Geomorphological Modelling in Coastal Waters

Morteza Kolahdoozan[1], Roger A. Falconer[2] (Fellow), Yiping Chen[3]

Abstract

Details are given herein of the development and application of a three dimensional layer integrated numerical model to predict geomorphological changes in estuarine and coastal waters. An Alternating Direction Implicit finite difference scheme has been used for solving the governing differential equations, which include the conservation of mass and momentum for flow, the transport equation for suspended sediment fluxes and mass conservation for bed level changes. Model predictions have been compared to predictions using other existing models and against laboratory measurements. A series of experiments have been carried out for a laboratory model harbour, with the model predictions being compared to laboratory measurements.

1. Introduction

Rivers, lakes, estuaries and coastal zones have been used as a means of navigation, disposal of waste, fishing and many commercial and economic activities for centuries. One of the most important phenomenon in these regions is the transport of sediments, which may cause erosion and deposition and produce problems for navigation and marine economic activities. In recent years there has been a growing interest in long term geomorphological processes in estuarine and coastal waters, which are related to the transport of sediment particles.

In modelling the geomorphological changes in coastal waters the main processes are the flow, sediment transport and bed level changes. These three independent processes also depend upon each other. The current structure causes sediment particles to erode from the bed and settle out through the water column, thereby causing bed level changes. The changed bed elevation can in-turn then effect the current structure and hence the sediment transport rate etc. Therefore, a complete numerical model to predict the geomorphological processes should contain a set of

[1] PhD Student, Department of Civil & Environmental Engineering, University of Bradford, UK
[2] Hyder Professor of Environmental Water Management, Cardiff School of Engineering, Cardiff University, UK
[3] Lecturer, Department of Civil & Environmental Engineering, University of Bradford, UK

coupled equations, solving for the flow and sediment transport processes and bed level changes in the region of interest.

Rivers, estuaries and coastal zones are continually undergoing geomorphological development, including sudden changes due to tectonic movement or human interference and gradual evolution as a natural process. In this study, geomorphological processes by means of gradual changes have been studied. Mathematical models now provide an efficient and relatively accurate method for studying many phenomena relating coastal and estuarine engineering problems, including geomorphological processes. Different models have been developed and used for predicting bed level changes. The theoretical aspects of one-dimensional geomorphological models were studied by de Vries, 1981. De Vriend (1986) worked on the theoretical basis of the behaviour of two-dimensional models. Van Rijn (1987) developed a two dimensional vertical model for predicting geomorphological changes in laboratory channels. Following a two-dimensional depth averaged asymptotic solution for the advective-diffusion equation, introduced by Galappatti and Vreugdenhil (1985), Wang (1989) developed a two-dimensional geomorphological model for tidal basins. Van Rijn (1987) used a two-dimensional depth integrated model, together with a logarithmic velocity profile in the vertical direction, to provide a quasi three-dimensional geomorphological model and applied it to steady state flows in straight channels.

In this paper details are given of the development and application of a three-dimensional layer integrated geomorphological model for coastal water studies, with the model predicting bed level changes as a result of both suspended and bed load transport. The hydrodynamic field has been modelled by using a combined layer integrated and depth integrated set of equations [see Falconer et al. (1991) and Lin and Falconer (1997a)]. The suspended sediment transport equation was solved using an operator splitting algorithm, as the ratio of the vertical to the horizontal length scale was generally very small in coastal waters [Lin and Falconer, 1996]. A depth integrated mass balance equation has been used for predicting bed and suspended load fluxes, and subsequently predictions in bed level changes. An Alternating Direction Implicit finite difference method has been used to solve the governing equation for bed level changes.

2. Mathematical Formulation

In order to simulate the geomorphological processes occurring in estuarine and coastal waters, the governing equations were divided into three categories, representing the hydrodynamic, sediment transport and bed level changes. The governing hydrodynamic equations include the continuity and momentum equations in three dimensions, together with a turbulence closure equation for the determination of the eddy viscosity and diffusivities. In the sediment transport model, the advective-diffusion equation for suspended sediment transport is solved. The bed load transport is determined using a box model originally developed by van Rijn (1984). With the velocity and sediment concentration fields predicted, and by using the mass balance equation for the bed material, then bed level changes can be determined accordingly.

