## DISTRIBUTION OF WAVE LOADS FOR DESIGN OF CROWN WALLS IN DEEP AND SHALLOW WATER

#### Jørgen Quvang Harck Nørgaard<sup>1</sup>, Thomas Lykke Andersen<sup>1</sup>

This paper puts forward a new method to determine horizontal wave loads on rubble mound breakwater crown walls with specific exceedance probabilities based on the formulae by Nørgaard et al. (2013) as well as presents a new modified version of the wave run-up formula by Van der Meer & Stam (1992). Predictions from the method are compared to measured horizontal wave loads from scaled model tests, and the new method provides results which are in agreement with measured values as long as the wave loads on the crown wall are relatively impulsive. Another aim of the paper has been to compare the displacements of a crown wall exposed to wave loads with different exceedance probabilities in an overload situation (in this case the loads exceeded by 0.1 % and 1/250 of the incident waves). The comparison is made using the assumption that the Eigenfrequency of the crown wall and breakwater is significantly higher than the dynamic wave load impulses on the structure. The relatively small difference in the evaluated exceedance probabilities has a significant influence on the displacement of the monolithic structure. Therefore, probabilistic design tools are recommended. Tools from the present paper will be used in future studies in more detailed investigations on the influence from the load exceedance probability of crown walls.

Keywords: rubble mound breakwater; crown wall; sliding; physical modeling; design wave load

#### INTRODUCTION

Monolithic concrete crown walls are installed on rubble mound breakwater crests to reduce wave overtopping and to provide access roads and installations on the crests. Various failure modes are typically considered when ensuring the stability of these superstructures, such as the stability against sliding, stability against rocking/tilting, and stability against geotechnical slip failure. However, for crown walls Nørgaard et al. (2012) concluded that if the crown wall is able to move freely on the rubble mound core material (without any passive pressure from the core material or filter material) and if the internal strength of the crown wall is sufficient to avoid failure (failure mode a in Fig. 1), the most important failure mode is horizontal sliding (failure mode b in Fig. 1). Predictions on simultaneous horizontal and vertical wave loads are needed to design for this failure mode.

Crown walls are typically designed to remain stable in the design sea state. However, Nørgaard et al. (2012) suggested allowing for small horizontal displacements of the superstructures in an overload situation (failure mode b) in order to reduce structure costs. Additionally, instead of upgrading existing rubble mound breakwater crown walls exposed to increasing wave loading due to climate changes, small displacements can e.g. be accepted. However, the present paper demonstrates that such displacements are a very non-linear function of the design wave height and that probabilistic design is needed.

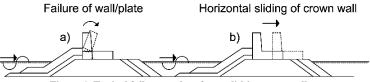


Figure 1. Typical failure modes of monolithic crown walls.

The horizontal wave load evolution on a monolithic structure during a wave run-up event originates from *dynamic loadings* and *reflective loadings*, cf. Martin et al. (1999). The dynamic peak is typically the highest with very short duration and appears due to a fast change in the direction of the run-up bore when the wave hits the structure. The reflective loading comes shortly after the dynamic loading and appears due to the water mass in the run-up bore rushing down the wall after the wave hits the structure. The sizes of the peaks are closely related to the wave steepness and the ratio between the parts of the wall which are protected and un-protected by the armour units.

Nørgaard et al. (2013) presented formulae for prediction of the horizontal dynamic wave loads with low-exceedance probabilities in deep and shallow water wave conditions. The formulae were updated versions of the Pedersen (1996) formulae, which are only valid for deep-water wave conditions. More specifically, Nørgaard et al. (2013) provided predictions for  $F_{H,0.1\%}$ ,  $M_{H,0.1\%}$ , and  $P_{b,0.1\%}$ , which are the horizontal wave load, overturning moment, and the front base pressure respectively (see e.g. Fig. 5c) exceeded by 0.1 % of the incident waves. The prediction formulae by

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Nørgaard et al. (2013) are based on  $H_{0.1\%}$  (i.e. the wave height exceeded by 0.1 % of the incident waves).

Instead of the 0.1 % exceedance level in the formulae by Nørgaard et al. (2013), other design formulae are based on exceedance levels defined by the average of the 1/250 highest waves, such as by Cuomo et al. (2010) for prediction of wave loads on vertical breakwaters. Additionally, Goda (2010) recommended using  $H_{max} = 1.80H_{1/3}$  (approximately corresponding to  $H_{1/250}$  in Rayleigh distributed waves) for prediction of design wave loads for vertical breakwaters, where  $H_{1/3}$  is the significant wave height in the time domain. However, Goda (2010) noted the possibility for obtaining one or two wave heights exceeding  $H_{max} = 1.80H_{1/3}$  during a design storm, but argued that the influence from these exceeding wave heights on the resulting sliding distance of a vertical breakwater would be small due to the highly impulsive nature of these low-exceedance loads. This argument is questioned for crown walls as the load is a very non-linear function of the wave height.

