PREDICTIVE MODEL OF THREE-DIMENSIONAL DEVELOPMENT AND DEFORMATION OF A RIVER MOUTH DELTA BY APPLYING CONTOUR LINE CHANGE MODEL

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Abstract

A predictive model of the three-dimensional development and deformation of a river mouth delta is developed. This model enables the prediction of not only the shoreline change but also the three-dimensional, long-term topographic changes around a river mouth, and offshore sand transport can be taken into account when the sea bottom slope exceeds a critical value given by the angle of repose of sand. Numerical simulation of three cases of initial beach slopes is carried out and compared. The calculated and actual beach changes measured around the Fuji and Tenryu River mouths are compared. It is concluded that calculated results can reproduce real phenomena measured around river mouths qualitatively.

I. INTRODUCTION

In order to evaluate the influence of the construction of a dam in a large river basin on coastlines, since the river supplies a large amount of sand to the surrounding coastline, the impact of a decrease in fluvial sand supply should be predicted quantitatively. In past studies, the shoreline change model has been widely used for this purpose (Hashimoto, 1975; Rafaat and Tsuchiya, 1991; Tsuchiya et al., 1995). This model is very simple and can be applied under any kind of practical conditions, but it cannot be used to predict profile changes caused by longshore sand transport and offshore discharge of sand. Recently, Uda and Kawano (1996) developed a new model called the "contour line change (CLC) model" which can be used to predict successive changes in each contour line position through numerical solution of the continuity equation of sand and longshore sand transport, with assumed values of depth change in the longshore sand transport rate. This model was applied to the prediction of the movement of a sand body on the Shizuoka coast in Japan, and it well reproduced this phenomenon quantitatively (Uda et al., 1997). This study is aimed at developing a new model to predict three-dimensional deformation of a river mouth delta.

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II. NUMERICAL MODEL OF DEVELOPMENT OF A RIVER MOUTH DELTA

2.1 Contour Line Change Model

Uda and Kawano (1996) and Uda et al. (1997) developed a model which enables the prediction of the spatial and temporal changes in contour line positions by assuming the depth distribution of the longshore sand transport rate. This model is designed for a coast with a steep slope near the shoreline, without bar/trough topography. It was originally developed as one of the applications of the analytical method in which temporal and spatial changes in offshore distance to some reference contours are investigated in place of simple comparison of sounding maps based on beach survey data collected on actual coasts. The fundamental idea of this model is as follows: on a steep coast without bar/trough topography the change in offshore distance to contours located between wave run-up height and the critical depth for beach changes is very closely related to the change in shoreline position if the beach changes are caused by longshore sand transport. This suggests predictability of subsequent locations of each contour due to longshore sand transport, if the relation between the changes in those contours can be determined. As schematically shown in Fig.1, this model can be used to predict the subsequent locations of contours y_1, y_2, \cdots , including the changes in longitudinal profile of the beach, by dividing the beach profile between h_R and -h_c into thin layers and assuming the depth distribution of the

longshore sand transport rate. Here, h_R and h_C are the wave run-up height and the critical depth for beach changes, respectively.

The introduction of a cross-shore distribution of longshore sand transport can be performed similar to the present model, but in this case. the cross-shore distribution must be transfered every time the shoreline changes its location, and in addition, even if the change in depth is calculated, complicated transformation is required to calculate the change in contours from the change in depth. For these reasons, in the present study we assume the depth distribution of longshore sand transport instead of the cross-shore distribution. In the CLC model the distribution of longshore sand transport



Fig.1 Schematic of the concept of the contour line change (CLC) model.

ranges between h_R and $-h_C$, as shown in Fig.1. Longshore change in wave height can easily be taken into account since h_C and h_R are assumed to be a function of breaker height H_b .

If the incident angle at the breaking point is assumed to be sufficiently small, Eq.(1) is satisfied using the Savage formula for longshore sand transport.

$$Q = F_0(\tan \alpha_0 - \frac{\partial y_s}{\partial x})$$
(1)

where Q: littoral transport rate, F_0 : a coefficient depending on wave energy flux, α_0 : incident angle at breaking point, x: longshore distance, and y_s : shoreline position measured normal to x axis. Eq.(1) is satisfied only if the beach profile undergoes parallel movement in time and space, and the rate of longshore sand transport is determined by the relationship between the shoreline and the incident angle of waves.