Governing Hydrodynamic Equations

The governing equations used to describe the velocity distribution in estuarine and coastal waters are generally based on the 3-D Reynolds equations for incompressible, unsteady turbulent flows. Usually in 3-D tidal models the assumption of a vertical hydrostatic pressure distribution is assumed, as is the case in the current model. According to this assumption the vertical acceleration of the flow must be much smaller than gravitational acceleration. As the water can be assumed to be well mixed in many estuarine and coastal zones, the water density can often be assumed to be constant throughout the domain. In applying these approximations, the governing three dimensional differential equations of mass and momentum can be written in their conservative form as follows [Lin and Falconer, 1997a]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=fv-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial vu}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}=-fu-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right) \tag{3}$$

$$\frac{\partial P}{\partial z}+\rho g=0 \tag{4}$$

where: t = time, x, y, z =Cartesian co-ordinates, u, v, w = components of velocity in x, y and z directions respectively, P = pressure, ρ = density of water, f = coriolis parameter, g= acceleration due to gravity and $\sigma_{xx}, \tau_{xy}, \tau_{xz}, \tau_{yx}, \sigma_{yy}, \tau_{yz}$ = components of stress tensor in x-z and y-z planes respectively.

Sediment Transport Equations

For the sediment transport sub-model both suspended load and bed load transport have been considered, with the sediment type depending upon the size and density of the bed material and the flow conditions. For predicting the suspended sediment concentration in estuarine and coastal waters, the advective-diffusion equation is usually used and which can be written as:

$$\frac{\partial s}{\partial t}+\frac{\partial}{\partial x}(us)+\frac{\partial}{\partial y}(vs)+\frac{\partial}{\partial z}\left((w-w_s)s\right)-\frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial s}{\partial x}\right)-\frac{\partial}{\partial y}\left(\varepsilon_y\frac{\partial s}{\partial y}\right)-\frac{\partial}{\partial z}\left(\varepsilon_z\frac{\partial s}{\partial z}\right)=0 \tag{5}$$

where: s = sediment concentration, w_s = particle settling velocity and $\varepsilon_x, \varepsilon_y, \varepsilon_z$ = sediment mixing coefficients in x, y and z directions respectively. For the case of bed load transport a box model, originally developed by van Rijn (1984), has been used giving:

$$q_b = 0.053\,T^{2.1}(\Delta g)D_{50}^{1.5}D_*^{-1} \tag{6}$$

where: T = transport stage parameter = $\dfrac{(u_*)^2 - (u_{*,cr})^2}{(u_{*,cr})^2}$, D_* = particle diameter = $D_{50}\left[\dfrac{(s-1)g}{\upsilon^2}\right]^{\frac{1}{3}}$, Δ = relative density and $u_{*,cr}$ = critical bed shear velocity = $(\theta_{cr}(s-1)gD_{50})^{\frac{1}{2}}$. For the critical mobility parameter (θ_{cr}), a novel unsteady criteria for the initiation of motion has been introduced and used in this study. Based on this hypothesis, the forces acting on a sediment particle can be divided into steady and unsteady flow forces. For unsteady flow effects, the significance of time dependency of the hydrodynamic parameters on the motion of sediment particles has been considered. Assuming a sediment particle rests on a horizontal bed as shown in Fig. (1), then the

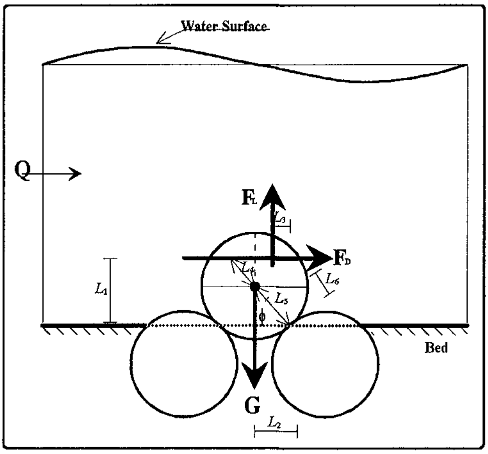


Fig. (1) Forces acting on the sediment particle

fluid forces acting on this particle are the pressure force and the skin friction. These forces can be divided into drag and lift forces, in the vertical and horizontal plane, and in the Cartesian co-ordinate system. For unsteady tidal flow the drag and lift forces consist of two components, i.e. steady state flow effects and time dependency effects of the fluid parameters. Therefore these drag and lift forces can be represented as follows:

