SOLUTE DISPERSION IN THE NEARSHORE DUE TO OBLIQUE WAVES

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An experimental study has been conducted in a large scale basin at Danish Hydraulic Institute (DHI). Simultaneous measurements of hydrodynamics using Laser Doppler Anemometry (LDA) and fluorescent tracer studies were undertaken within the surfzone under a regular wave condition with waves approaching the shore at 20°. Through a series of hydrodynamic and tracer measurements and their comparison with the existing theoretical values, this study quantifies the physical processes and their integrated effects on a solute tracer in the nearshore zone subject to combined waves and the induced longshore currents. A theoretical dispersion model has been developed, adopting both experimental and theoretical velocimetry approaches. The results of theoretical model have been compared to the tracer data. Using the results from this study together with all known previous studies of dispersion measurements within the surfzone, good agreement exists.

**Keywords:** Coastal Mixing, Surfzone, Oblique Waves, Pollution

**INTRODUCTION**

In the UK, a small, but significant number of coastal waters fail to satisfy the minimum standard for Faecal Indicator Organisms. Around 10% of UK beaches (which are routinely tested) are predicted to fail the revised Bathing Water Directive standards, and another 12% are predicted to be classified as sufficient. Near-shore coastal waters receive pollutant loading through both the shoreline (predominately diffused pollution from farmland and urban areas) and seaward boundaries (sewer outfalls). From the seaward boundary, pollutant loading is transported landward towards the surfzone by the so-called Stokes drift effect (Stokes, 1847). From the shoreline boundary, runoff pollution, which can contain Faecal Indicator Organisms and human viruses (Grant et al, 2005) can drain into the surfzone. Consequently, these pathogens can congregate in the nearshore region, where the water quality can affect the health of the general public. The problem is particularly acute following rainfall storm events. Air masses pushed by wind cause frontal and orographic rainfall, which is predominant in the UK. In coastal waters, these winds also generate wave activity; hence larger wave activity normally occurs during and after storm events. Therefore, it is very essential to understand the mixing processes due to the effect of wave activities in the nearshore region. Although many efforts has been made during the past decades to study different aspects of mixing yet little new information is available about the mixing under wave and current activities in the coastal zone.

Water quality numerical models used to aid management decisions are usually 2D depth averaged, exclude wave processes and require as input, a value for the dispersion coefficient. Predicting dispersion coefficients in this complex three-dimensional flow field is difficult and is caused by the interaction of the periodic orbital motions of the waves, the variable depth, longshore current induced vertical and lateral shear effects, the effects of Stokes drift and the bed and free surface boundary sources of turbulence.

**PREVIOUS WORK**

In riverine flows the analogy of turbulent mixing processes to Fickian diffusion has been made by several researchers. Reviews of this field are provided in Fischer et al. (1979) and more recently, Rutherford (1994). The depth averaged advective-dispersion equation given by Rutherford (1994)

\[
\frac{\partial c_d}{\partial t} + \frac{\partial}{\partial x}(du_d c_d) + \frac{\partial}{\partial y}(dv_d c_d) = \frac{\partial}{\partial x}\left[d(e_x + \varepsilon_x) \frac{\partial c_d}{\partial x}\right] + \frac{\partial}{\partial y}\left[d(e_y + \varepsilon_y) \frac{\partial c_d}{\partial y}\right]
\]  

(1)

Where;

\[x, y = \text{longitudinal and transverse directions,}\]
\[u_d, v_d = \text{depth averaged longitudinal and transverse velocities,}\]
\[d = \text{depth of flow,}\]
\[c_d = \text{depth averaged concentration,}\]
\[e_x, e_y = \text{diffusion coefficients in the x and y directions,}\]
\[\varepsilon_x = \text{the longitudinal dispersion coefficient which accounts for the effects of the vertical variations in the longitudinal velocity,}\]
\[& \quad \varepsilon_y = \text{the transverse dispersion coefficient which accounts for the effects of the vertical variations in the transverse velocity,}\]

