A Multiple-Sized Transport Formula for Nonuniform Sediments under Current and Waves

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INTRODUCTION

The influence of nonuniform sediment properties on coastal processes is commonly underestimated due to the difficulty in characterizing and quantifying these types of sediments. The tendency of many empirical formulas of coastal sediment transport, such as Bailard (1981), Van Rijn (1984), Dibajnia and Watanabe (1992), Ribberink (1998), and Camenen and Larson (2007), is to assume uniform or homogeneous sediments (e.g. a well-sorted fine sand). However, nonuniform sediment transport exhibits difference from uniform sediment, even when the mean grain size is the same for both cases. The hiding, exposure, and armoring among different size classes in the nonuniform bed material may significantly affect sediment transport, morphological change, bed roughness, wave dissipation, etc.

In this study, the Wu et al. (2000) formula which was developed for river sedimentation is extended to multiple-sized sediment transport with current and waves for coastal applications. Methods are developed to determine the bed shear stress due to waves only and combined current and waves, and in turn applied to compute the bed-load and suspended-load transport rates.

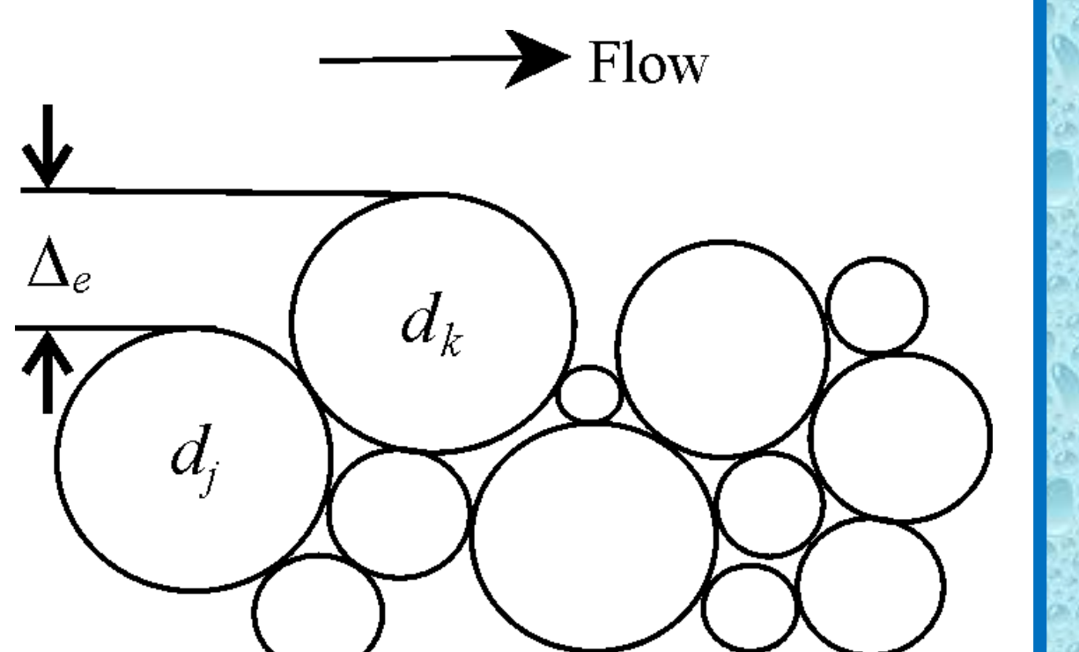
WU ET AL. (2000) FORMULA UNDER CURRENT ONLY

❑ Bed-load Transport Rate

$$\frac{q_{bk}}{p_{bk}\sqrt{(\gamma_s/\gamma-1)gd_k^3}} = 0.0053\left(\frac{\tau'_b}{\tau_{cri,k}}-1\right)^{2.2}$$

❑ Suspended-load Transport Rate

$$\frac{q_{sk}}{p_{bk}\sqrt{(\gamma_s/\gamma-1)gd_k^3}} = 0.0000262\left[\left(\frac{\tau}{\tau_{cri,k}}-1\right)\frac{U}{\omega_{sk}}\right]^{1.74}$$



- q_{bk}, q_{sk}: bed-load and suspended-load transport rates of sediment size class k (m^2s^{-1});
- d_k: representative diameter of size class k of the sediment mixture;
- p_{bk}: fraction of sediment size class k in the bed material;
- g: gravitational acceleration;
- γ_s, γ: specific weights of sediment and water, respectively;
- τ: shear stress on the wetted perimeter of the cross-section including bed and banks;
- τ'_b: bed shear stress corresponding to grain roughness;
- $\tau_{cri,k}$: critical shear stress for the incipient motion of sediment size k on the bed;
- ω_{sk}: settling velocity;
- U: depth-averaged flow velocity.

