EQUILIBRIUM TERRACED AND BARRED BEACHES

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ABSTRACT

The results of two sediment transport models based on the sheet flow approach and the energetics approach are compared with experimental data of random waves over fine sand beaches in equilibrium in a wave flume. Two tests were conducted with a terraced profile (test 1) and a barred profile (test 2). The energetics-based model predicts the equilibrium profile better than the sheet flow model for both tests. However, the energetics model predicts offshore sediment transport and bar migration, whereas the sheet flow model predicts onshore sediment transport and bar migration. These models cannot predict zero net sediment transport rate on these equilibrium profiles with negligible profile changes, whereas the standard equilibrium profile cannot explain the existence of the terrace and bar.

INTRODUCTION

At the present time, there are no models capable of accurately predicting the development and final shape of terraced and barred equilibrium beach profiles. A profile of the form $d = Ay^{2/3}$, with d = still water depth, y = seaward distance from the shoreline, and A = sediment scale parameter, has been shown to represent typical profiles of natural beaches [e.g., Dean (1991)]. Perturbations on the simple equilibrium profile in the form of barred and terraced beaches have also been observed. However, the processes involved in the creation of these perturbed beach profiles are not presently well understood. Trowbridge and Young (1989) developed a sheet flow sand transport model based on the time-averaged bottom shear stress and showed that this model could explain the measured onshore movement of a nearshore bar in Duck, North Carolina during the mild wave conditions between February and August, 1982. Thornton *et al.* (1996) and Gallagher *et al.* (1998) used an energetics-based sediment transport model to explain the offshore movement of a bar on the same beach during storms in 1990 and 1994, respectively. Yet no existing model can predict both onshore and offshore bar migrations satisfactorily.

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This study conducted two detailed tests with a concave fine-sand beach in a laboratory wave tank, on one barred and one terraced profile in quasi-equilibrium. This ensured that the net cross-shore sediment transport rate on these profiles would be zero. The velocity and surface elevation measurements made during these tests were used to calculate bottom elevation changes predicted by the sheet flow and energeticsbased models. These tests will help to identify the cross-shore sediment transport processes that take place on these quasi-equilibrium profiles and to suggest effects that have been neglected by existing transport models.

EXPERIMENT

The two tests were conducted in a wave tank that was 30 m long, 2.44 m wide, and 1.5 m high with a constant water depth of 61.0 cm. Repeatable irregular waves, based on the TMA spectrum (Bouws *et al.* 1985) using linear wave theory and random phases, were generated with a piston-type wave paddle. The beach was composed of fairly uniform fine sand with an initial slope of 1:12. For the random waves chosen for tests 1 and 2, the incident wave spectral peak periods were $T_p =$ 2.8 s and 1.6 s, respectively, and the spectral significant wave heights were $H_{mo} =$ 0.203 m and 0.182 m, respectively. The experimental setup is shown in Figure 1. Quasi-equilibrium profiles were obtained after repeatedly running the selected waves in bursts of 400 s for several days.

Ten capacitance wave gauges were used to measure the temporal variations of free surface elevations. Velocity statistics were measured in multiple cross-shore positions using a single Acoustic-Doppler Velocimeter (ADV) (Kraus et al. 1994). After equilibrium had been established for each test, the corresponding random wave burst was run 21 times over the stable profile, repositioning the velocimeter and several wave gauges between runs. The wave paddle motion, controlled by a computer, was approximately identical for all the runs in each test. The sampling rate for the free surface and fluid velocity measurements was 20 Hz. The measurement positions for the wave gauges and the ADV are shown in Figure 2. At two of its positions (x =7.35 m and x = 9.85 m) the 2D probe was adjusted vertically to obtain three-point profiles of the cross-shore velocity over depth. At the remaining positions the velocity was measured as close as possible to mid-depth. Longshore velocities associated with three-dimensional turbulence were small compared with cross-shore velocities, and thus only cross-shore velocities were analyzed in these tests. The duration of each run was 400 s, from which the initial transient duration of 75 s was removed before the following analysis.