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This equation shows how both turbulent diffusion processes and dispersion, due to the spatial averaging of velocities, influence the spreading of a conservative tracer. Molecular diffusion is assumed to be negligible. For a continuous line source in unbounded flow where the transverse dispersion is dominant, the concentration distribution may be described as

\[
c_c(x,y) = \frac{Q}{u_d \sqrt{4\pi} \left( y / u_d \right)} \exp \left( \frac{-y^2 u_d}{4e x} \right)
\]  

(2)

Where, \( Q \) is the mass inflow rate per unit length. Fischer (1967) utilized this result to estimate values of transverse diffusion coefficients from measurements of transverse concentration distributions. From a continuous point source injection in wide channels, the transverse mixing coefficient was obtained from

\[
D_y = \frac{u_d}{2} \frac{d\sigma_y^2}{dx}
\]

(3)

Where the spatial variance, \( \sigma_y^2 \) of the transverse concentration distribution \( c_d(y) \) is given by

\[
\sigma_y^2 = \frac{\int \left( y - \mu \right)^2 c_d(y) dy}{\int c_d(y) dy}
\]

(4)

Where \( \mu \) is the position of the centroid of the distribution. This technique provides an estimate of the transverse diffusion coefficient which although termed a diffusion coefficient, includes all the effects present within the flow which contribute to the observed spreading of the tracer plume. It perhaps more correctly should be termed a transverse dispersion coefficient, as secondary flows are rarely absent.

There have been comparatively few experimental studies to investigate aspects of mixing under waves in the coastal zone. Some site specific field studies have been undertaken, however, the contribution to mixing due to wave activity is difficult to interpret as all the associated parameters responsible for the transport processes were not measured.

**Tracer Studies:** Inman et al. (1971) referred to the studies of Harris et al. (1963) who undertook a series of experiments in both the field and laboratory to investigate the mixing of a solute when released into the surf zone. Both field and laboratory-based experiments produced results which suggested that the mixing across the surf zone is proportional to \( H^2/T \), where \( H \) is the crest to trough wave height and \( T \) is the wave period.

In the nearshore zone, where water depths are small, wave processes dominate the mixing of soluble pollutants. Pearson et al. (2002) quantified the on-offshore dispersion for monochromatic waves of different heights and of fixed period, in non-breaking wave regions, just seawards of the breaker point. Based on this work, a technique to predict the magnitude of the dispersion coefficients was suggested. A full-scale fieldwork experimental study of surfzone dye dispersion on a plain 1:50 sandy beach was reported by Clark et al. (2010). Three potential theoretical mechanisms for on-offshore tracer dispersion in the surf zone were examined, breaking wave induced dispersion, and undertow shear induced dispersion [developed by Pearson et al (2009)] were both shown to have correlations to the measured values (undertow shear, 94% correlation). 2D horizontal rotational velocities (surfzone eddies) were found to be the primary contributor to the overall measured mixing.

**Hydrodynamic Studies:** The pioneering works of Longuet-Higgins (1960, 1964) [radiation stress] still form the basis of many theoretical applications of near-shore hydrodynamics to this day. In terms of measurements, much of the early work focused on phase-averaged velocities (e.g. Hansen & Svendsen (1984), Cox et al (1995), Svendsen (1986)) following the quantitative analysis of undertow by Dyhr-Nielsen & Sorensen (1970). Early measurements by Stive (1984) and Nadaoka & Kondah (1982) using Laser Doppler Anemometry (LDA) techniques in the surfzone produced significant data sets. Ting & Kirby (1994, 1995, 1996) demonstrated that the turbulent kinetic energy generated around the breaker or plunge point, is transported seaward under a spilling breaker, but landward under a plunging breaker.

Svendsen (1987) adopted the analysis of Prandtl (1952) to suggest that the turbulence generated mixing in the surfzone was dominated by the breaking wave. Svendsen (1987) summarized a number of detailed velocity measurement studies, which determined the on-offshore variation of eddy viscosity.
both in the surfzone and seawards of the breaker point. He proposed that the length scale \( l_m \) of the eddies were closely related to the water depth \( d \) and suggested that under laboratory conditions, the variation of eddy viscosity \( (\nu_e) \) across the surfzone generated by the breaking wave could be characterized by the empirical relationship; \( \nu_e = Md\sqrt{gd} \), where \( M \) is a constant which lies in the range \( 0.01 < M < 0.03 \), \( d \) is the water depth, and \( g \) is the acceleration due to gravity.

