wave runup shown in Fig. 4 is attributed to the wave attenuation caused by the vegetation field. The effect of Weibull distribution of wave runup with vegetation is expected to lead to further reduction in $R_{2\%}$, which is being investigated.

CONCLUSIONS
The effects of vegetation on wave setup and runup are investigated by numerical experiments. In order to solve for MWL changes, a new parametric model of $f_e$ is proposed. Numerical experiments guided by laboratory measurements are carried out to determine the index $m$ in Eq. (4) for random waves. Three models of $f_e$ are used in CSHORE for reproducing the MWL change in the flume tests by Wu et al. (2011). Numerical results show that the modeled MWL using the proposed model of $f_e$ agrees well with the laboratory measurements. With respect to the wave runup, in the numerical tests, vegetation leads to 20%-50% reduction in wave runup. The reduction rates strongly depend on the wave conditions and vegetation properties. A larger vegetation field and taller plant height will lead to greater reductions in wave height, wave setup and wave runup. Although the modeling of the vegetation effects on wave setup and runup presented in this paper is promising, further tests of the model against laboratory and field measurements of wave setup and runup with vegetation are needed.

REFERENCES
Dean and Bender (2006). “Static wave setup with emphasis on damping effects by vegetation and bottom friction.” Coastal Eng., vol. 53, pp. 149-156.