Preceding the experiment, a grain size analysis showed the beach sand to be relatively uniform, with mean diameter $d_{50} = 0.18$ mm. The average fall velocity for a spherical particle of diameter d_{50} was calculated to be w = 1.89 cm/s and the measured fall velocity was 1.9 cm/s. During each of the two tests detailed measurements were made of the beach profile using a manual vernier pointer in the swash zone and a Panametrics 22DLHP ultrasonic depth gauge in deeper water. In both tests fairly uniform ripples (approximately 1–3 cm in height and 10–15 cm in length) were established in the regions offshore of the bar and along the terrace between the bar and the



Figure 1: Wave Tank Experimental Setup



Figure 2: Cross-Shore Positions of Wave Gauge and ADV Measurements Over Final Profiles (- - Test 1, -.- Test 2)



Figure 3: Final Equilibrium Profiles (- - Test 1; -.- Test 2) and $d = Ay^{2/3}$



Figure 4: Undertow Velocity Fields for Tests 1 and 2

swash zone. The waves in test 1 plunged intensely in a small region slightly shoreward of the terrace edge, creating a large amount of suspended sediment. Spilling breakers were more common in test 2, and breaking occurred more continuously over the bar and in shallower water up to the swash zone. Figure 3 shows the measured equilibrium profiles for tests 1 and 2 in comparison with the equilibrium profile $d = Ay^{2/3}$ with $A = 0.09m^{1/3}$ estimated for this sand using the empirical formula given by Dean (1991). The standard equilibrium profile represents the profile inside the surf zone fairly well, but there are significant deviations from the profile in the region of the terrace edge (test 1) or the bar crest (test 2) where the incident random waves broke intensively. The terraced and barred beaches for tests 1 and 2 are similar to beach profiles measured in Duck, except that the offshore slope in these laboratory tests was much steeper (*e.g.*, Thornton *et al.* 1996).

Free Surface and Velocity Statistics

Measured wave statistics were presented in detail by Kobayashi *et al.* (1997), and in comparison with the corresponding results for a 1:16 slope by Kobayashi *et al.* (1998). The wave setup, $\overline{\eta}$, gradually increased as waves moved shoreward in shallower water, becoming tangential to the beach slope in the swash zone in both tests. The measured values of $H_{\rm rms}$ were approximately constant in the region seaward of the bar/terrace. They reached a slight peak after passing onto the terrace, then decreased steadily in the surf zone and more rapidly in the swash zone. The measured values of $H_{\rm rms}/\overline{h}$ reflected the beach profile fairly strongly in deeper water, then began to rise rapidly as they approached the still water shoreline. Undertow was strong over most of the terrace region in both tests, indicating the existence of other mechanisms required to maintain the equilibrium profile against undertow. The measured undertow velocity fields are shown over the corresponding profiles in Figure 4.

Figure 5 shows profiles of velocity statistics measured at two locations (x = 7.35 m and 9.85 m) for the two tests. In general, the mean velocity, \overline{u} , and standard deviation, σ_u , remain relatively constant over the depth ranges measured. The measured undertow was within 20% of its mid-depth value for all profiles. The vertical variation of \overline{u} for irregular waves appears to be less than that for regular waves (Cox and Kobayashi 1998a). The standard deviation remains within 5% of the value calculated at mid-depth for both tests. The figures also show relative uniformity of values for the skewness, s_u , and the kurtosis, K_u , over the depth. The greater variability in these results at x = 9.85 m for test 1 may be explained as a consequence of the intense wave breaking. This uniformity of the cross-shore velocity and its moments over depth provides support for our decision to calculate cross-shore sediment transport using a mid-depth velocity at each cross-shore location.

SHEET FLOW MODEL FOR ONSHORE BAR MOVEMENT

The sheet flow model proposed by Trowbridge and Young (1989) is presently the only existing model that attempts to explain onshore bar movement outside the surf zone in the absence of undertow. While application of this model has been limited to plane beds without ripples, the profiles obtained in these experiments did include rippled sand beds in the offshore and surf zones for both tests. However, both profiles were free of ripples in the region over the bar with the most intense wave breaking and in the swash zone. The following comparison should be interpreted in light of these limitations.

Theory

The measured cross-shore velocity is represented here by u(t), with t =time. The overbar is used to indicate time averaging. The time-averaged rate of onshore sediment transport, q, is assumed to be expressible as

$$\overline{q} = K \frac{w\overline{\tau}_b}{\rho g(s-1)} \tag{1}$$

where w = sand fall velocity, $\rho = \text{density of fluid}$, $s = \rho_s/\rho = \text{specific gravity of sand}$ with $\rho_s = \text{sand density}$, $\overline{\tau}_b = \text{time-averaged bottom shear stress}$, g = gravitational acceleration, and K = an empirical coefficient. Trowbridge and Young (1989) analyzed the wave boundary layer and derived the following expression for the mean bottom shear stress, $\overline{\tau}_b$:

$$\overline{\tau}_b = +\frac{\rho f_w}{2} \frac{|\tilde{u}|^3}{\sqrt{g\bar{h}}} \tag{2}$$

where $f_w =$ friction coefficient and $\tilde{u} = (u - \overline{u}) =$ oscillatory part of first-order wave velocity. Their wave boundary layer analysis did not account for undertow, which was