**A Simplified Mixing Mechanism:** Using Svendsen’s (1994) methodology, the turbulent diffusion can be estimated by \( e_z = e_y = \nu_e = Md\sqrt{gd} \). In the surfzone, Pearson et al. (2009) adopted the above analogy to describe a theoretical advection-dispersion transport process for on-off shore mixing generated by undertow shear dispersion driven by the vertical variation in on-off shore velocity in the surfzone. Using suitable estimates for the turbulent diffusion (diffusion the same in all directions) and on-off shore wave-induced velocity, a theoretical approximation to the overall mixing within the surfzone was obtained. They showed that the on-off shore mixing (ensuring that the lengths and times are measured in meters and seconds) could be given by:

\[
D_y \approx \frac{gH_b^4}{768d\epsilon} \approx \frac{gH_b^4}{768d(0.01d\sqrt{gd})} \approx 0.13g^{1/2}H_b^{4/2}d^{5/2} = 0.13g^{1/2}\gamma^{5/2}H_b^{3/2} \tag{5}
\]

Where, \( D_y \) is the depth averaged on-off shore dispersion coefficient in the surfzone, \( g \) is acceleration due to gravity, \( \gamma \) is the breaker index and \( H_b \) is the wave height at breaking.

Figure 1 shows the relationship between \( H_b^{3/2} \) and the measured on-off shore dispersion \( (D_y) \) [Equation (3)]. As the experimental studies are from a number of sources, both within the laboratory and within the field, for simplicity, it has been assumed that the breaker index can be characterised by the commonly adopted value of \( \gamma = 0.78 \) [Galvin (1972)]. This result suggests that theoretical on-off shore mixing within in the surfzone is a function of \( H_b^{3/2} \).
LABORATORY STUDY OF OBLIQUE WAVES IN THE NEARSHORE

The experimental work was undertaken in the shallow water basin at DHI, Denmark [Figure 2]. In this study mixing of buoyant pollutant in the nearshore region has been investigated from both Eularian and Lagrangian perspective. Detailed hydrodynamic measurements in the nearshore region performed by using Laser Doppler Anemometry (LDA). Fluorometric study has been carried out by use of Rhodamine Water Tracing Dye. The measurement section measured 18m x 8m, with an offshore water depth of 0.5m. The bed of the facility consisted of concrete screed with an assumed roughness element 1mm high. The facility is equipped with an absorbing piston-type wave-maker and all experiments were performed on a 1:20 plain beach, with waves approaching the shore at 20°. A regular wave \( [H_o=0.1, T=1.85s] \) was generated and after significant endeavours, the facility was finely tuned by re-circulating the longshore current at the down-stream end of the facility and re-introducing the flow at the up-stream end of the facility. A Mike 21 computational model was used in the experimental design to assist in determining the re-circulation discharge, and appropriate outlet weir conditions [Figure 3].

**Hydrodynamic Measurements:** The velocity field in the surfzone and seaward of the breaker point for plunging breaking waves were investigated from a Eularian point of view by using Laser Doppler Anemometry (LDA). The turbulence structure within the nearshore region has been determined by studying two dimensional velocity field, which was obtained from hydrodynamic measurements in the vertical plane from 120 consequent monochromatic waves that were measured at various cross-shore locations in the nearshore region.

Wave height measurements were simultaneously undertaken across the basin at 0.5m intervals to determine the wave characteristics in the region of the dye plume. The wave conditions were logged continuously throughout the duration of both the concentration and velocity data collection periods. Sixteen wave monitor modules connected to twin wire wave probes were utilized to measure the water surface elevation at selected locations within the facility. The voltage output from the modules was logged by the data acquisition system. The wave probes were re-zeroed and calibrated over 4 points...
usually (0,+100,-100,0)m] each day by driving the probe up or down. The output voltage from a wave probe monitor is directly proportional to the probes depth of immersion.

For this study the region near to the paddles has been termed ‘offshore’, although due to the water depth \((d=0.50\text{m})\), the waves are in the transitional region. The ‘offshore’ wave steepness, \(S_{op}\) has been determined using the inshore wave celerity given by \(\sqrt{g d}\).