$$F_D = F_{DST} + F_{DUST} \tag{7}$$
$$F_L = F_{LST} + F_{LUST} \tag{8}$$

where: F_{DST} = steady drag force effect, F_{LST} = steady lift force efffect, F_{DUST} = unsteady drag force effect and F_{LUST} = unsteady lift force effect. For steady state flows the drag and lift forces can be shown to be:

$$F_{DST} = \frac{1}{2} C_D A \rho u^2 \tag{9}$$

$$F_{LST} = \frac{1}{2} C_L A \rho u^2 \tag{10}$$

Similarly, for unsteady flows the force due to the time dependency of hydrodynamic parameters can be shown to be:

$$F_{UST} = m\left(-\frac{A_w}{\omega^2} Sin(\omega t + \varphi) + u\frac{\partial u}{\partial x} \right) \tag{11}$$

where: m = mass of sediment particle, A_w = amplitude of tidal wave, ω = angular velocity and φ = tidal phase. Combining equations (9), (10) and (11), the critical mobility parameter for unsteady tidal flows can be shown to be:

$$\theta_{cr} = \frac{4}{3} \frac{1}{C^2 (L_1 C_D + L_3 C_L)} \left\{ - \left[-\frac{A_w}{\omega^2} Sin(\omega t + \varphi) + u\frac{\partial u}{\partial x} \right] (Sin\psi + Cos\psi) + gL_2 \right\} \tag{12}$$

where the coefficients L_1, L_2 and L_3 are distances from the contact point of the particle, as shown in Fig. (1).

Bed Level Changes Equation

Bed level changes can be described mathematically using the mass balance equation for sediment fluxes. In the three dimensional computational domain a control volume is assumed in the vertical direction of the water column as shown in Fig. (2). The depth integrated mass balance equation for sediment in this case can be shown to be given as:

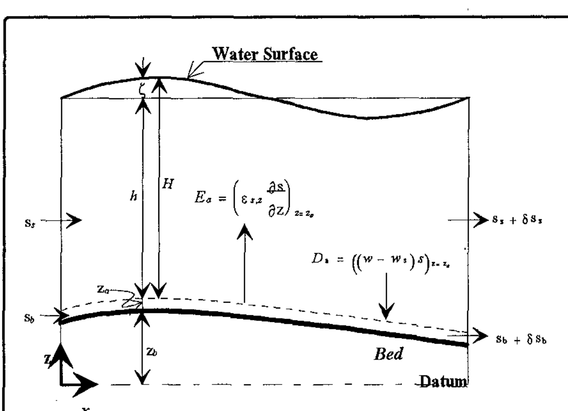


Fig. (2) A control volume in the water body

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-p}\left\{\frac{\partial}{\partial t}(h\bar{s}) + \frac{\partial}{\partial x}(S_{t,x}) + \frac{\partial}{\partial y}(S_{t,y})\right\} = 0 \tag{13}$$

where: z_b = bed level above datum, $\bar{s}$ = depth averaged concentration, S_t = total transport = $s_s + s_b$ and s_s = depth integrated suspended load flux which given as:

$$s_{s,x} = \int_a^h \left(us - \varepsilon_{s,x}\frac{\partial s}{\partial x}\right)dz \tag{14}$$

$$s_{s,y} = \int_a^h \left(vs - \varepsilon_{s,y}\frac{\partial s}{\partial y}\right)dz \tag{15}$$

s_b = bed load transport.

3. Numerical Procedure

The 3-D layer integrated TRIVAST model, originally developed by Falconer et al., (1991) and refined by Lin and Falconer, (1997b), has been used for both the hydrodynamic and sediment transport part of the model. For the hydrodynamic model the depth integrated continuity and momentum equations are first solved to define the water elevations across the domain. Then the three dimensional continuity and momentum equations are solved for each layer giving the velocity components in three dimensions. In solving the to the above set of equations the Alternating Direction Implicit (ADI) finite difference method was used.