❑ Hiding and Exposure Effects

$$\frac{\tau_{cri,k}}{(\gamma_s-\gamma)d_k} = \Theta_{cri}\left(\frac{p_{ek}}{p_{hk}}\right)^{-m} \qquad p_{ek} = \sum_{j=1}^{N} p_{bj}\frac{d_k}{d_k+d_j} \qquad p_{hk} = \sum_{j=1}^{N} p_{bj}\frac{d_j}{d_k+d_j}$$

- Θ_{cri}: critical Shield number for incipient motion of the mean or median diameter of bed material;
- m: empirical parameter;
- p_{hk}, p_{ek}: hiding and exposure probabilities of particles d_k in the bed material;
- N: total number of particle size classes in the nonuniform sediment mixture.

BED SHEAR STRESS UNDER CURRENT AND WAVES

To apply the Wu et al. formula to sediment transport under current and waves in coastal context, the most important step is to determine the bed shear stress.

❑ Bed Shear Stress due to Current

$$\tau_{b,c} = \frac{\rho g n^2}{h^{1/3}}U_c^2 \qquad n = \frac{h^{1/6}}{18\log(12h/k_s)} \qquad k_s = k'_s + k''_s$$

$$\tau'_{b,c} = \frac{1}{2}\rho f'_c U_c^2 \qquad f'_c = 2\left(\frac{n'}{n}\right)^{1.5}\frac{gn^2}{h^{1/3}} \qquad n' = d_{50}^{1/6}/20$$

- $\tau_{b,c}$, $\tau'_{b,c}$: total and grain bed shear stresses due to current;
- U_c: depth-averaged current velocity;
- h: water depth;
- n: Manning's roughness coefficient;
- k_s: equivalent roughness height
- k'_s, k''_s: grain and bed form roughness (Soulsby 1997);
- ρ: fluid density;
- f'_c: current friction coefficient for grain bed shear stress;
- n': grain Manning's roughness coefficient;
- d_{50}: median diameter of the sediment mixture.

❑ Bed Shear Stress due to Waves (Soulsby 1997)

$$\tau_{b,wm} = \frac{1}{4}\rho f_w U_w^2 \qquad f_w = 0.237\left(A_w/k_s\right)^{-0.52}$$

$$\tau'_{b,wm} = \frac{1}{4}\rho f'_w U_w^2 \qquad f'_w = 0.237\left(A_w/k'_s\right)^{-0.52}$$

- $\tau'_{b,wm}$, τ'_{bwm}: mean total and grain bottom wave stresses averaged over a wave cycle;
- f_w, f'_w: total and grain bed wave friction coefficients;
- A_w: bottom wave excursion $U_wT_w/2\pi$.

Asymmetric waves: $u_w(t) = U_w(\cos\omega t + r_w\cos 2\omega t)$

$$\tau'_{b,wm.on} = \frac{1}{2}\rho f'_w\frac{U_w^2}{2}\left(1+r_w^2+\frac{13}{6}r_w\frac{\sin a_c}{a_c}+\frac{1}{6}\frac{\sin 2a_c}{2a_c}\right)$$

$$\tau'_{b,wm.off} = \frac{1}{2}\rho f'_w\frac{U_w^2}{2}\left(-1-r_w^2+\frac{13}{6}r_w\frac{\sin a_t}{a_t}-\frac{1}{6}\frac{\sin 2a_t}{2a_t}\right)$$

- $\tau'_{b,wm,on}$, $\tau'_{b,wm,off}$: bed grain shear stresses in the crest and trough half wave cycles; $r_w = u_{w,max}/U_w - 1$; $a_c = \pi T_{wc}/T_w$; $a_t = \pi T_{wt}/T_w$.