Now, for a region divided by n contour lines, and if the longshore sand transport at a water depth corresponding to $k = 1 \dots n$ is assumed to be q_k and if it is also assumed that a similar relationship is established between the contour line distance y_k and q_k in analogy with Eq.(1), the following equation is obtained.

$$q_{k} = F_{0k} \left(\tan \alpha_{0} - \frac{\partial y_{k}}{\partial x} \right)$$
(2)

where $F_{0k} = F_0 \cdot \mu_k$, $\Sigma \mu_k = 1$. Eq.(2) assumes that the longshore sand transport in each layer is governed by the relationship between the location of each contour line and incident wave direction. This model, therefore, does not require parallel movement, unlike the shoreline change model.

 μ_{k} is a coefficient to give the longshore sand transport rate at each water depth and is calculated using Eq.(3) by giving the depth change of the longshore sand transport rate.

$$\mu_{k} = \int_{z_{k}}^{z_{k+1}} \xi(z) dz / \int_{-h_{c}}^{h_{c}} \xi(z) dz$$
(3)

z is the vertical distance with the still water level as the reference. The continuity equation of the longshore sand transport is given as

$$\frac{\partial q_k}{\partial x} + h_k \frac{\partial y_k}{\partial t} = 0, \quad k = 1 \cdots n \tag{4}$$

where h_k (k = 1 ... n) is the characteristic height of beach changes related to the topographic change represented by the contour lines, and is given by

$$h_k = z_k - z_{k-1} \tag{5}$$

If the functional form of ξ (z) is given, μ_k is calculated using Eq. (3), and so the change in contour for each water depth is calculated by simultaneously solving Eqs.

(2) and (4). If the vertical distribution of the longshore sand transport rate is assumed to be between the wave run-up height on the foreshore and beach changes, the following relation can be assumed based on the field observation.

$$z^* = z / H_h, \ h_c^* = h_c / H_h$$
 (6)

When $-h_C < z < h_R$,

$$\xi(z)^* = 2/h_c^{*3}(h_c^*/2 - z^*)(z^* + h_c^*)^2$$
⁽⁷⁾

When $z < -h_c$ and $z > h_R$,

$$\xi(z^*) = 0 \tag{8}$$

Uda and Kawano (1996) showed a correction method for the overhanging state of contours, namely, the state that the offshore distance to some reference contour becomes larger than that to a deeper contour. This induces offshore sand movement due to the gravity effect, and the sea bottom slope becomes more stable. Accordingly,

the physical meaning of this correction is equivalent to the profile adjustment in order for the local sea bottom slope to maintain this critical value if the sea bottom slope exceeds a critical value, and eroded and accreted areas in the profile are always equivalent, while satisfying the continuity of sand volume per unit distance. Figure 2 shows a schematic of this process.

Assuming that the local sea bottom with a slope exceeding the critical value is equivalent to the critical slope and the **Fig.2** point of intersection between BC and B'C' is set to be M, a procedure for transformation from point B to B' and from C to C' is carried out so as to satisfy the equivalence of areas ABMB' and DCMC'. Since this mechanism is included in the model, sand movement to a far-offshore zone deeper than the critical depth for beach



ig.2 Schematic of stabilization mechanism of beach profile change due to Offshore sand movement.

Table 1 Conditions for calculation.

Case No	initial bottom slope	fluvial sand supply (m^3/yr) (t $\ge 2yrs$)
1	1/5	0
2	1/10	0
3	1/20	0
4	1/5	2.5×10^{5}
5	1/10	2.5×10^{5}
6	1/20	2.5×10^{5}

changes is possible.

2.2 Conditions of Numerical Simulation

Six cases of numerical simulation of the development and deformation of a river mouth delta were investigated as shown in Table 1. The effect of the initial sea bottom slope on the formation of a foreset slope is investigated first for three different sea bottom slopes of 1/5, 1/10 and 1/20. The initial sea bottom slope is a key parameter because it influences the amount of accumulation space of fluvial sand supplied from the river mouth. Cases ①, ② and ③ aim at the investigation of this point. In each case a constant sand supply of $5.0 \times 10^5 \text{m}^3/\text{yr}$ from a river mouth is assumed from the initial stage up to two years, and thereafter, sand supply from the river is stopped completely (cases ①~③) and in another three cases, ④, ⑤ and ⑥, prediction is carried out for an additional 2 years after sand supply from the river was cut to half, in order to study the topographic changes around the river mouth due to the decrease in fluvial sand supply. Accordingly, the conditions in cases ①, ② and ③ are the same as those in ④, ⑤ and ⑥, except for the boundary condition at the river mouth after two years.