## **Objectives in Present Paper**

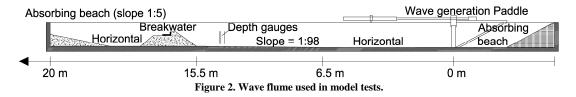
So far, no studies have evaluated the statistical distribution of wave loads on crown walls in deep and shallow water wave conditions. Additionally, no studies have investigated the sensibility of the crown wall stability to the exceedance probability of the design wave load.

This paper has two main objectives. The first objective is to provide a simple tool which can describe the statistical distribution of wave loads on crown walls and which can be used to predict loads with other exceedance probabilities than e.g.  $F_{H,0.1\%}$  predicted by using the design formulae by Nørgaard et al. (2013). Data from 2D small-scale laboratory tests, described in Nørgaard et al. (2013), is used for the verification of the tool.

The second objective is to perform an initial evaluation on the influence from wave loads exceedance probability on the sliding stability of crown walls. A simplified one-dimensional approach described in Burcharth et al. (2008) and Nørgaard et al. (2012) is used for this analysis, together with the model test data by Nørgaard et al. (2012, 2013, and 2014).

## PHYSICAL MODEL TEST STUDY

The physical model test setup in the present paper is further described in Nørgaard et al. (2012, 2013, and 2014). The 2D model tests are performed at Aalborg University in a 25 *m* long and 1.5 *m* wide wave flume. Tests are performed in Froude scale  $\approx$  1:30 where irregular JONSWAP spectra with peak enhancement factor  $\gamma$ =3.3 are generated from a hydraulic piston mode generator using the software AWASYS 6 (2013). At least 1000 waves are generated in each tests series using simultaneous active absorption of reflected waves during the wave generation. Fig. 2 illustrates the wave flume.



Both deep  $(H_{m0}/h \le 0.2)$  and shallow water wave conditions  $(H_{m0}/h > 0.2)$  are evaluated in the model test study and two different rubble mound breakwater geometries are considered. *h* is the water depth at the toe of the structure, and  $H_{m0}$  is the significant wave height in the frequency domain. Table 1 gives the ranges of the tested structure dimensions for the deep and shallow water structures. Fig. 3 illustrates the structural geometry.

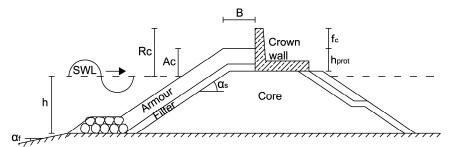


Figure 3. Symbol definitions for specification of tested model geometry.

Table 1. Ranges of structure dimensions for the breakwater model (Nørgaard et al., 2013).								
Test series	<i>R</i> <sub>c</sub> [ <i>m</i> ]	A <sub>c</sub> [ <i>m</i> ]	α <sub>s</sub> [-]	α <sub>f</sub> [-]	B [m]	R <sub>c</sub> /A <sub>c</sub> [-]	A₀⁄B [-]	f_/ A_c [-]
H <sub>m0</sub> /h>0.2	0.20 - 0.29	0.20 - 0.24	1:1.5	1:100	0.24	1.00 – 1.33	0.83 – 1.00	0 - 0.35
<i>H</i> <sub>m0</sub> /h≤0.2	0.10 - 0.19	0.10 - 0.14	1:1.5	1:100	0.17	1.00 – 1.70	0.59 – 0.82	0 - 0.70

All materials used in the breakwater models are quarried rock with specifications given in Table 2.

	Table 2. Material sizes used in the rubble mound breakwater model (Nørgaard et al., 2013).					
Core material		Filter material	Armour material			
	$D_{n50} = 5 mm$	$D_{n50} = 20 \ mm$	$D_{n50} = 40 \ mm$			

Table 3 lists the tested wave conditions in the deep and shallow water tests series.  $T_{m,-I,0}$  is the spectral wave period  $(m_{-I}/m_0)$ .

Table 3. Ranges of target conditions in deep water and shallow water test series (Nørgaard et al., 2013).					
Test series	h [m]	H <sub>m0</sub> [ <b>m</b> ]	T <sub>m-1,0</sub> [s]	H <sub>m0</sub> /h [-]	A <sub>c</sub> /H <sub>m0</sub> [-]
H <sub>m0</sub> /h>0.2	0.300 - 0.360	0.150 - 0.180	1.826	0.500	1.00 - 1.600
<i>H</i> <sub>m0</sub> /h≤0.2	0.500 - 0.560	0.100	1.826	0.179 – 0.200	0.800 - 1.400

20 to 30 Drück PMP UNIK pressure transducers with diameters of 20 mm are installed in the crown wall model, depending on the tested breakwater geometry in the specific test. The Drück transducers have high Eigenfrequency with correct response of up to 5 kHz. Signals from the transducers are initially sampled with 1500 Hz and hereafter low-pass filtered to 250 Hz and 100 Hz for the horizontal and vertical pressures, respectively. The appropriate sampling frequencies in the setup are based on the pressure peak celerity in relation to the spatial resolution of the pressure transducers by following the recommendations in Lamberti et al. (2011). The positioning of the transducers and the integration of pressures are described in details in Nørgaard et al. (2012, 2013).