**Fluorometric Measurements:** A constant head injection of Rhodamine WT dye was introduced to the basin from a small brass tube at approximately mid depth, at various distances from the shoreline. The subsequent spreading of the tracer was then recorded by pumping one liter samples into sterilized containers for later analysis. The facility has a re-circulating flow system which causes the continual build-up of background dye concentrations. Hence, to minimize the temporal build-up during testing, ten discrete samples were collected simultaneously with their sample tubes placed at approximately mid depth and spaced at an on-offshore distance of 50mm apart. To eliminate additional mixing generated by extracting the samples within the facility, the flow rate of the pumps were adjusted so that the velocity at the inlet of the pipe matched the longshore current velocity. The ten samples took approximately two minutes to collect, and the array of tubes was stepped across the plume at half-metre intervals. An additional background sample was collected upstream of the injection point whilst collecting the 10 discrete samples. Approximately 70 samples were collected for each on-offshore section.

**RESULTS**

In this section the results of hydrodynamic and tracer measurements for the case of monochromatic oblique waves inside the surfzone are presented. Hydrodynamic data has been adopted to quantify the turbulent kinetic energy (TKE) and determine turbulent diffusion coefficient. The shear dispersion coefficient obtained by employing the vertical velocity profile and the eddy viscosity value. Fluorometric data has been analyzed by using Taylor’s (1953) analogy. The results of hydrodynamic model have been compared to the dye tracer measurements.

**Hydrodynamic Data:** Mixing processes in the nearshore region has been investigated from a Eulerian perspective by use of the hydrodynamic data obtained from detailed LDA measurements across the nearshore. An analytical model has been developed based on hydrodynamic data to quantify the advective-dispersive mechanisms inside the surfzone.

From the Fickian advection-diffusion equation the total on-offshore mixing coefficient \(E_{xx}\) inside the surfzone can be written as a summation of turbulent diffusion \((v_t)\) and shear dispersion \(D_x\) [equation 6]. Surface generated turbulence due to the wave breaking phenomena inside the surfzone is the major contributor to the turbulent diffusion and combination of bed generated turbulence; non-uniform wave period and secondary velocity profile over the depth are bestowing shear dispersion in the nearshore region.

\[
E_{xx} = v_t + D_x
\]  
(6)

In the first step, an attempt has been made to detect and remove the spikes in the LDA datasets using Acceleration Theresholding Method (Goring & Nikora, 2002). Spikes in LDA data are common problem that need to be dealt with carefully. Spikes are usually due to aliasing of Doppler signals as well as the air entrainment into the probe as a result of wave breaking phenomena inside the surfzone and bubble generation due to intensive turbulence dissipation.

Diffusivity inside the surfzone has been calculated based on hydrodynamic data by adopting Svendsen & Putrevu (1994) methodology, which relates eddy viscosity \(v_t\) to Turbulent Kinetic Energy \(k\) and characteristic length scale of turbulence \(l_m\).

\[
v_t = l_m\sqrt{k}
\]  
(7)
Turbulent Kinetic Energy has been determined following the spectral analysis procedures. Figure (4) compares the vertical structure of TKE determined from hydrodynamic data with Nadaoka & Kondoh (1982) measurements of monochromatic waves on a 1/20 beach slope. $\zeta$ is the distance from the bottom bed and $d$ is the local mean water depth. The position of each set of measurements is indicated by the value of $d/d_oB$ where $d_o$ is the undisturbed local water depth and subscript B indicates the breaking condition. The results show good agreement with the existing experimental data.

The eddy viscosity obtained based on equation (7) and by adopting the depth-averaged turbulent kinetic energy and taking the characteristic length-scale as Svendsen & Putrevu (1994) suggestion. For the case of $H_o=0.10m$, $T=1.85s$ regular wave condition at a 20° approach, the eddy viscosity at 1.0m from the shoreline determined as $10^{-6}m^2/s$. Comparison of $u_1$ calculated for this study with the theoretical relation proposed by Svendsen & Putrevu (1994) for the case of waves inside the surfzone, shows that the turbulent diffusion coefficient determined in this study is in-line with the existing data.