For the sediment transport sub-model an operator splitting algorithm was used to solve the advective-diffusion equation for suspended load transport. The basic principle of this method is to split the advective-diffusion equation into several smaller and simpler sub-equations, with each sub-equation being solved using the most efficient numerical algorithm (Lin and Falconer 1996). For more details about the solution procedure of this part of the model see Lin and Falconer, (1997b). As the accuracy of the sediment flux predictions is significant in the bed level change predictions, it is therefore important to use a highly accurate scheme for solving the sediment transport equation. Since the advection terms (of order Δx^{-1}) are likely to dominate over the diffusion terms (of order Δx^{-2}) in estuarine and coastal waters, then for this part of the equation it was particularly important that an accurate scheme was used to discretize the advective-diffusion equation. Based on a study carried out by Cahyono (1992), the ULTIMATE scheme, originally proposed by Leonard (1991), was particularly attractive since it was more general than the other schemes considered and easier to apply. Lin and Falconer (1996) in their 2-D depth integrated estuarine model, used both splitting and non-splitting methods for the third order QUICKEST scheme, combined with the ULTIMATE limiter, to produce solute and sediment flux predictions [see Lin and Falconer, 1997b]. In this study, the ULTIMATE QUICKEST scheme was used to represent the advective terms in the sediment transport sub-model given by equation (5).

For the bed level sub-model, the depth integrated mass balance equation was solved using an Alternating Direction Implicit finite difference method. It is assumed that in the three-dimensional computational domain the sediment flux is integrated over each layer and then over the whole water depth, to calculate the depth integrated suspended sediment flux. These three sets of equations have been solved together in an uncoupled model resulting in predictions of the geomorphological changes occurring in coastal waters.

4. Experimental Set-up

For the validation of the model a laboratory experimental programme was carried out in the tidal flume [Falconer and Chapman, 1996] at the University of Bradford. This model was first set-up to study the tidal currents and flushing characteristics within a rectangular harbour. The overall working area of the tidal basin was 5.34×3.68 m, with the square harbour being positioned on a level platform covering the full width of the tank and extending 3 m out from the rear wall of the basin, as illustrated in Fig. (3). The square harbour had a plan-area of 1.08×1.08 m with varying entrance widths being considered in this study. Tides were generated using a variable elevation waste weir driven by a computer and producing a sinusoidal wave for this study. The flat bed of the harbour was first covered with a uniform depth of non-cohesive bed material. After 24 repetitive model tides, the bed level changes inside the harbour were measured. These changes mainly occurred along the entrance

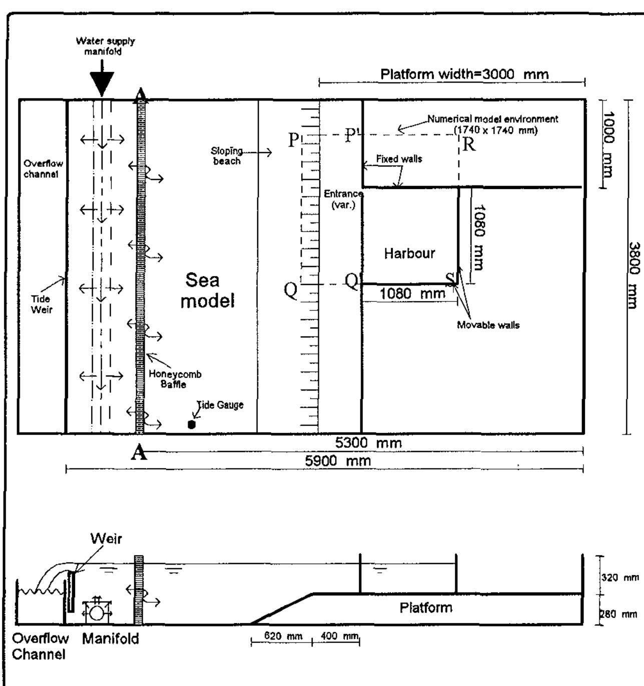


Fig. (3) Schematic illustration of the tidal basin centreline of the harbour, where a strong jet-flow was produced due to the narrow harbour entrance. Measurements were taken for a number of parameters, including: the depth inside the harbour and the tidal range. A comparison of the results for different tidal ranges is shown in Fig. (4), with increased erosion occurring as the tidal range increased and the mean water depth decreased.