Figure 5: Depth variations of \overline{u} , σ_u , s_u , and K_u at x=7.35 and 9.85 m

significant in the present experiment. Note that while Trowbridge and Young defined the x-axis as positive offshore, the present analysis defines x as positive onshore. Substituting (2) into (1):

$$\bar{q}_u = \left(\frac{Kf_w}{2}\right) \frac{w |\bar{u}|^3}{g(s-1)\sqrt{g\bar{h}}} \tag{3}$$

Trowbridge and Young calibrated the value of Kf_w for $d \simeq 0.2$ mm and recommended $Kf_w = 0.5$. In the present tests, $d_{50} = 0.18$ mm and $Kf_w = 0.5$ is adopted as well. For sheet flow conditions, (3) may be used to predict the time-averaged cross-shore transport rate, \bar{q}_u , based on the velocity data as indicated by the subscript u.

Trowbridge and Young used linear long wave theory to relate \tilde{u} to $\tilde{\eta} = (\eta - \bar{\eta})$, with the assumption that $\tilde{\eta}$ is Gaussian. Since the velocity data is available here, it is assumed instead that \tilde{u} is Gaussian. This assumption yields

$$\overline{\tilde{u}|^3} \simeq \sqrt{\frac{8}{\pi}} (\overline{|\tilde{u}|^2})^{1.5} \tag{4}$$

which is better than the result of Trowbridge and Young because the skewness, s_u , of the velocity is generally smaller than the skewness, s, of the free surface (Kobayashi *et al.* 1997, 1998). Linear long wave theory is then assumed to obtain

$$\overline{|\tilde{u}|^2} = \overline{|(u-\overline{u})|^2} \simeq \overline{|\sqrt{\frac{g}{\bar{h}}}(\eta-\overline{\eta})|^2} = \frac{g}{\bar{h}}\overline{|\tilde{\eta}|^2}$$
(5)

Combining (4) and (5), one obtains

$$\overline{|\tilde{u}|^3} \simeq \sqrt{\frac{8}{\pi}} \left(\frac{g}{\bar{h}} \overline{|\tilde{\eta}|^2}\right)^{1.5} = \sqrt{\frac{8}{\pi}} \frac{g}{\bar{h}} \sqrt{\frac{g}{\bar{h}}} \sigma^3 \tag{6}$$

where σ is the standard deviation of the free surface elevation. Substitution of (6) into (3) yields

$$\overline{q}_{\eta} = \frac{K f_w}{16\sqrt{\pi}} \frac{w H_{\rm rms}}{(s-1)} \left(\frac{H_{\rm rms}}{\tilde{h}}\right)^2 \tag{7}$$

where the root-mean-square wave height $H_{\rm rms}$ is defined as $H_{\rm rms} = \sqrt{8}\sigma$. Eq. (7) may be used to predict the time-averaged cross-shore transport rate, \bar{q}_{η} , using free surface data as indicated by the subscript η .

An expression for the predicted rate of change of the sand bottom elevation is derived by applying the conservation equation of sediment. This yields an expression for the erosion (negative) or accretion (positive) rate $\partial z_b/\partial t$ in terms of the gradient of the cross-shore transport rate \bar{q} :

$$\frac{\partial z_b}{\partial t} = \frac{-1}{(1-n_p)} \frac{\partial \overline{q}}{\partial x} \tag{8}$$

in which the porosity n_p was 0.4 in this experiment. Eq. (8) is combined with (3) or (7) to determine the erosion/accretion rate based on the velocity data or free surface data, respectively, where $(\partial z_b/\partial t)_u$ and $(\partial z_b/\partial t)_\eta$ are used for the computed values of $(\partial z_b/\partial t)$ corresponding to \overline{q}_u and \overline{q}_η , respectively. In addition, $\overline{q}_u \simeq 0$ and $\overline{q}_\eta \simeq 0$ for the quasi-equilibrium profiles if the sheet flow model is applicable to these tests.

Analysis of Experimental Results

The velocity and free surface data from tests 1 and 2 were analyzed to obtain the quantities necessary for the evaluation of (3) and (7) at the 17 cross-shore measurement locations. At the locations of the three point velocity profiles the time series from only the middle position were used. A cubic spline interpolation was performed on each set of \bar{q} values to obtain the corresponding derivatives, $\partial \bar{q}/\partial x$. Eq. (8) was then used to calculate $\partial z_b/\partial t$ at each location. The results of these calculations are displayed graphically in Figure 6 and detailed in Orzech and Kobayashi (1997).