Advection shear dispersion in the nearshore region is mainly due to bed frictional effects from the oscillatory wave motions ($D_{\zeta}$), bottom friction on tidal currents, wind driven currents and secondary velocity profile ($D_{\Phi}$). Hence the total dispersion coefficient in the on-offshore direction can be given by:

$$D_x = D_{\zeta} + D_{\Phi}$$

The dispersive shear mechanisms are generally more significant contributors to the overall mixing compared to turbulent diffusion alone. In this paper advective shear dispersion has been investigated with extracting the vertical variation of on-offshore velocity from the hydrodynamic data collected across the surfzone. The shear dispersion coefficient $D_x$ determined from hydrodynamic data using the method of Zones, originally proposed by Chikwendu (1986). N-zone model divides the two-dimensional flow in the surfzone into $N$ zones of parallel flow with the thickness $t_j$, the average velocities $u_j$ and longitudinal diffusivities $D_{xj}$, where $j = 1, 2, \ldots, N$. Figure (5) is a schematic sketch of N-zone model. Each zone is assumed to be well-mixed with the concentrations $c_1$ to $c_N$ and the shear dispersion coefficient can be obtained from advection-diffusion equation by dividing it into $N$ coupled dispersion equations:

$$\partial_t c_1 = D_{x1} \partial_z^2 c_1 - u_1 \partial_z c_1 + b_{12} \beta_1 (c_2 - c_1)$$

$$\partial_t c_j = D_{xj} \partial_z^2 c_j - u_j \partial_z c_j + b_{(j-1)j} \beta_j (c_{j-1} - c_j) + b_{jj+1} \beta_j (c_{j+1} - c_j)$$

$$\partial_t c_N = D_{xN} \partial_z^2 c_N - u_N \partial_z c_N + b_{(N-1)N} \beta_N (c_{N-1} - c_N)$$

The system of equations introduced in (9) can be analyzed through use of large-time exponent and Fourier transformation, hence the longitudinal dispersivity can be written at large times:
In this paper a modified N-zones model with varying eddy viscosity and phase-averaged velocity for each layer has been employed to determine the shear dispersion coefficient ($D_x$). For the case of monochromatic waves with $S_{op}=3.5\%$ a shear dispersion coefficient ($D_x$) of 0.0148 m$^2$/s was obtained at 1.0 m from SWL. Therefore, total on-off shore mixing coefficient of 0.015 m$^2$/s achieved based on equation (6). In the following section the on-off shore mixing coefficient based on tracer studies will be determined and the results will be comparing to the analytical model described in this part.

\[ D(N) = \sum_{j=1}^{N-1} (q_1 + q_2 + \cdots + q_j)^2 \left[ 1 - (q_1 + q_2 + \cdots + q_j) \right]^{1/2} \times \left[ u_{12\ldots j} - u_{(j+1)\ldots N} \right]^{1/2} / b_{(j+1)} + \sum_{j=1}^{N} q_j D_{xj} \]

(10)

Tracer Data: The longshore current profile over the measurement section is shown in Figure (6) for the $H_o=0.10m$, $T=1.85s$ regular wave condition at a 20° approach. The facility has a horizontal bed in the longitudinal direction, thus it was not possible to produce longitudinally uniform flow. Further study of Figure (6) shows that the velocity appears to vary by approximately 5% over the 8m control section of the facility. Additionally, it is evident that there is an up-stream recirculation in the system (outside the measurement section), which couldn’t be eliminated during set-up of the facility.

Figure 5. Schematic sketch of N-Zones dispersion model in the nearshore

Figure 6. Longshore velocity distribution over measurement section
Figure (7) shows the resultant above background concentration profiles for the $H_0=0.1$, $T=1.85s$ wave condition. The spatial variance ($\sigma^2$) of each concentration profile has been determined, and results are shown in Figure (8). Over the width of the plume the velocity distribution varies [Figure (6)], Equation (3) does not account for a variation in the longitudinal velocity and hence the validity in this case may be questionable. Nevertheless, results presented in Figure (8), demonstrate that the increase in on-off shore variance with distance, although showing some small scatter, is approximately linear.