As a calculation domain, a 3km stretch, in the longshore direction, of uniform sandy beach is considered, and this domain is divided into 61 points at 0.05km intervals alongshore. A river is located at the center of this region and the openboundary condition is set at both ends of the calculation domain. In the model, landward and offshore limits are given by the maximum run-up height, h_R and the critical depth for beach changes, h_c , respectively. Furthermore, the critical slopes of the beach on land and in the sea are assumed to be 1/1.7 and 1/2, respectively. As the wave condition, a simple condition of a constant breaker height of 3m alongshore is selected as an example, and the wave direction is assumed to be perpendicular to the initial shoreline in order to facilitate the understanding of the mechanism of the development and deformation of a river mouth delta. Furthermore, h_R and h_c are set to be 4m and -7m, respectively. The calculation period is four years, which requires the total number of time steps of around 4.15×10^5 , because one time step is set to be 300s.

Sand is supplied from three points located at the distance of $2\Delta x$ in the longshore direction and the amount of supplied sand per unit time is divided into three portions; $0.6Q_0$ at the central point and $0.2Q_0$ at a point on either side, where Q_0 is the amount of fluvial sand supply from the river per unit time. The supplied sand is assumed to be rapidly divided in each layer separated vertically in correspond to the depth change in longshore sand transport by wave action.

III. RESULTS OF NUMERICAL SIMULATION

(1) Initial sea bottom slope of 1/5 (cases ① & ④)

Figures 3 (a), (b), (c) and (d) show the results of the calculations, for case (1), in which a river supplies sand at a rate of $Q_0=5.0\times10^5 \text{m}^3/\text{yr}$ to the surrounding sandy coast with the initial sea bottom slope of 1/5 and parallel contours. Under the condition of a constant sand supply, the development of a river mouth delta continues for two years. As the river mouth delta protrudes, the sea bottom slope gradually exceeds the critical slope given by the angle of repose of sand, leading to the discharge of sand into a deeper zone. As a result, the interval of contours in the deep



Fig.3 Results of calculation for development and deformation of a river mouth delta (initial beach slope 1/5: cases ① & ④).

zone is greatly narrowed. In the two years after sediment supply from the river is totally cut off, the protruding contours of a triangular shape in the zone shallower than $-h_c$ become milder. In this case the redistribution of sand due to longshore sand transport is impossible in the zone deeper than $-h_c$, and therefore a gentle slope is formed by erosion in the zone shallower than $-h_c$ with the formation of a mildly protruding shoreline. In case (4), as shown in Figs.3 (e) and (f), where sediment supply had been cut in half, sand further accumulates to build a river mouth delta while crutosis of the protruding contours in the vicinity of the center of the river mouth delta decreases gradually.

(2) Initial sea bottom slope of 1/10 (cases 2 & 5)

Figure 4 shows the results of the calculation for cases 2 and 5 with initial sea bottom slope of 1/10. The only difference between cases 1 and 4, and cases 2and 5 is the initial sea bottom slope. Figures 4 (a)~ 4 (d) show that the aerial range, in which offshore sand movement is observed with the formation of a very



Fig.4 Results of calculation for development and deformation of a river mouth delta (initial beach slope 1/10: cases 2 & 5).

steep angle of repose of sand, is narrowed, since the sea bottom slope in cases 2 and 5 becomes gentler than that in cases 1 and 4. Therefore the area expressed by dense contours in the offshore zone decreased. The rate of protrusion of the river mouth delta, however, is increased with a decrease in the average water depth of the accumulation zone of sand in cases 2 and 5, as shown in Fig.4, compared with that in cases 1 and 4, as shown in Fig.3. Furthermore, in case 4, in which sediment supply from the river is cut in half, as shown in Fig.4 (e) and (f), sand further accumulates to build a river mouth delta, while crutosis of the protruding contours in the vicinity of the center of the river mouth delta decreases gradually.