Fig. 4 shows a photo of the positioning of the pressure transducers and the moveable part of the crown wall.



Figure 4. (left) Front view of breakwater, (right) Pressure transducers in fixed part of crown wall. (Nørgaard et al., 2012)

## ANALYSIS AND REMAINING CONTENT OF PAPER

This paper follows four steps to evaluate the influence from the design wave exceedance probability on the sliding stability of a monolithic rubble mound breakwater crown wall. Initially, the incident wave height distribution (*a* in Fig. 5) is used to describe the statistical distribution of run-up heights (*b* in Fig. 5), which again is used to describe the statistical distribution of wave loads on the crown wall (*c* in Fig. 5). In the end, the wave loads with different exceedance probabilities are used to evaluate the stability of the monolithic crown wall (*d* in Fig. 5). The analysis uses measured incident wave heights and wave loads from the physical model tests.  $\alpha$  in Fig. 5 is the exceedance probability level in relation to the incident waves (as an example  $R_{u,\alpha}$  is the wave run-up height exceeded by  $\alpha$ % of the incident waves).

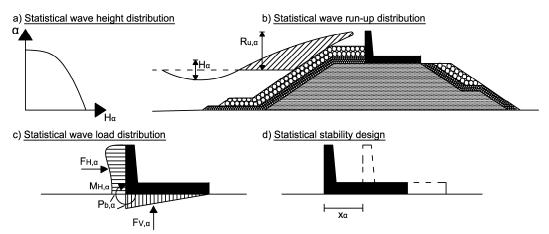


Figure 5. Illustration of analysis performed in the following sections where the wave height distribution is used to predict the statistical distribution of run-up heights, which again is used to predict the wave loads and the stability of the crown wall.

#### WAVE HEIGHT DISTRIBUTION IN DEEP AND SHALLOW WATER

Design wave loads on coastal protection structures are closely related to the low-exceedance wave heights during a design storm. Typically, the design storm conditions are defined based on the significant wave height and the peak period at a deep water location. It is thus often required to translate these wave parameters to the toe of the structure in shallow water and transform the parameters to low-exceedance design conditions, such as  $H_{1/250}$  or  $H_{0.1\%}$ , cf. Nørgaard et al. (2013) and Goda (2010).

Nørgaard (2013) evaluated the performance of different existing state-of-art wave height distributions by Longuet-Higgins (1952), Goda (2010), Battjes & Groenendijk (2000), and Glukhovskii (1966).

Longuet-Higgins (1952) suggested that wave heights in deep water  $(H_{1/3}/h\leq 0.2)$  are Rayleigh distributed, which however is not the case for shallow water wave conditions where the largest wave heights in the spectrum are depth limited.

Glukhovskii (1966) suggested including the depth-limitation effects in the largest wave heights in shallow water by modifying the Rayleigh distribution to include the mean wave height to water depth ratio  $H_m/h$ .

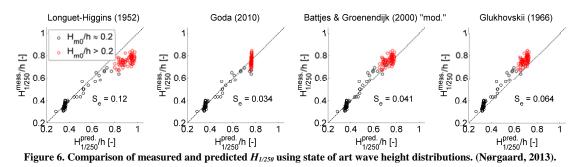
Battjes & Groenendijk (2000) suggested a so-called composite Weibull-distribution, i.e. a Rayleigh distribution above a certain transition exceedance probability and a Weibull distribution for lower wave height exceedance probabilities, which are affected by depth-limitation. Caires & Gent (2012), however, observed that the Battjes & Groenendijk (2000) study on deep water did not converge towards the Rayleigh distribution but provided an overestimation. Caires & Gent (2012) thus recommended using the original Battjes & Groenendijk (2000) distribution if the predictions were not exceeding those of the Rayleigh distribution, and otherwise use the predictions from the Rayleigh distribution.

Goda (2010) suggested a so-called breaker index, which can be used to directly propagate the waves from deep water towards the breaker zone.

The different wave height distributions are evaluated in Fig. 6 for both deep and shallow water wave conditions, where predictions from the wave height distributions are compared to the measurements from the model tests by Nørgaard et al. (2012, 2013, and 2014). For this case solely the  $H_{1/250}$  wave heights are evaluated. The so-called standard error  $S_e$  in Eq. 1 is used for comparison of the performances of the different methods where *meas*. are the measured values, *pred*. are the predicted values, and *n* is the number of values.

$$S_e = \sqrt{\frac{1}{\nu} \sum_{i=1}^{n} (meas_i - pred_i)^2}$$

$$\nu = n - 2$$
(1)



As seen, relatively good performances are obtained in deep and shallow water wave conditions from both the Goda (2010) breaker index method and the Battjes & Groenendijk (2000) distribution. The Rayleigh distribution significantly over-predicts the low-exceedance wave heights in shallow water wave conditions.