The sheet flow model predicts large onshore transport rates and rapid profile change in the surf zone, especially for test 1. For perfectly equilibrium profiles, $\bar{q} = 0$ and $\partial z_b/\partial t = 0$, but the measured equilibrium profiles for test 1 and 2 had uncertainties on the order of 1 cm/hr. For test 2, $\partial z_b/\partial t$ based on u is somewhat smaller than $\partial z_b/\partial t$ based on η , though still greater than 1.0 cm/hr over the bar. For both tests the bar is predicted to move further shoreward. Change of this magnitude was of course not observed in any part of either profile.

ENERGETICS-BASED MODEL FOR OFFSHORE BAR MOVEMENT

Unlike the sheet flow model, the energetics-based model developed by Bowen (1980) and Bailard (1981) attempts to account for both onshore/offshore transport due to wave asymmetry and offshore transport due to undertow, as well as slope effects due to gravity. This model separates sediment transport into bed load and suspended load components, including separate terms for each of the above transport mechanisms. Because these experiments were conducted in a wave flume there was no longshore current to contribute to sediment transport. This model does not account for the initiation of sediment movement, although the fine sand particles were observed to move constantly during this experiment.

Theory

In the energetics model, the time-averaged cross-shore sediment transport rate per unit width can generally be expressed by equation (2) in Thornton *et al.* (1996). For the present analysis with zero longshore velocity, however, the net onshore sediment transport rate \overline{q} is simplified as:

$$\overline{q} = K_b \left(\overline{|u(t)|^2 \tilde{u}(t)} \right) + K_b \left(\overline{|u(t)|^2 \overline{u}} \right) - K_{bg} \left(\overline{|u(t)|^3} \right) + K_s \left(\overline{|u(t)|^3 \tilde{u}(t)} \right) + K_s \left(\overline{|u(t)|^3 \overline{u}} \right) - K_{sg} \left(\overline{|u(t)|^5} \right)$$

$$\equiv \overline{q}_{bw} + \overline{q}_{bu} + \overline{q}_{bg} + \overline{q}_{sw} + \overline{q}_{su} + \overline{q}_{sg}$$
(9)

where u(t) = cross-shore horizontal velocity, which is positive shoreward in this analysis. In (9), the first three terms represent bed load (subscript b) produced by wave asymmetry (subscript w), undertow (subscript u), and the effects of gravity (subscript g) on the bottom slope, respectively. The final three terms represent suspended load (subscript s) produced by the same three respective effects. The coefficients in (9)



Figure 6: Net Transport Rates \overline{q}_u , \overline{q}_η and Bottom Elevation Change Rates $(\partial z_b/\partial t)_u$, $(\partial z_b/\partial t)_\eta$, Over Test 1 and 2 Profiles (Sheet Flow Model)

are expressed as

$$\begin{split} K_b &= \frac{1}{(s-1)g} C_f \frac{\varepsilon_b}{\tan(\phi)} \quad ; \quad K_{bg} = K_b \frac{\tan(\beta)}{\tan(\phi)} \\ K_s &= \frac{1}{(s-1)g} C_f \frac{\varepsilon_s}{w} \quad ; \quad K_{sg} = K_s \frac{\varepsilon_s}{w} \tan(\beta) \end{split}$$

where $s = \rho_s/\rho$ = specific gravity of sand, C_f = drag coefficient, ϕ = internal friction angle of sand, ε_b = bed load efficiency factor, ε_s = suspended load efficiency factor, $\tan(\beta)$ = local bed slope, and w = fall velocity. In this experiment, s = 2.66 and w = 1.9 cm/s, while the other parameters are given the same values as those used in Thornton *et al.* (1996): C_f = 0.003, $\tan(\phi)$ = 0.63, ε_b = 0.135, and ε_s = 0.015. The local slope, $\tan(\beta) = dz_b/dx$, is computed using the equilibrium bottom profile, $z_b(x)$, for each test.

After \overline{q} is obtained using (9), the predicted erosion or accretion rate, $\partial z_b/\partial t$, for the profile at given locations is found by use of (8). This corresponds to equation (1) in Thornton *et al.*, except that here the value of $\mu = (1 - n_p)$ is taken as 0.6 rather than 0.7 because the measured porosity $n_p = 0.4$ in this experiment.