❑ Bed Shear Stress due to Current and Waves

❑ Grain shear stress for bed-load formula

$$\tau'_{b,on} = \frac{1}{2}\rho f'_{cw}\left(U_c^2+U_{wm,on}^2+2U_cU_{wm,on}\cos\varphi\right)$$

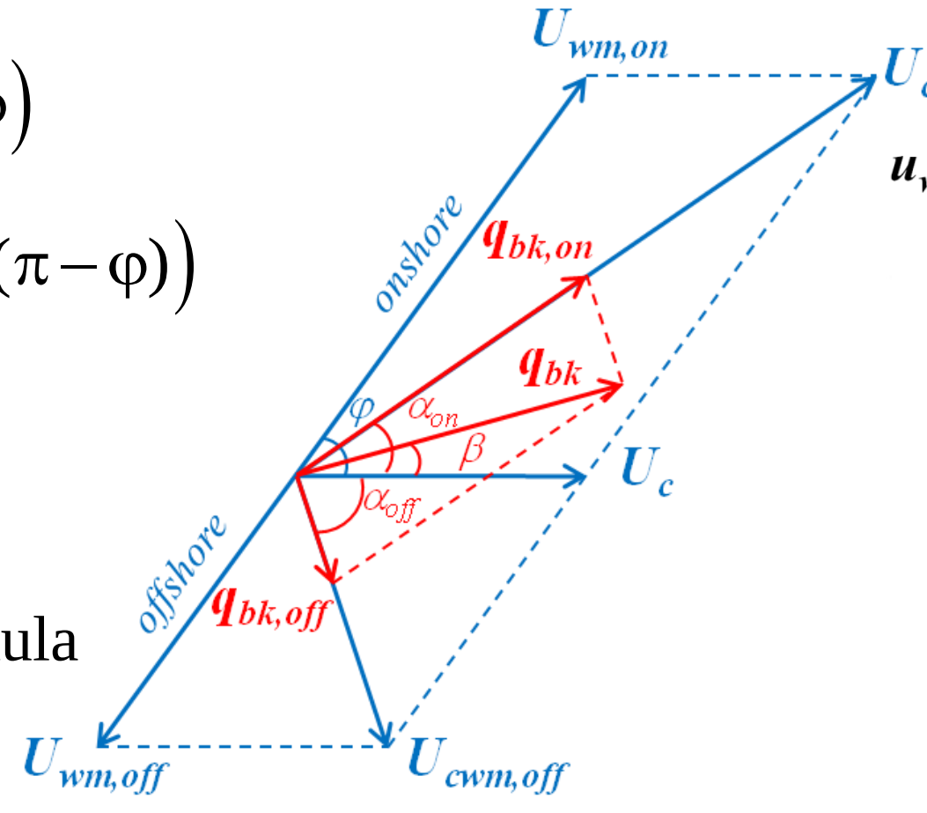
$$\tau'_{b,off} = \frac{1}{2}\rho f'_{cw}\left(U_c^2+U_{wm,off}^2+2U_cU_{wm,off}\cos(\pi-\varphi)\right)$$

