ABSTRACT: An algorithm for representing seasonal variations in shoreline position as caused by on-offshore sediment transport in a one-line model is presented. The study was based on an analysis of an 11-year long time series of waves and shoreline location obtained at Duck, North Carolina. Two different approaches were used: one was based on wave steepness and dimensionless fall speed and the other on ratio of the maximum near-bottom orbital velocity to the critical velocity required to initiate sediment movement. Both applications showed that it is possible to reproduce the overall spatial and temporal behavior of seasonal cross-shore transport related shoreline variation using two simple one-line model compatible approaches. Preliminary calculations show that the method may be used for analyzing the seasonal behavior of a beach. At the same time, the impact of individual storms are not represented to any significant degree.

INTRODUCTION

One-line models of shoreline response have demonstrated their predictive capabilities in numerous projects (Hanson et al. 1988). This class of models calculates shoreline position changes that occur over a period of years to decades. Changes in shoreline position are assumed to be produced by spatial and temporal differences in the longshore sand transport rate (Hanson 1989, Hanson and Kraus 1989). Thus, this type of model is best suited to situations where there is a systematic trend in long-term change in shoreline position, such as recession down-drift of a groin. Cross-shore transport effects, such as storm-induced erosion and cyclical movement of shoreline position associated with seasonal variation in wave climate, are assumed to cancel over a long enough simulation period or are accounted for through external calculation.

1Assoc. Prof., Dept. of Water Res. Engrg., Univ. of Lund, Box 118, S-22100 Lund, Sweden. Fax +46 46 222 8987. Email Hans.Hanson@tvrl.lth.se, Magnus.Larson@tvrl.lth.se.
Cross-shore models, on the other hand, typically predict beach change as a result of cross-shore transport produced by storms (Larson and Kraus 1989). This type of models is simplified by omitting longshore transport processes. Although these models have been relatively successful in reproducing short-term profile response to individual storm, they have been less suitable for long-term predictions. In principle, the two types of models could be combined to obtain both long- and short-time changes in shoreline position. Such attempts have been done (e.g. Bakker 1969, Perlin and Dean 1979, Larson et al. 1990), but these approaches have not yet found their way into engineering practice.

Nevertheless, it is recognized that seasonal variations can play a significant role in the long-term evolution of the shoreline. Therefore, it would be a significant step forward if, at least in a schematic way, the main features of cross-shore related seasonal shoreline variations could be represented in one-line shoreline response models. If successful, it would constitute a model for simulating long-term cross-shore related coastal evolution for engineering use and at the same time eliminate one of the main constraints of the use of one-line models.

An important criterion for the formulation of the cross-shore contribution was that it should be compatible with the one-line formulation in terms of independent variables and level of sophistication. This may seem like a paradox because numerous studies (e.g. Larson and Kraus 1989) have shown that the profile shape is a key parameter for the cross-shore transport magnitude and direction. At the same time, it is recognized that one-line models does not provide any information about the profile shape. Thus, the relatively simple one-line model cannot be expected to represent the complex impact of individual storms, but rather to capture the more long-term effects. Consequently, the objective of this study was to represent seasonal variation due to cross-shore transport using a one-line model.

**PROCEDURE**

**Transport Magnitude**

Because the one-line model does not require or provide any information about the actual shape of the bottom profile, the cross-shore transport magnitude must be calculated using a relation that is independent of the profile shape. Numerous formulae for the cross-shore sand transport rate may be found in the literature (Horikawa 1988, p. 196 ff.). Many of these may be written in the generic form

\[
\frac{q_o}{wd} = K_q (\Psi - \Psi_c)^a
\]

where \(q_o\) = cross-shore sediment transport rate per unit width, \(w\) = sediment fall speed, \(d\) = median grain size, \(K_q\) = transport coefficient, \((\Psi_c)\) \(\Psi\) = (critical) Shields parameter, and \(a\) = exponent. The value of the exponent \(a\) varies from 1 to 3 in the different proposed
relationships. In the present study a value of 1.5 was selected in accordance with Watanabe (1982). For simplicity, the value of the critical Shields parameter was set to zero. Thus the expression may be written:

$$q_o \over wd = K_q \Psi^{1.5}$$  \hspace{1cm} (2)

Recognizing that the Shields parameter may be expressed in the form (using shallow water approximations),

$$\Psi = f_w u_{b,m}^2 \over 2sgd = K_q u_{b,m}^2 \over gd = K_q gH \over gd = K_q H \over d$$  \hspace{1cm} (3)

where $f_w$ = Jonsson's (1966) wave friction factor, $u_{b,m}$ = the maximum horizontal near-bottom orbital fluid velocity, $s$ = sediment specific density in water, $g$ = acceleration due to gravity, $K_q$ = coefficient, $H$ = the local wave height. In order to be consistent with the one-line theory, it is presumed that the conditions at the break point may be used to compute the overall cross-shore sediment transport, i.e., a fixed transport distribution is assumed that scales with the breaker height. Thus, the cross-shore sediment transport rate per unit length alongshore, $q_o$, here regarded as a potential rate, may be calculated, with $K = K_q K_r^{1.5}$:

$$q_o = wd \left( K_q \left( \frac{H_b}{d} \right) \right)^{1.5} = w \left( \frac{H_b^2}{d} \right)^{0.5}$$  \hspace{1cm} (4)

where $H_b$ is the breaking wave height. This relationship will be used in the following application of a numerical model with $K$ used as a calibration parameter. However, because this relation does not give a transport direction, this will have to be resolved separately.