Analysis of Experimental Results

Each of the time-averaged velocity expressions in (9) was evaluated for the 17 velocity locations, with the middle position again selected from the two three-point velocity profiles. The values of the six sediment load components are plotted over the test 1 and test 2 equilibrium profiles in Figure 7 (tabulated in Orzech and Kobayashi (1997)). In both tests the largest values predicted for all the terms in (9) occur around the location of the bar or terrace edge, in the vicinity of $x \simeq 7-8$ m. The more intense breaking of test 1 is clearly visible from the comparison of sediment loads in this region. At x = 7.85 m in this test, a large volume of sediment is transported seaward by both wave asymmetry and undertow effects; however, just 0.5 m shoreward at x = 8.35 m, the wave asymmetry suspended load \bar{q}_{sw} suddenly reverses and becomes strongly positive onshore. In general, the predicted suspended load values are larger than the corresponding bed load terms, as would be expected from the relatively large waves and fine sand used in the experiment.

For the two velocity profile locations, the cross-shore sediment transport rate \overline{q} in (9) was found to be largely insensitive to the elevation of the cross-shore velocity, u(t), used for its prediction. Predicted sediment loads due to undertow and bottom slope remained nearly constant over the depth, while predictions of wave-asymmetry-induced loads \overline{q}_{sw} and \overline{q}_{bw} varied somewhat more, especially for test 1. It may thus be reasonable to predict the sediment load due to undertow and bottom effects by measuring velocities at mid-depth (as done in this study) instead of immediately outside the bottom boundary layer as specified by the theory of Bailard (1981).

The total sediment loads and rates of profile change predicted by the energetics model at each cross-shore location are plotted over the equilibrium profiles in Figure 8 (tabulated in Orzech and Kobayashi (1997)). Unlike Thornton *et al.* (1996), the



Figure 7: Cross-Shore Variation of Suspended Load and Bed Load Quantities Over Test 1 and 2 Profiles (Energetics Model)



Figure 8: Predicted Bed Load \overline{q}_b , Suspended Load \overline{q}_s , and Total Load \overline{q} with $\partial z_b/\partial t$ Over Test 1 and 2 Profiles (Energetics Model)

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suspended load, \overline{q}_s , is rarely an entire order of magnitude larger than the bed load, \overline{q}_b , in this laboratory experiment. In test 2, the total bed load \overline{q}_b is actually comparable to the total suspended load, \overline{q}_s , at most locations. This may be reasonable under the milder wave conditions of this test. If the energetics model could predict the existence of these equilibrium profiles, one would expect that the net sediment transport $\overline{q} = 0$. In test 1, \overline{q} is predicted to be very weakly onshore at locations seaward of the breaker zone but strongly offshore at almost all locations on the terrace. The magnitude of the parameter $\partial z_b/\partial t$ is not greater than 1 cm/hr except in the region of intense breaking. In test 2, all locations are predicted to have a weak offshore transport, with $\partial z_b/\partial t$ consistently small and within measurement errors of 1 cm/hr. These values are smaller than those in test 1 because waves broke less intensely, leading to slower profile changes. For both tests the energetics model predicts the growth and offshore movement of a bar near x = 7 m and the deepening of a trough near x = 8 m. The energetics model is unable to predict quasi-equilibrium profiles with no net sediment transport, although it predicts the bottom profile change within 1 cm/hr for test 2.

CONCLUSIONS

The general failure of state-of-the-art sediment transport models to predict the equilibrium profiles illustrates the limitations on such models at the present time. While the sheet flow model predicted onshore sediment transport and bar migration, the energetics model predicted offshore sediment transport and bar migration. The energetics model, which attempts to account for effects of wave asymmetry, undertow, and bottom slope, predicted smaller profile change than the sheet flow model, which did not include the effects of wave breaking and undertow. The standard $Ay^{2/3}$ equilibrium profile represents the profile inside the surf zone fairly well, although it does not predict the existence of a bar and terrace.

An improved sediment transport model will need to account for the combined onshore and offshore transport effects of the above two models. In this study, the energetics model was more accurate than the sheet flow model in both tests 1 and 2, very likely because of the inclusion of a greater number of sediment transport mechanisms. The assumptions involved in simulating each transport mechanism must also be carefully reviewed and questioned. In this experiment, both models assume the instantaneous response of bed load *and* suspended load particles to the horizontal velocity immediately outside the bottom boundary layer. However, the water was observed to be consistently very cloudy during both tests; thus the assumption of the instantaneous response of suspended load appears to be questionable. A better model of sediment suspension might need to consider the instantaneous vertical velocity associated with the coherent intermittent fluid motion, as suggested in Cox and Kobayashi (1998b).

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