### WAVE RUN-UP DISTRIBUTION IN DEEP AND SHALLOW WATER

One of the governing terms in the design formulae for prediction of wave loads on crown walls by Nørgaard et al. (2013) is the so-called wave run-up height exceeded by 0.1 % of incident waves,  $R_{u,0.1\%}$ . Nørgaard et al. (2013) used the prediction formula by Van der Meer & Stam (1992) for prediction of  $R_{u,0.1\%}$ . The formula provides predictions of run-up heights with exceedance probabilities of 0.13 %, 1 %, 2 %, 5 %, and 10 % on permeable rough straight rock slopes in conditions with head-on waves and Rayleigh distributed wave heights. Nørgaard et al. (2013) assumed that  $R_{u,0.1\%} \simeq R_{u,0.13\%}$ .

The idea in the present paper is to couple the wave height with given exceedance probability,  $H_a$ , to  $R_{u,a}$ , and the wave load with the same exceedance probability,  $F_{H,a}$ .

## Prediction of Wave Run-up Heights on Rubble Mound Breakwaters in Deep Water

Van der Meer & Stam (1992) suggested the formula in Eq. 2 for estimation of the wave run-up height,  $R_{u,Meer,\alpha}$ , where  $T_m$  is the mean wave period in the time domain, and  $a_{Ru,\alpha}$ ,  $b_{Ru,\alpha}$ ,  $c_{Ru,\alpha}$ ,  $d_{Ru,\alpha}$  are empirical coefficients, which are given in Table 4.

Table 4. Coefficients for estimation of wave run-up height on Run-up. (Van der Meer & Stam, 1992)					
α [%]	<b>a</b> <sub>Ru,α</sub> [-]	<b>b</b> <sub>Ru,а</sub> [-]	С <sub><i>Ru,а</i> [-]</sub>	<b>d</b> <sub>Ru,α</sub> [-]	
0.13	1.12	1.34	0.55	2.58	
1	1.01	1.24	0.48	2.15	
2	0.96	1.17	0.46	1.97	
5	0.86	1.05	0.44	1.68	
10	0.77	0.94	0.42	1.45	

$$\frac{R_{u,Meer,\alpha}}{H_{1/3}} = \begin{cases} a_{Ru,\alpha} \cdot \xi_{m0} & \xi_{m0} \le 1.5 \\ b_{Ru,\alpha} \cdot \xi_{m0}^{\ c_{Ru}} & \xi_{m0} > 1.5 \end{cases} \quad \text{where} \quad \xi_{m0} = \frac{\tan \alpha_s}{\sqrt{\frac{2\pi}{g} \cdot \frac{H_{1/3}}{T_m^2}}}, \quad \frac{R_{u,Meer,\alpha}}{H_{1/3}} \le d_{Ru,\alpha} \tag{2}$$

#### Prediction of Wave Run-Up Height on Rubble Mound Breakwaters in Shallow Water

Nørgaard et al. (2013) suggested modifying the run-up formula by Van der Meer & Stam (1992) to include the effects of depth-limited wave conditions. The modification is initially performed based on the conclusion by Kobayashi et al. (2008) that run-up levels are Rayleigh distributed in deep water wave conditions,  $H_{1/3}/h \le 0.2$ , (i.e. the distribution of run-up levels follows the incident wave height distribution). Nørgaard et al. (2013) demonstrated that the run-up levels also follow the incident wave height distribution in shallow water wave conditions,  $H_{1/3}/h \ge 0.2$ . Eq. 3 presents the modified  $R_{u,Nørgaard,0.1\%}$ .

$$\frac{R_{u,Norgaard,0.1\%}}{H_{0.1\%}} = \begin{cases} a_{Ru,0.1\%} \cdot \left(\frac{H_{1/3}}{H_{0.1\%}}\right)_{Rayleigh} \cdot \xi_{m0} & \xi_{m0} \leq 1.5\\ b_{Ru,0.1\%} \cdot \left(\frac{H_{1/3}}{H_{0.1\%}}\right)_{Rayleigh} \cdot \xi_{m0} & \xi_{m0} > 1.5 \end{cases}, \quad \frac{R_{u,Norgaard,0.1\%}}{H_{1/3}} \leq d_{Ru,0.1\%} \tag{3}$$

The depth-limitation effects are included in Eq. 3 based on the ratio  $H_{1/3}/H_{0.1\%}$  (using the Rayleigh distribution), which is significantly reduced in shallow water wave conditions compared to deep water wave conditions. As mentioned earlier,  $H_{1/3}/H_{0.1\%}$  can e.g. be predicted with relatively good accuracy using the Battjes & Groenendijk (2000) wave height distribution. Fig. 7 presents the influence from the significant wave height to water depth ratio,  $H_{1/3}/h$ , on the wave run-up height using Eq. 3. As seen,  $R_{u,0.1\%}/H_{1/3}$  is significantly reduced in shallow water compared to deep water wave conditions.