5. Model Application and Verification

In verifying the mathematical model for the case with suspended sediment transport, a straight channel, 3 km long and 1 km wide, under uni-directional and steady state flow conditions was chosen [van Rijn, 1987 and Wang, 1989]. A 400 m long headland was sited in the middle of the channel as illustrated in Fig. (5). The upstream and downstream boundary conditions were set to a constant discharge of 4000 cumecs and a flow depth of 6 m respectively. In the three-dimensional numerical model the water column was divided into 10 layers in the vertical. The predicted velocity and streamline fields are shown in Figs. (6) and (7) respectively. The predicted depth averaged

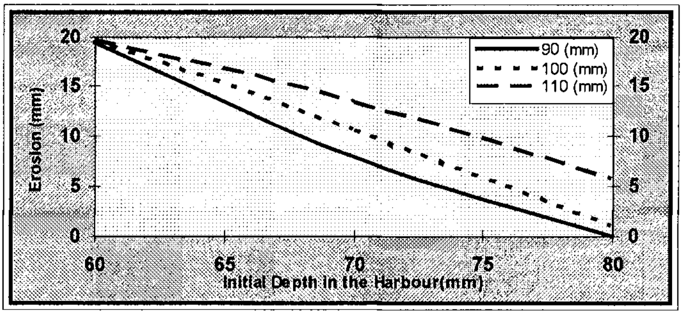


Fig. (4) Influence of depth and tidal range on erosion

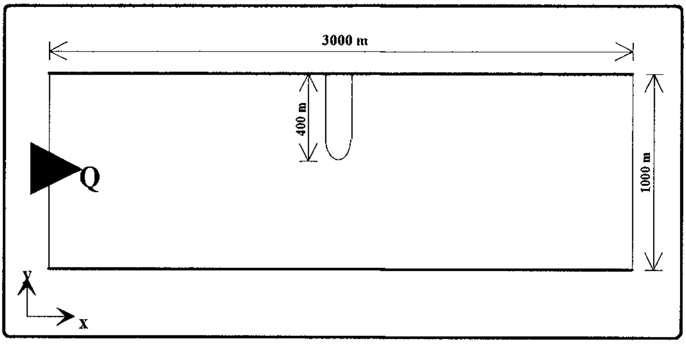


Fig. (5) Layout of channel and boundaries

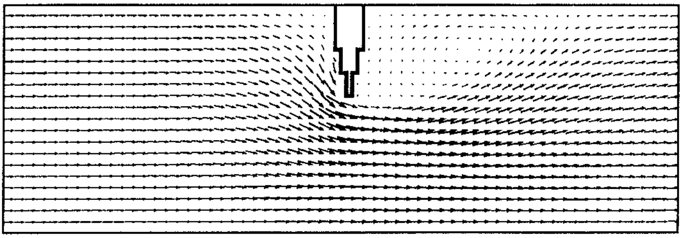


Fig. (6) Predicted depth averaged velocity field

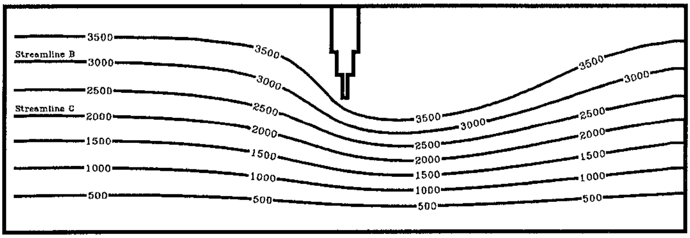


Fig. (7) predicted streamline pattern

velocity field has been compared with predictions from the SUTRENCH and ESMOR models, developed by Delft Hydraulics (van Rijn, 1987 and Wang, 1989). The results obtained from these two models were compared along streamlines B and C with the current model, with the comparisons being shown in Figs. (8) and (9) respectively. As can be seen from Figs. (8) and (9), the depth averaged velocity prediction obtained from the three-dimensional model were in good agreement with the Delft Hydraulic model predictions.