(3) Initial sea bottom slope of 1/20 (cases 3 & 6)

Figure 5 shows the results of the calculations for cases (3) and (6) with initial sea bottom slope of 1/20. The width of the region, where offshore sand movement is observed with the formation of a very steep angle of repose of sand, is greatly narrowed, since the sea bottom slope in cases (3) and (6) becomes much gentler



Fig.5 Results of calculation for development and deformation of a river mouth delta (initial beach slope 1/20: cases ③ & ⑥).

than that in cases ① and ④. For example, the foot depth of the foreset slope, which was formed by the successive deposition of fluvial sand, increases to 8m, 14m and 26m in cases ③, ② and ①, respectively, as shown in Figs.5 (b), 4 (b) and 3 (b).

Figure 6 shows the relation between the reciprocal of the initial beach slope and the foot depth of the foreset slope. The gentler the initial beach slope is, the shallower this foot depth becomes.

Here consider the case



Fig.6 Relation between foot depth of foreset slope and initial beach slope.

where sand is supplied to a coast with a gentle initial slope, as in case ③ shown in Fig.5. After cut-off of the fluvial sand supply, deposited sand will be carried away by longshore sand transport, resulting in less protruding contours, since a large portion of the sand had been deposited in a zone shallower than the critical depth for beach changes. In contrast, in case ⑥, as shown in Figs.5(e) and (f), the development of a river mouth delta further continues, although crutosis of the river mouth contours are decreased.

(4) Comparison of beach profiles along the centerline of the river mouth delta

Figures 7 (a), (b) and (c) show the profile changes along the centerline of the river mouth delta. After the sand supply from the river was cut off 0, a gentle slope is formed in the zone shallower than $-h_c$ in each case. In the case of the slope of 1/5, as shown in Fig.7 (a), a large portion of the sand carried into the sea is deposited at a depth where sand movement is impossible due to wave action, so that the nourishment effect to the surrounding coastline of fluvial sand is minimum. In the case of the slope of 1/10, as shown in Fig.7 (b), a large portion of fluvial sand can be redistributed by longshore sand transport, since the water depth of the accumulation zone is sufficiently small. In the case of the slope of 1/20, as shown in Fig.7 (c), a large portion of the supplied sand is eroded due to wave action, since almost all the sand is deposited in the zone shallower than $-h_c$. It is concluded that if the initial sea bottom slope is sufficiently gentle, the protrusion of the river mouth delta, and therefore shoreline advance, will be large with a constant sand supply from the river. On the contrary the influence of a decrease in fluvial sand supply to the surrounding coastline becomes strong in terms of shoreline recession.

Figure 8 shows the temporal change in shoreline position along the centerline crossing the river mouth delta, where Figs.8 (a) and (b) are for cases in which sand supply from the river is totally cut off and reduced to half over two years, respectively.

The results in both cases are the same up to two years; the advancement rate of the shoreline position gradually decreases, though the change in shoreline position monotonically increases. This is because the water depth in the deposition zone of



Fig.7 Temporal change in beach profile along centerline crossing the river Mouth delta.



sand gradually increases with the seaward advance of the river mouth, and sand supplied from the river mouth is carried offshoreward and deposited in a deeper zone. In the case that the sand supply from the river is cut off over two years, shoreline recession starts immediately, and this response is expressed by an exponential curve. In contrast, in the case that the sand supply from the river is reduced to half, the shoreline initially retreats but later gradually advances.

Consider the case in which the river mouth delta being accretive for a long time is eroded due to a decrease in the fluvial sediment supply. In this case, a steep sea bottom slope formed by the successive deposition of sand is distinguished very well from the very gentle slope reformed by erosion upon a sudden change of slope, as shown in Fig.7. Figure 9 shows the offshore distance from the shoreline position to the location of the sudden change in slope at each time. This offshore distance gradually increases with time after the sand supply from the river is totally cut off over two years, whereas it increases immediately after the sand supply from the river is reduced to half but then changes to a gradual decrease with time. These findings indicate that the offshore distance from the shoreline position at each time to the location of sudden change of the slope is a useful index for expressing the elapsed period of erosion of the river mouth delta. This point shows the importance of fluvial sand supply to nourish the surrounding shoreline even if the sand supply rate has been decreased.