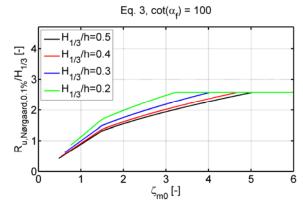


Figure 7. *R<sub>u,Norgaard,0.1%</sub>* in deep to shallow water wave conditions predicted using Eq. 3.

The modified  $R_{u,0.1\%}$  in Eq. 3 was used in Nørgaard et al. (2013) to include the effects of shallow water wave conditions in the design formula for wave loads on the crown wall.

# Suggested Statistical Distribution of Wave Run-Up Heights on Rubble Mound Breakwaters in Deep and Shallow Waters

Eq. 4 shows the statistical distribution of run-up heights. The distribution is based on Eq. 2 and the coefficients in Table 4 by Van der Meer & Stam (1992).

$$\frac{R_{u,Norgaard,\alpha}}{H_{0.1\%}} = \begin{cases}
a_{Ru,\alpha} \cdot \left(\frac{H_{1/3}}{H_{0.1\%}}\right)_{Rayleigh} \cdot \xi_{m0} & \xi_{m0} \leq 1.5 \\
b_{Ru,\alpha} \cdot \left(\frac{H_{1/3}}{H_{0.1\%}}\right)_{Rayleigh} \cdot \xi_{m0}^{c_{Ru,\alpha}} & \xi_{m0} > 1.5
\end{cases}, \quad \frac{R_{u,Norgaard,\alpha}}{H_{1/3}} \leq d_{Ru,\alpha} \quad (4)$$

Alternatively,  $R_{u,\alpha}$  can be predicted using Eq. 5 since, as demonstrated, the run-up levels follow the incident wave height distribution.  $H_{\alpha}$  is the incident wave height exceeded by  $\alpha$  of all wave heights. Unlike Eq. 4, this method is not limited to the specific exceedance probabilities for *a*, *b*, *c*, and *d* in Table 4.

$$R_{u,N\sigma rgaard,\alpha} = R_{u,N\sigma rgaard,0.1\%} \frac{H_{\alpha}}{H_{0.1\%}}$$
(5)

Fig. 8 illustrates the variations of  $R_{u,\alpha}$  obtained from Eq. 4 and Eq. 5 as function of  $\zeta_{m0}$ , and in case of shallow water wave conditions.

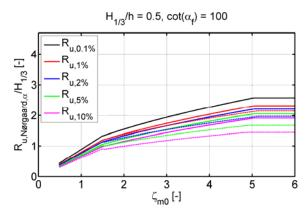


Figure 8. Modified  $R_u$  in shallow water wave conditions with various exceedance probabilities,  $\alpha$ , predicted using Eq. 4 (*dashed line*) and Eq. 5 (*solid line*).

## WAVE LOAD DISTRIBUTION IN DEEP AND SHALLOW WATER

The formulae by Nørgaard et al. (2013) are given in Eqs. 6, 7, and 8 for prediction of horizontal wave loads,  $F_{H,0.1\%}$ , wave-induced horizontal overturning moments,  $M_{H,0.1\%}$ , and base front corner pressures,  $P_{b,0.1\%}$ . Fig. 9 illustrates the volumes  $V_1$  and  $V_2$  of the run-up wedges in (6). *a*, *b*, *d*, *e*<sub>1</sub>, and *e*<sub>2</sub> are empirical coefficients calibrated in Nørgaard et al. (2013).

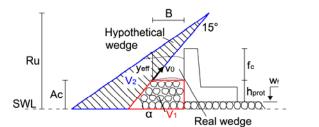


Figure 9. Volumes  $V_1$  and  $V_2$  in the assumed run-up wedges in (6). (Nørgaard et al., 2013)

$$F_{Hu,0.1\%} = a \cdot \sqrt{\frac{L_{m0}}{B}} \cdot p_m \cdot y_{eff} \cdot b , \qquad a = 0.21, b = 1.0$$

$$F_{HI,0.1\%} = \frac{1}{2} \cdot a \cdot \sqrt{\frac{L_{m0}}{B}} \cdot p_m \cdot V \cdot h_{prot}$$

$$F_{H,0.1\%} = F_{Hu,0.1\%} + F_{HI,0.1\%} = a \cdot \sqrt{\frac{L_{m0}}{B}} \cdot \left( p_m \cdot y_{eff} \cdot b + \frac{p_m}{2} \cdot V \cdot h_{prot} \right)$$

$$p_m = g \cdot \rho_w \cdot (R_{u,0.1\%} - A_c)$$

$$V = \begin{cases} \frac{V_2}{V_1} & \text{for } V_2 < V_1 \\ 1 & \text{for } V_2 \ge V_1 \end{cases}$$
(6)

$$M_{H,0.1\%,\text{mod.}} = \left(h_{prot} + \frac{1}{2} \cdot y_{eff} \cdot e_2\right) \cdot F_{Hu,0.1\%} + \frac{1}{2} \cdot h_{prot} \cdot F_{HI,0.1\%} \cdot e_1$$

$$e_1 = 0.95, \ e_2 = 0.40$$
(7)

$$P_{b,0.1\%} = d \cdot V \cdot g \cdot \rho_{w} \cdot (R_{u,0.1\%} - A_{c}) \qquad , \qquad d = 1.0$$
(8)

The formulae by Nørgaard et al. (2013) are limited to the structural dimensions and wave conditions in Table 5.