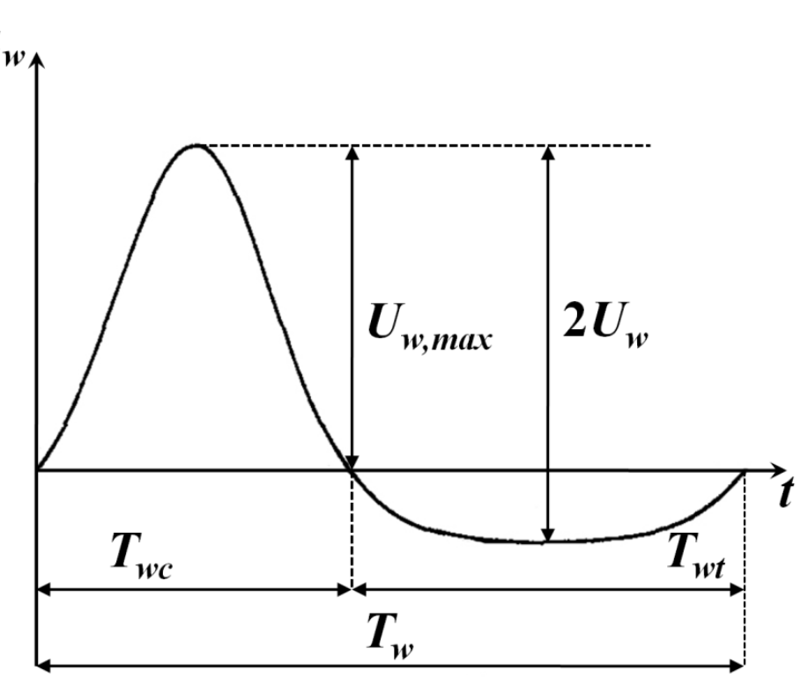
$$f'_{cw} = X_u f'_c + (1-X_u)f'_w$$

$$X_u = U_c^2\big/\left(U_c^2+0.5U_w^2\right)$$

❑ Total shear stress for suspended-load formula

$$\tau_b = \sqrt{\tau_{b,c}^2+\tau_{b,wm}^2+2\tau_{b,c}\tau_{b,wm}\cos\varphi}$$





- $\tau'_{b,on}$, $\tau'_{b,off}$: onshore and offshore resultant grain shear stresses due to the combined current and waves;
- f'_{cw}: friction coefficient for grain bed shear stress under combined current and waves;
- $U_{wm,on}$, $U_{wm,off}$: root-mean-square values of the wave velocity over the onshore and offshore half cycles;
- φ: angle between current and waves during onshore half cycle;
- τ_b: total shear stresses due to the combined current and waves.

MULTIPLE-SIZED SEDIMENT TRANSPORT RATE UNDER CURRENT AND WAVES

❑ Bed-load Transport Rate

The onshore and offshore bed-load transport rates $q_{bk,on}$ and $q_{bk,off}$ are:

$$q_{bk,on} = 0.0053p_{bk}\sqrt{(\gamma_s/\gamma-1)gd_k^3}\left(\frac{\tau'_{b,on}}{\tau_{cri,k}}-1\right)^{2.2} \qquad q_{bk,off} = 0.0053p_{bk}\sqrt{(\gamma_s/\gamma-1)gd_k^3}\left(\frac{\tau'_{b,off}}{\tau_{cri,k}}-1\right)^{2.2}$$

The net transport rate is thus calculated by summing the two vectors corresponding to the onshore and offshore bed-load transport rates:

$$\vec{q}_{bk} = \frac{T_{wc}}{T_w}\vec{q}_{bk,on} + \frac{T_{wt}}{T_w}\vec{q}_{bk,off}\begin{cases}\textbf{Magnitude: } q_{bk} = \sqrt{\left(\frac{T_{wc}}{T_w}q_{bk,on}\right)^2+\left(\frac{T_{wt}}{T_w}q_{bk,off}\right)^2+2\frac{T_{wc}T_{wt}}{T_w^2}q_{bk,on}q_{bk,off}\cos(\alpha_{on}+\alpha_{off})} \\ \textbf{Direction: } \beta = \alpha_{on} - \arccos\left\{\left[\frac{T_{wc}}{T_w}q_{bk,on}+\frac{T_{wt}}{T_w}q_{bk,off}\cos(\alpha_{on}+\alpha_{off})\right]\Big/q_{bk}\right\}\end{cases}$$

- α_{on}, α_{off}, and β are the onshore, offshore, and resultant bed-load transport angles with respect to the current direction.

❑ Suspended-load Transport Rate

The net transport rate of the kth size class of suspended load along the current direction is:

$$q_{sk} = 0.0000262p_{bk}\sqrt{(\gamma_s/\gamma-1)gd_k^3}\left[\left(\frac{\tau_b}{\tau_{cri,k}}-1\right)\frac{U_c}{\omega_{sk}}\right]^{1.74}$$