![Figure 7. Above background transverse concentration profiles for $H_0=0.1$, $T=1.85s$ wave condition](image)

Employing the concept of Taylor’s (1953) turbulent diffusion analogy (Equation 3), an estimate of the transverse mixing coefficient $D_y$, combining both diffusion and dispersion or differential advection processes can be obtained. The spatial variance ($\sigma^2$) of each concentration profile has been determined, which results in a transverse mixing coefficient of 0.0119 m$^2$/s. Comparison of this mixing coefficient from tracer data with the theoretical model based on hydrodynamic data ($E_{xx}=0.0150$ m$^2$/s) shows that the result is in-line with expectations.

Dye concentration measurements were confined to the surfzone region with an injection points at $y = 1.0m$ from the shore-line. At this location, estimates of wave set-up ($\eta$) and set-down for the tested conditions are small relative to the water depth, and have been neglected in further calculations. An important feature of surfzone hydrodynamics which influence the resultant wave-induced velocity is the depth of water below the wave trough level [$d_\eta$]. According to DeVriend & Stive (1987) and Svendsen (1987), the depth can be approximated by $0.8(d + \eta)$. Due to the nature of the waves in the surfzone, all LDA measurements were performed below the wave trough level, hence only the undertow velocities were recorded and analyzed.
Development of Theoretical Model: To test the validity of a 2-layered advective-dispersive mechanism generated by the bore of the breaking wave, the measured velocities were compared with the theoretical depth mean velocities within the bore of the breaking wave. For simplicity wave transformations have been neglected, and the ‘offshore’ wave heights measured towards the paddle have been adopted as the incident wave condition at the breaker point, and these velocity profiles have been combined with estimates of turbulent mixing [Svendsen & Putrevu (1994)] and integrated to establish an overall depth averaged dispersion coefficient.

A literature review has indicated that the only known experimental studies which incorporate the overall on-off shore mixing within the surfzone were studies undertaken by Harris et al. (1963), Inman et al. (1971), Tanaka et al. (1980), Pearson et al. (2009), and Clarke et al. (2010). Figure (9) shows the relationship between measured on-off shore dispersion ($D_y$) using tracer studies, and the predicted on-off shore dispersion identified within this study. As the experimental studies are from a number of sources, both within the laboratory and within the field, for simplicity, it has been assumed that the breaker index can be characterised by the commonly adopted value of $\gamma = 0.78$ [Galvin (1972)]. For steeper sloping beaches ($<1:50$) it has been assumed that $\sqrt{\gamma}$; and for gentle sloping beaches, it has been assumed $\sqrt{\gamma}$. Using these assumptions, a theoretical on-off shore mixing has been estimated using equation (5) and is shown Figure (9). The new measured results from this present study have been included, and the wave height at the breaker point ($H_b$) has been adopted for all calculations. For clarity the 45 degree line is also shown. It is noticeable that the measured and theoretically predicted depth averaged dispersion coefficients are strongly correlated, with a coefficient of 94%. This suggests that the prediction method described within this study is not only applicable in both laboratory and field based studies, but is also demonstrates that the vertical variation on-off shore velocities are important in nearshore mixing problems within the surfzone.
CONCLUSION

Within the surfzone, a theoretical advection-dispersion mixing mechanism has been identified. It has been shown by using suitable estimates for the turbulent diffusion and on-off shore wave-induced velocity that a reasonable approximation to the measured solute transport processes can be obtained. When the results of previous studies are compared to new measured results, it has been shown that the mixing coefficient obtained follows the general trend of results from the previous studies. It appears that a simplified two-dimensional on-off shore mixing model derived in this study can be used for both laboratory and field studies. It is noticeable that the measured dispersion coefficient from this present study is consistent with predicted results.

A dispersion model has been developed based on hydrodynamic data. Turbulent diffusion and shear dispersion have been quantified for the case of regular oblique waves. The total mixing coefficient obtained from hydrodynamic model is in-line with the tracer data and the existing theoretical model. For the condition investigated (at given location within surfzone), it has been shown that the shear dispersion is the dominant source of mixing.

Through theoretical approximations, it has been demonstrated that within the surfzone, the on-off shore mixing is highly correlated by the effects of the on-off shore velocity, however additional mixing may still be generated by 2D horizontal rotational velocities described by Clark et al. (2010), as the measured mixing was consistently higher than the predicted mixing using the on-off shore shear dispersion mechanism described within this study.

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