Table 5. Investigated parameter ranges in modified design formulae for $F_{H,0.1\%}$ , $M_{H,0.1\%}$ , and $P_{b,0.1\%}$ . (Nørgaard et al., 2013)				
Parameters	Ranges $f_c = 0$	Ranges <i>f<sub>c</sub></i> > 0		
ξm	2.3 - 4.9	3.31 - 4.64		
H <sub>m0</sub> /A <sub>c</sub>	0.5 - 1.63	0.52 - 1.14		
R√Ac	0.78 - 1	1 - 1.7		
A₀⁄B	0.58 - 1.21	0.58 - 1.21		
H <sub>m0</sub> /h	0.19 – 0.55	0.19 – 0.55		
H <sub>m0</sub> /L <sub>m0</sub>	0.018 - 0.073	0.02 - 0.041		

Statistical Distribution of Wave Loads on Rubble Mound Breakwater Crown Walls in Deep and Shallow Waters

Predicted  $F_{H,\alpha}$  and  $M_{H,\alpha}$  using the Eqs. 6 and 7 formulae are compared to the measured loads from the model tests by Nørgaard et al. (2012, 2013, and 2014) in Figs. 10 and 11, respectively.  $R_{u,\alpha}$  is predicted using Eqs. 4 and 5. For comparison, the performances of the two suggested methods for prediction of  $R_{u,\alpha}$  are evaluated based on  $S_e$ , calculated using Eq. 1. The 0.1 % exceedance probability approximately corresponds to the highest wave in the evaluated test series.

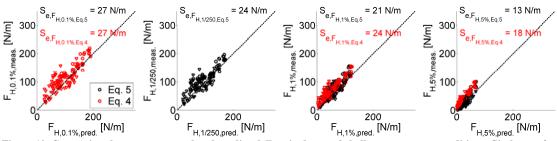


Figure 10. Comparison between measured and predicted  $F_{H,a,\%}$  in deep and shallow water wave conditions. Circles are for deep water wave conditions and triangles are for shallow water wave conditions.

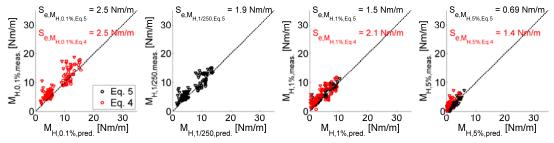


Figure 11. Comparison between measured and predicted  $M_{H,a\%}$  in deep and shallow water wave conditions. Circles are for deep water wave conditions and triangles are for shallow water wave conditions.

As seen, the two methods for prediction of  $R_{u,\alpha}$  both provide relatively good results for the evaluated exceedance probabilities. The tendency is, however, that  $R_{u,\alpha}$  obtained from Eq. 5 gives results which are in slightly better agreement with the measured values compared to Eq. 4 (i.e. generally smaller  $S_e$ ). The method in Eq. 5 further has the advantage that it is not limited to the specific exceedance probabilities in Table 4. Therefore, it is advised to predict  $R_{u,\alpha}$  by using Eq. 5.

## Limitations to Suggested Statistical Distribution of Wave Loads on Crown Walls based on Lowexceedance Design Formula

Since the formulae by Nørgaard et al. (2013) are originally providing wave loads with 0.1 % exceedance levels (i.e. load events which are dominated by the dynamic load peak,  $F_{dyn}$ , cf. Martin et al. (1999)), the formulae may not provide reliable predictions for large exceedance levels where only the reflective load peak is present. To illustrate this, the load incidents with various exceedance probabilities are plotted in Fig. 12 from a test with fully protected crown wall front  $f_c=0$ , c.f. Fig. 3, and wave steepness  $s = H_m/L_{m-1,0} = 0.028$ . As seen, especially the dynamic load peak is visible in the low-

exceedance load events, which however decreases for increasing exceedance probability due to decreasing wave steepness.

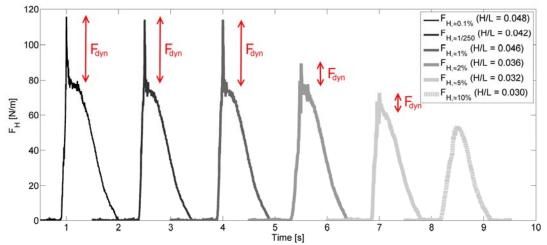


Figure 12. Dynamic pressures in pressure peaks with different exceedance probabilities during a specific test with fully protected crown wall,  $f_c = 0$ , and wave steepness  $s = H_m 0/L_{-l,0} = 0.028$ .