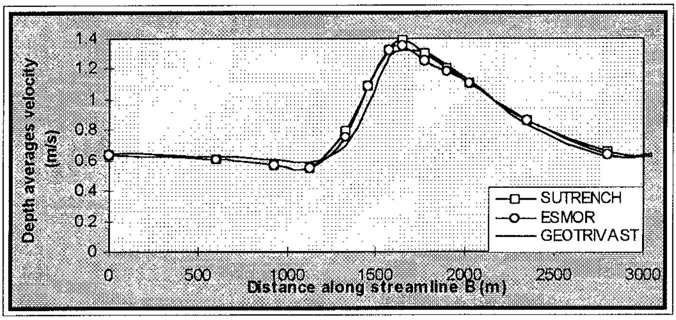


Fig. (8) Comparison of velocities along streamline "B"

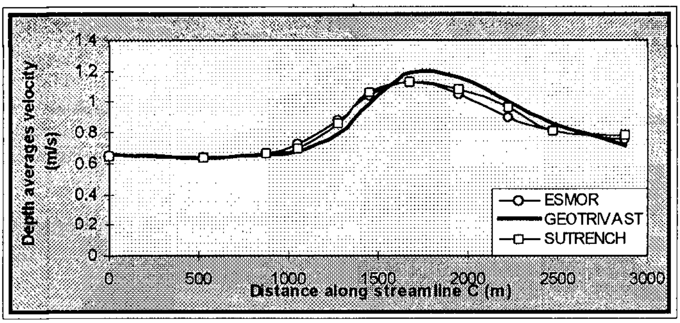


Fig. (9) Comparison of Velocities along streamline "C"

A comparison of the depth averaged suspended sediment fluxes for five different numerical model predictions are shown in Figs. (10) and (11) along streamlines B and C respectively. As can be seen from the results, the current model predictions are again in good agreement with the other models. Finally, the sediment concentration

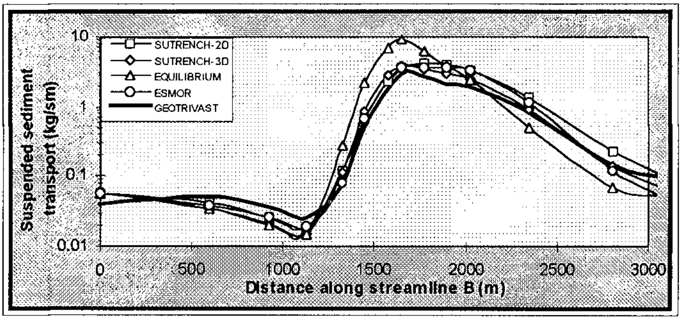


Fig. (10) Comparison of sediment fluxes along streamline "B"

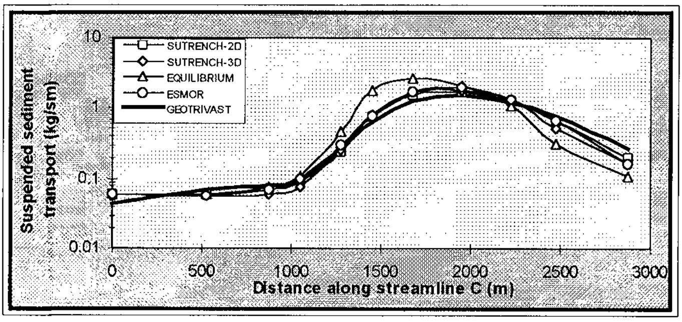


Fig. (11) Comparison of sediment fluxes along streamline "C"

patterns using this model are shown in Fig. (12), with the rate of bed level changes being predicted as shown in Fig. (13).