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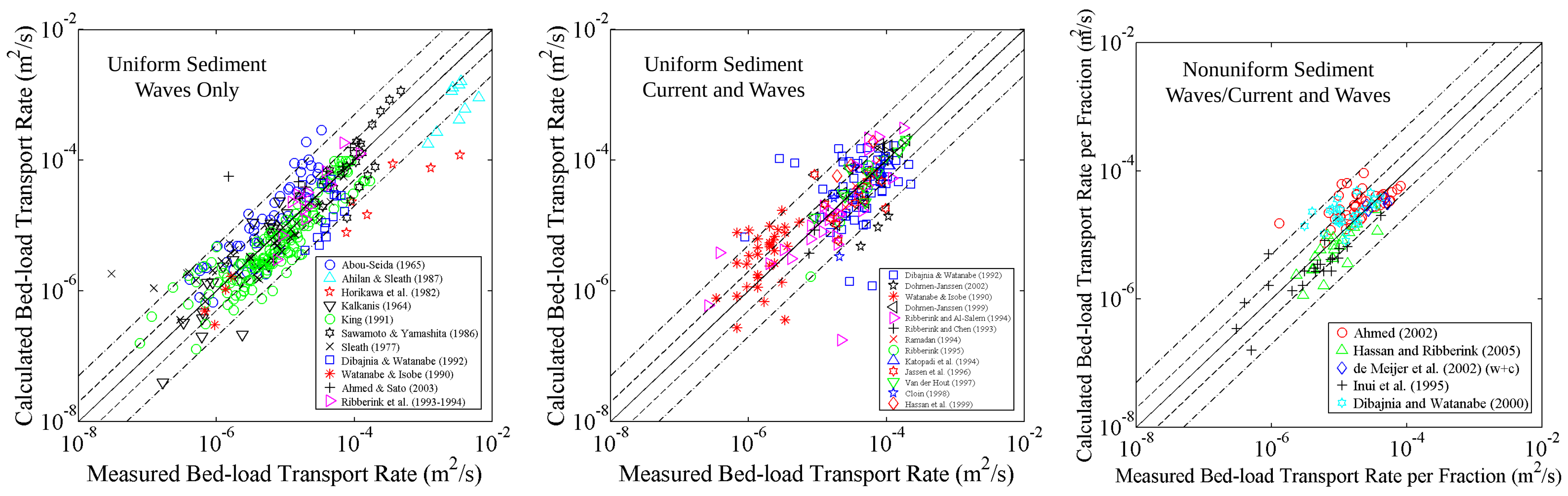
COMPARISON WITH EXPERIMENTAL DATA AND EXISTING FORMULAS

A wide range of existing data sets of single-sized bed-load and suspended-load transport under current and waves compiled by Camenen and Larson (2007) are used to test the developed sediment transport formula in this study. Several sets of data on nonuniform bed-load and suspended-load collected from other literature are also used. In addition, the developed formula is compared with several existing formulas for their performances.

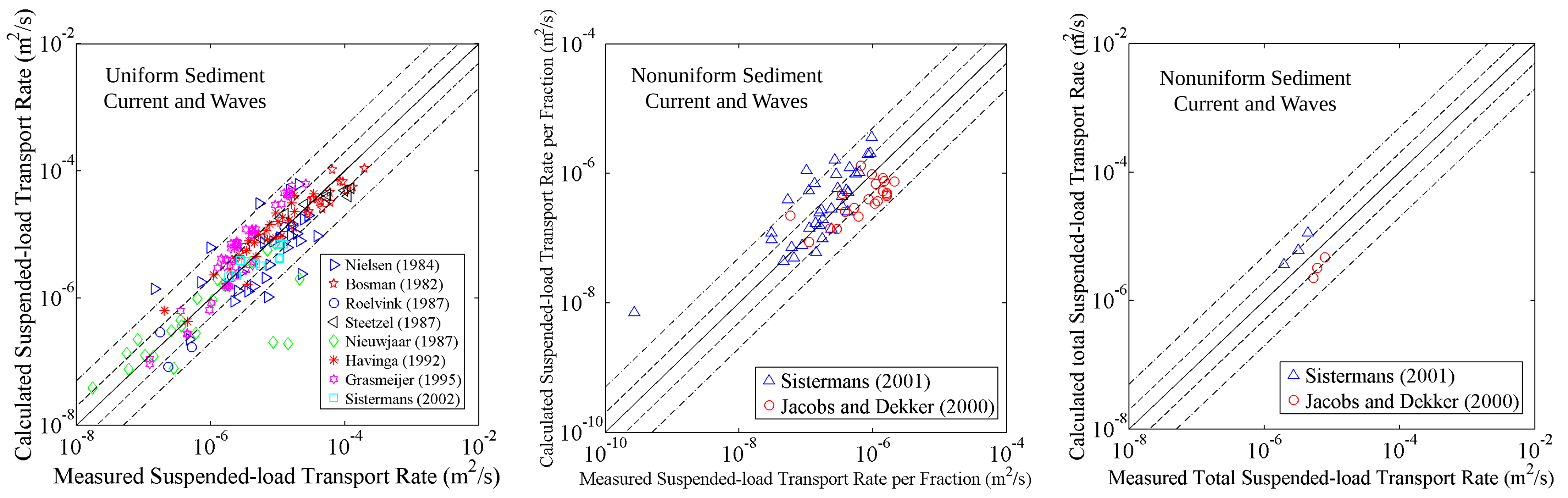
❑ Calculated (Wu et al. formula) vs. Measured Bed-load Transport Rate