In the specific test case in Fig. 12, the dynamic peaks,  $F_{dyn}$ , appear for the load incidents with up to  $\approx 5\%$  exceedance probability. Measured and predicted loads (using the formulae in Eqs. 5, 6, 7 and 8) are compared in Fig. 13 with  $\alpha = 10\%$ . As seen, the general tendency is that the wave loads are overpredicted since they are based on design formulae which are initially constructed to predict dynamic low-exceedance design loads.

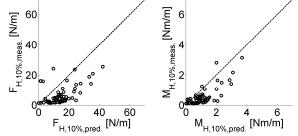


Figure 13. Comparison between measured and predicted wave loads in shallow water wave conditions with  $\alpha = 10\%$ .

## STABILITY OF MONOLOITHIC CROWN WALLS

Nørgaard et al. (2012) compared the predicted sliding of a monolithic crown wall structure obtained from a simple one-dimensional dynamic model based on integration of the equation of motion as well as from a Finite-Element model to the measured sliding distance obtained from model tests. The comparison showed that the results from the simplified analysis were in agreement with the results from the sophisticated Finite Element model due to the relatively high Eigenfrequency of the monolithic crown wall structure. Thus, it was concluded that a model with only a single degree of freedom (horizontal sliding) was sufficient to model the displacement of such structures.

The one-dimensional model by Nørgaard et al. (2012) is given in Eq. 9, where  $M_{crownwall}$  is the mass of the crown wall reduced for buoyancy,  $M_{added}$  is the added mass on the structure, F(t) is the reduced load time series acting on a fixed structure,  $F_{H}(t)$  is the horizontal load time series measured on a fixed structure,  $F_{V}(t)$  is the vertical load time series acting on a fixed structure, x is the horizontal displacement of the crown wall, G is the dead load of the crown wall corrected for buoyancy, g is the failure function for horizontal displacement, and  $\mu$  is the friction coefficient between crown wall and foundation material.

$$F(t) = \left(M_{crownwall} + M_{added}\right) \frac{d^2 x}{dt^2} = \left(-\left(G - F_V(t)\right)\mu + F_H(t)\right) \cdot \lambda_{sliding}(t) \cdot \lambda_{dyn}(t) = -g$$
(9)

The structure is sliding when  $g \leq 0$ .

 $\lambda_{sliding}$  and  $\lambda_{dyn}$  are reduction factors which are related to the increasing crest width *B* when the structure slide backwards and to the reduced relative velocity between the run-up bore and the crown wall where the loads are measured on a fixed structure, respectively. The two reduction factors  $\lambda_{sliding}$  and  $\lambda_{dyn}$  are derived by Nørgaard et al. (2012) based on comparison between the predicted and measured horizontal displacements in model tests. The reduction factors are given in Eqs. 10 and 11 where *B* is the crest width during sliding,  $B_{initial}$  is the initial crest width ( $B_{initial} = B(t=0)$ ),  $u_{bore}$  is the horizontal velocity of the run-up bore, and  $u_{crowwall}$  is the horizontal velocity of the crown wall.

$$\lambda_{sliding}\left(t\right) = \sqrt{\frac{B_{initial}}{B(t)}}, \quad B(t) = B_{initial} + x(t), \quad B_{initial} = B(0)$$
(10)

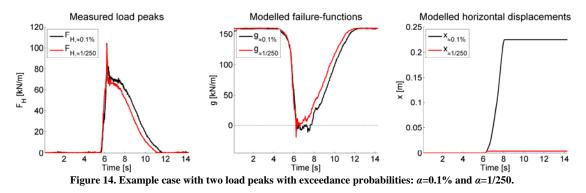
$$\lambda_{dyn}(t) \approx \frac{\left(u_{bore} - u_{crownwall}\right)^2}{u_{bore}^2} \tag{11}$$

The sliding distance of a load event  $(x(t_2) - x(t_1))$  is determined based on double time integration in Eq. 12 where  $t_1$  is the instance where destabilizing loads are bigger than stabilizing forces and  $t_2$  is the instance where the structure has come back to rest.

$$x(t) = x(t_2) - x(t_1) = \int_{t_1}^{t_2} \int_{t_1}^{t} \frac{1}{M_{crownwall} + M_{added}} \cdot F(t) dt dt$$
(12)

#### Evaluation of Wave Load Exceedance Probability for Stability Design of Crown Walls

Under the hypothesis that the Eigenfrequency of the crown wall is much higher than the dynamic wave load impulses, the sliding model in Eq. 12 is used to evaluate the influence from the exceedance probability in the design wave load when performing stability design of the crown wall. The deep water structure  $(H_{m0}/h \le 0.2)$  in Table 1 is evaluated with a crown wall and base slab thickness of 1 *m*. The measured  $F_{H,\approx 0.1\%}$  and  $F_{H,\approx 1/250}$  together with their corresponding vertical wave loads,  $F_V$ , are evaluated in the stability analysis. The two considered load events are adjusted so the maximum loads in the peaks appear at the same time. A friction factor of  $\mu = 0.6$  is evaluated in the example, which is assumed unchanged in the static and dynamic case. Fig. 14 illustrates the considered load peaks and their corresponding displacements in prototype scale. As seen, there is a significant difference in the modelled displacements of the two load series.