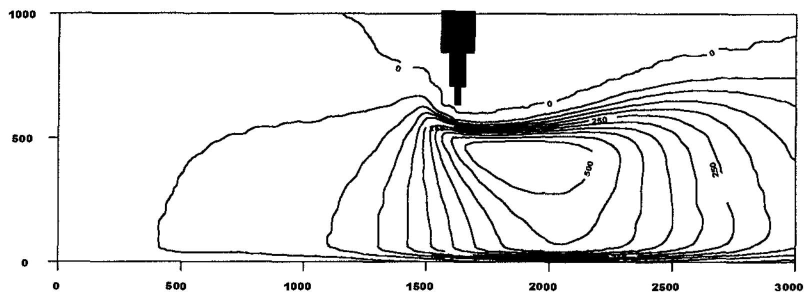


Fig. (12) Suspended sediment concentration in 10[th] layer

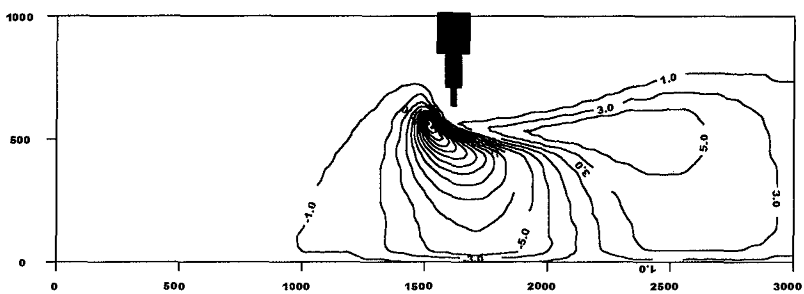


Fig. (13) Rate of bed level change using 3-D model (mm/hr)

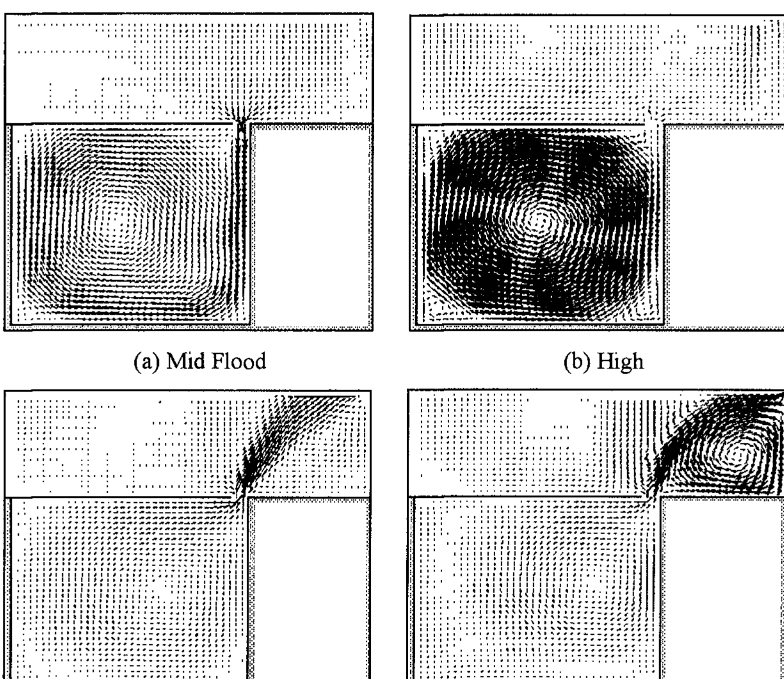
The maximum erosion rate near the tip of the headland was predicted to be 44.6 mm/hr and the maximum deposition rate was 6.9 mm/hr. In contrast, the results using a 2-D model gave lower values for the erosion and deposition rates. The values reported by Wang and van Rijn for maximum erosion were 43 and 100 mm/hr respectively, and the maximum deposition rates were 10.5 and 25 mm/hr respectively. A comparison of the above results shows that the results using the current 3-D model were in close agreement with the results of Wang (1989).

In terms of bed load transport, the model was then set-up to predict the bed level changes in the laboratory model harbour. A tidal period of 300s, with a tidal range of 10cm and an entrance width of 60mm gave rise to the predicted depth averaged velocity distribution for different tidal phases and using a no-slip boundary condition as shown in Fig. (14).

By including the sediment transport part of the model in predicting the bed load transport in the tidal basin, it was found that the Shield's criterion for the initiation of motion-which is based on extrapolation of the bed shear stresses obtained from experimental measurements for steady state flow conditions in flumes - failed to predict any changes in the bed level for the laboratory model harbour. Therefore a new criterion for the initiation of motion under unsteady flow conditions had to be developed and used in this model. Results from the refined model for the bed level changes along the harbour entrance centreline are shown in Fig. (15), together with the experimentally measured results. As can be seen from this comparison, the refined model was capable of accurately predicting the bed level changes in the laboratory model harbour.