❑ Statistical Analysis for Calculated Bed-load Transport Rate

Conditions		Formulas	% of Calculated Transport Rates in Error Range				E_{rms}
			0.8-1.25	0.67-1.5	0.5-2.0	0.2-5.0	
Uniform	Waves	Bailard (1981)	11.88	18.23	33.98	78.73	0.631
		Camenen and Larson (2007)	20.44	32.32	56.08	89.78	0.451
		Dibajnia and Watanabe (1992)	17.65	30.75	51.07	77.01	0.625
		Ribberink (1998)	7.32	12.74	26.43	76.11	0.629
		Van Rjin (2007)	23.01	40.80	57.98	89.57	0.468
		Present Formula	18.38	37.33	56.82	93.04	0.421
	Current+Waves	Bailard (1981)	20.86	34.97	53.37	85.28	0.467
		Camenen and Larson (2007)	22.50	35.63	64.38	93.13	0.558
		Dibajnia and Watanabe (1992)	14.58	27.08	43.23	70.31	0.752
		Ribberink (1998)	7.79	13.64	24.68	44.16	1.115
		Van Rjin (2007)	14.57	30.65	50.75	78.89	0.560
		Present Formula	25.63	40.70	57.79	88.94	0.458
Nonuniform	Current+Waves (Fractional Rate)	Camenen and Larson (2007)	13.73	22.55	32.35	68.63	0.637
		Dibajnia and Watanabe (1996)	18.56	40.21	56.70	86.60	0.498
		Hassen et al. (2001)	4.27	7.69	11.97	51.28	0.914
		Van Rjin (2007)	22.22	34.19	54.70	89.74	0.435
		Present Formula	29.91	48.72	72.65	95.73	0.307

❑ Calculated (Wu et al. formula) vs. Measured Suspended-load Transport Rate



❑ Statistical Analysis for Calculated Suspended-load Transport Rate

Conditions		Formulas	% of Calculated Transport Rates in Error Range				E_{rms}
			0.8-1.25	0.67-1.5	0.5-2.0	0.2-5.0	
Uniform	Current+Waves	Bailard (1981)	20.19	33.65	46.15	79.81	0.581
		Camenen and Larson (2007)	14.35	24.40	40.67	77.99	0.610
		Van Rjin (2007)	10.26	16.67	26.92	52.56	1.051
		Present Formula	22.75	38.32	58.68	95.21	0.388
Nonuniform	Current+Waves (Fractional Rate)	Camenen and Larson (2007)	16.07	23.21	46.43	91.07	0.431
		Van Rjin (2007)	12.00	22.00	30.00	62.00	0.976
		Present Formula	17.86	28.57	55.36	91.07	0.432
	Current+Waves (Total Rate)	Camenen and Larson (2007)	0.00	0.00	83.33	100.00	0.277
		Van Rjin (2007)	16.67	16.67	50.00	50.00	1.036
		Present Formula	0.00	0.00	66.67	100.00	0.308

CONCLUSIONS

The Wu et al. (2000) formula has been extended to calculate multiple-sized sediment transport under current and waves for coastal applications. It can calculate the onshore and offshore bed-load transport rates separately and then derive the net transport rate, whereas the enhanced suspended-load formula calculates only the net transport rate due to the limited available data. Future development may consider onshore and offshore suspended-load transport rates.

The enhanced Wu et al. formula has been tested using a wide range of single-sized sediment transport data sets and several sets of nonuniform sediment transport data collected from literature. The formula provides reliable predictions in both fractional and total transport rates. More than half of the test cases are predicted within a factor of 2 of the measured values, and more than 90% of the cases are within a factor of 5. This accuracy is generally acceptable for sediment transport, particularly under current and waves, which are very complex and little understood. Uncertainties may exist in both formulation and measurement. Additional errors are from the bed shear stress, which is difficult to measure when waves are included and has to be determined using empirical models of bed roughness. The developed formula is desirable to be further validated and improved using more data.

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