It should be mentioned that the influence from the design load exceedance probabilities on the displacements in Fig. 14 is solely evaluated based on the maximum load in the two considered load peaks (the dynamic peaks). However, also the reflective peaks are different (cf. Fig. 14), which significantly influences the displacement.

To obtain a better estimate on the influence from the exceedance probability on the stability of the crown wall, the same structure is evaluated again but with a load time series from 43 model test cases with deep water wave conditions (ranges are given in Table 3). Fig. 15 illustrates the comparison

between  $x_{\approx 0.1\%}$  and  $x_{\approx 1/250}$  (i.e. displacements corresponding to  $F_{H,\approx 0.1\%}$  and  $F_{H,\approx 1/250}$ ) in prototype scale. Failure is assumed for x>1~m (i.e. relatively small displacements as suggested by Nørgaard et al. (2012)). As seen, the horizontal displacements are significantly higher when  $\alpha=0.1\%$  than  $\alpha=1/250$ . If the crown wall is initially designed based on  $F_{H,1/250}$ , as suggested by Goda (2010) for caisson breakwaters, failure is obtained for  $F_{H,0.1\%}$ . This indicates that the design safety factor for stability should be higher for crown walls than for caisson breakwaters.

It is recommended applying the exceedance probability corresponding to the maximum wave height in the design storm or adopting a fully probabilistic design approach.

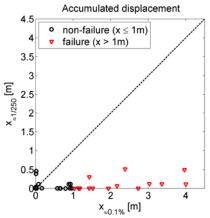


Figure 15. Comparison of displacements from loads with different exceedance probabilities:  $\alpha$ =0.1% and  $\alpha$ =1/250.

It should be mentioned that the load peaks evaluated in the present paper are more impulsive compared to the considered wave load in Nørgaard et al. (2012). Ongoing investigations are evaluating the influence of the wave load exceedance probability (and thereby the dynamic wave loads) on the displacements of crown walls using physical models and more sophisticated analytical and numerical models.

#### CONCLUSIONS

Different failure modes can be considered when designing a monolithic rubble mound breakwater crown wall to remain stable in the design conditions. However, Nørgaard et al. (2012) concluded that horizontal sliding of the crown wall is the most important failure mode if the crown wall is able to move freely on the rubble mound core material without any influence from passive pressure from core material or filter material. Nørgaard et al. (2012) further suggested allowing for relatively small displacements of the crown wall in the overload conditions in order to reduce initial construction costs.

The design condition for wave loads on crown walls is not solely defined based on the return period but the load exceedance probability is also of high importance. Different researchers have suggested different design wave load exceedance probabilities such as the 0.1 % exceedance probability by Nørgaard et al. (2013) for wave loads on crown walls and the 1/250 exceedance probability by Cuomo et al. (2010) and Goda (2010) for wave loads on vertical walls.

The present paper has suggested a new method for predicting wave loads on monolithic crown walls with other exceedance probabilities than 0.1 % (as suggested by Nørgaard et al. (2013)) and has evaluated the influence of the load exceedance probability on the displacement of the crown wall in the design conditions.

Since wave loads on crown walls are fully correlated with the run-up height, the present paper suggests further modification of the already modified run-up formula by Nørgaard et al. (2013) (initially proposed by Van der Meer & Stam (1992)) to predict run-up heights with specific exceedance probabilities. Nørgaard et al. (2013) initially modified the run-up formula to include the effects of shallow water wave conditions. The modification in the present paper is performed by coupling the wave run-up height with a specific exceedance probability to the incident wave heights with the same exceedance probability. The wave heights with specific exceedance probabilities in deep and shallow water wave conditions can e.g. be predicted with relatively good accuracy using the Battjes & Groenendijk (2000) distribution.

Predicted wave loads with specific exceedance probabilities using the formulae by Nørgaard et al. (2013) and the modified run-up formula were compared to results from the model tests described in

Nørgaard et al. (2012, 2013, and 2014). Good agreement was obtained between measured and predicted wave loads as long as dynamic loads were obtained on the crown wall (exceedance probabilities up to around  $\alpha = 5\%$  for the evaluated load time series in the present case).

Under the assumption of a high Eigenfrequency of the crown wall compared to the wave load impulses, the influence from the load exceedance probability on the stability of the crown wall was studied by applying a simple one-dimensional model based on the integration of the equation of motion. Measured wave load time series with 0.1 % and 1/250 exceedance probabilities were evaluated, and significant differences were obtained from the comparison between predicted sliding distances. This clearly demonstrates the importance of the applied exceedance probability in the design of the crown wall structure. Generally, it is recommended to apply the exceedance probability corresponding to the maximum wave height in the design storm or to adopt a fully probabilistic design approach.

Ongoing study is aiming at deriving a formula for prediction of non-impulsive load peaks for design of structures in long waves and is aiming at providing more detailed knowledge and design guidelines on the choice of wave load exceedance probability for stability design of crown wall superstructures.

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