6. Conclusion

A three-dimensional layer integrated geomorphological model has been developed to predict bed level changes in coastal and estuarine waters. As the accuracy of the bed level predictions highly depends upon the accuracy of the sediment transport predictions, an accurate finite difference scheme - named the ULTIMATE QUICKEST scheme - was used to predict the suspended sediment fluxes. For bed load transport, the criterion for the initiation of motion for steady and unsteady flow conditions has been investigated. A comparison between laboratory measurements and the predictions obtained from the numerical model, for both steady and unsteady initiation of motion, was undertaken. These comparisons showed that a new criterion for the initiation of motion, based on a tidal flow, was capable of predicting bed level changes accurately in comparison with the physical model measurements.



(a) Mid Flood (b) High

(c) Mid Ebb (d) Low

Fig. (14) Predicted velocity distribution for different tidal phases

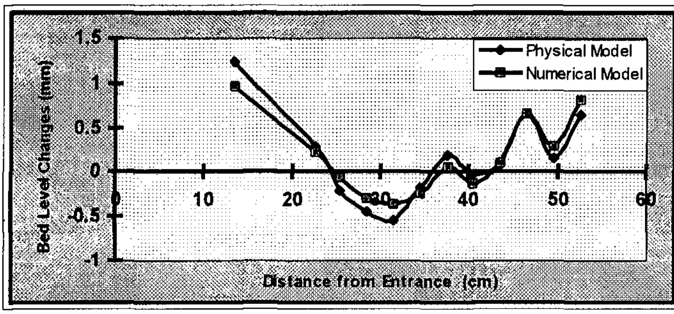


Fig. (15) Bed level changes along harbour entrance centreline

References

Cahyono, M., (1992), "Three Dimensional Numerical Modelling of Sediment Transport Processes in Non-stratified Estuarine and Coastal Waters", PhD Thesis, Department of Civil Engineering, University of Bradford, Bradford, UK.

de Vriend, H.J., (1986), "Two and Three Dimensional Mathematical Modelling of Coastal Morphology", Report No. H284-2, Delft Hydraulics Laboratory, Delft, The Netherlands.

Falconer, R.A. and Chapman, B.C., (1996), "Design and Operation of a Laboratory Tidal Basin for Model Harbour Studies", Dock and Harbour Authority, Vol. 746, No. 864, pp. 175-179.

Falconer, R.A., George, G.D. and Hall, P., (1991), "Three Dimensional Numerical Modelling of Wind Driven Circulation in a Shallow Homogeneous Lake", Journal of Hydrology, Vol. 124, pp. 59-79.

Galappatti, R. and Vreugdenhil, C.B. (1985), "A Depth Integrated Model for Suspended Sediment Transport", Journal of Hydraulic Research, Vol. 23, No. 4, pp. 359-377.

Leonard, B.P. (1991), "The ULTIMATE Conservative Difference Scheme Applied to Unsteady One-Dimensional Advection", Computer Methods in Applied Mechanics and Engineering, Vol. 88, pp. 17-74.

Lin, B. and Falconer, R.A., (1996), "Numerical Modelling of Three-Dimensional Suspended Sediment for Estuarine and Coastal Waters", Journal of Hydraulic Research, Vol. 34, pp. 435-456.

Lin, B. and Falconer, R.A., (1997a), "Three Dimensional Layer Integrated Modelling of Estuarine Flows with Flooding and Drying", Journal of Coastal and Shelf Research, Vol. 44, No. 6, pp 737-751.

Lin, B. and Falconer, R.A., (1997b), "Tidal Flow and Transport Modelling Using ULTIMATE QUICKEST Scheme",Journal of Hydraulic Engineering ASCE, Vol. 123, No. 4, pp. 303-314.

van Rijn, L.C., (1984), "Sediment Transport, Part I: Bed Load Transport", Journal of Hydraulic Engineering, ASCE, Vol. 110, No. 10, pp. 1431-1456.

van Rijn, L.C., (1984), "Mathematical Modelling of Morphological Processes in the Case of Suspended Sediment Transport", PhD Thesis, Delft University of Technology, Delft, The Netherlands.

Wang, Z.B., (1989), "Mathematical Modelling of Morphological Processes in Estuaries", PhD Thesis, Delft University of Technology, Delft, The Netherlands.