IV. DISCUSSION

In the present study, the topographic changes around a river mouth delta were



Fig.10 Sea bottom contours around Fuji River mouth measured in 1989.



Fig.11 Comparison of beach profiles around Fuji River mouth.

predicted under the conditions that sand supply from a river is totally cut off and reduced to half over two years, after a constant supply of sand for two years. For actual rivers it is generally considered that a large amount of sand is supplied to the sea by infrequent, large floods through the river mouth and thereafter, sand supply to the sea decreases for a considerably long period. Taking this into account, it is considered that for an actual river mouth, a cyclic mode of topographic change is induced from the stage shown in Fig.3 (b) via the stage shown in Fig.3 (c) or Fig.3 (e), depending on the decrease rate of sand supply, back to the stage shown in Fig.3 (b), if a large amount of sand is supplied from the river mouth.

An example of a case ① river mouth is the Fuji River discharging at the Fuji coast in Suruga Bay facing the Pacific Ocean. The median diameter of river bed materials at the river mouth is around 75mm and the riverbed slope near the river mouth is very steep at 1/400. Figure 10 shows the beach topography around the Fuji River mouth measured in 1989. Close examination of contours between the shoreline and -20m depth reveals that the contour line intervals between the shoreline and -12m depth are very wide, whereas the contour intervals in the zone deeper than -12m are abruptly narrowed west of the mouth. Inversely, there is a very gentle slope near the depth of -4m east of the mouth, but beyond this gentle slope, the seabed slope becomes very steep. In short, contours around this river mouth show remarkable asymmetry between the east and west directions with respect to the centerline of the river mouth as well as the eastward extension of the river mouth barrier. All these findings indicate that over the long term, eastward longshore sand transport is



Fig.12 Sea bottom contours around Tenryu River mouth measured in 1984.



Fig.13 Comparison of deformation process of the terrace topography off Tenryu River mouth.

predominant at the Fuji River mouth with waves obliquely incident from the south. Profiles along the three crosssections shown in Fig.10 are shown in Fig.11 by superimposing crosssections off the critical depth for beach changes given at around -8m (Uda, 1997). There is a terrace with a gentle slope and a very steep seabed with a slope of 1/4 off this terrace. Profiles of the east and west crosssections of the river mouth are approximately the same, whereas the sea bottom elevation along the centerline crossing the mouth is higher than those along both sides, and the profile is upward convex, implying excess sand deposition from the river in front of the river mouth. These topographic characteristics are very similar to the results of the calculation for case (1), although temporal changes in profiles are not known in field data.

An example of a case ③ river mouth is the Tenryu River discharging at the Enshu coast facing the Pacific Ocean. The riverbed slope near the mouth of this river is 1/1,230, the median diameter of riverbed materials is around 13.9mm and the mean sea bed slope of this coast is 1/90.

Figure 12 shows river mouth topography around the Tenryu River measured in 1984. There is a flat terrace at -2m depth west of the river mouth. The contour of -10m shows that the western area is concave and the eastern area convex with respect to the centerline of the river mouth. This asymmetry of the contours in the two directions is the same as that observed at the Fuji River mouth, and implies the predominance of an oblique wave incidence from clockwise direction.

Figure 13 shows profile changes along the centerline of the river mouth. In

September 1970, the water depth at the flat river mouth terrace was about 2m and the terrace was as wide as around 400m from the shoreline. In July 1984, the offshore slope of the terrace retreated to a large extent compared with the one in September, 1970. In July 1986, the river mouth terrace was eroded and narrowed. These changes are considered to be caused by the imbalance between the longshore sand transport flowing away from the river mouth and sand supply from the Tenryu River which was decreased due to river bed excavation before 1967. These results agree well qualitatively with the topographic changes shown in Fig.5.

V. CONCLUSIONS

A predictive model of the three-dimensional development and deformation of a river mouth delta was developed. This model enables the prediction of not only the shoreline change but also the three-dimensional topographic changes around a river mouth. In this model, offshore sand movement is taken into account, with constant critical slope, when the sea bottom slope exceeds a critical slope given by the angle of repose. This has the advantage that the shoreline change, the progression rate of which gradually decreases with time can be simulated, since the depth of the sand deposition zone increases with the progression of the shoreline position.

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