**Transport Direction using Fall Speed**

Kraus et al. (1991) presented several criteria for discriminating between erosive and accretive conditions on the basis of a classification of profile response to breaking waves in large wave tank (LWT) model tests as well as from the field. The LWT data consisted of two data sets. One was from US Army Corps of Engineers, compiled by Kraus and Larson (1988) and referred to in the following as the CERC data. The other set was from the Central Research Institute for Electric Power Industry (CRIEPI) in Japan (Kajima et al. 1982) and referred to here as the CRIEPI data. The field data set (Kraus and Mason 1991) consisted of observations of well-documented responses to small and large storms.

In the present study, on the basis of shoreline (MSL) movement alone, the results from the LWT and field tests were re-evaluated in terms of erosion and accretion. Figure 1 shows
Figure 1. Discriminating erosive and accretive shoreline conditions on the basis of wave steepness and dimensionless fall speed (modified from Kraus et al. 1991).

the data plotted on wave steepness $H_o/L_o$ versus dimensionless wave steepness $H_o/wT$, where $H_o$ = deep-water wave height, $L_o$ = deep-water wavelength, $w$ = fall speed, and $T$ = wave period. The proposed criteria,

$$\frac{H_o}{L_o} = M_1 \left( \frac{H_o}{wT} \right)^3 = 0.00054 \left( \frac{H_o}{wT} \right)^3 \quad (5)$$

where $M_1$ = discriminator coefficient, discriminates between erosion and accretion with a skill of 0.92, defined as the ratio of correct predictions to total observations (Seymour and Castel 1989).

By assuming a Rayleigh probability distribution function (pdf) for the wave height, the smallest (critical) erosional deep-water wave height $H_{oc}$ may be derived from Eq. (5) as (Larson 1996):

$$H_{oc} = \sqrt[3]{\frac{1}{M_1} \frac{(wT)^3}{L_o}} \quad (6)$$
At a location \( x \), a certain portion \( \delta_e \) of the broken waves will be erosional:

\[
\delta_e = e^{\frac{\left(\frac{H}{H_{\text{reo}}}\right)^2}{e^{\frac{\left(\frac{H_{\text{bo}}}{H_{\text{reo}}}\right)^2}}}} \quad \delta_e \leq 1
\] (7)

Thus, if evaluated at the shoreline where \( H_{\text{bo}} = 0 \), the portion will be

\[
\delta_e = e^{\frac{\left(\frac{H}{H_{\text{reo}}}\right)^2}{e^{\frac{\left(\frac{H_{\text{bo}}}{H_{\text{reo}}}\right)^2}}}} \quad \delta_e \leq 1
\] (8)

By substituting Eq. (6) into Eq. (8) we obtain

\[
\delta_e = e^{\frac{1}{M} \frac{H_{\text{reo}}}{L_o} \left(\frac{u}{H_{\text{reo}}}\right)^3} \quad \delta_e \leq 1
\] (9)

If a portion \( \delta_e \) of the broken waves are erosional the rest \( \delta_a = 1 - \delta_e \) of the waves must be accretionary. Giving each single wave equal weight when summing up to determine the net direction yields,

\[
\xi = \delta_a - \delta_e = 1 - 2\delta_e = 1 - 2\xi \quad -1 \leq \xi \leq 1
\] (10)

where \( \xi \) gives the net direction and a weight that includes the variability in wave height defined by the Rayleigh pdf. Substituting Eq. (9) into Eq. (10) gives the net transport direction as:

\[
\xi = 1 - 2e^{\frac{1}{M} \frac{H_{\text{reo}}}{L_o} \left(\frac{u}{H_{\text{reo}}}\right)^3} = 1 - 2\xi \quad -1 \leq \xi \leq 1
\] (11)

**Transport Direction using Velocity Ratios**

Based on the same field data as above, Ahrens and Hands (1998) parameterizes beach erosion and accretion processes on the basis of the ratio of the maximum near-bottom orbital fluid velocity \( u_{b,m} \) to the critical velocity \( u_{cr,\text{eq}} \) required to initiate sediment movement under the wave. Whereas Ahrens and Hands (1998) use stream function wave theory, the present study will focus on linear wave theory, again to be consistent with traditional one-line modeling procedures. The critical velocity \( u_{cr,\text{eq}} \) was defined as (Hallermeier 1980),

\[
u_{\text{crit}} = \sqrt{8\Delta g d}
\] (12)

where \( \Delta = (\rho_s - \rho)/\rho_s \), \( \rho_s \) (\( \rho \)) is the density of the sediment (water).
Figure 2 shows the data plotted on wave steepness $H_s/L_o$ versus relative velocity $u_{b,m}/u_{crit}$. The horizontal full line is given by $u_{b,m}/u_{crit} = 9.85$ and represents the criterion that separates most of the accretion and erosion events. Thus, in this analysis the discriminating criterion may be written as

$$u_{b,m} = M_2 u_{crit} = 9.85 u_{crit}$$

where $M_2$ = discriminator coefficient, that discriminates with a skill of 0.92 if only field data are considered and 0.85 if all data are considered. The application of the random wave concept leads to relationships quite similar to Eqs. (6) to (11) in the previous section and will not be discussed here.

**Slope Effects**

Because the proposed Eq. (4) gives the transport magnitude for a horizontal bottom, it has to be corrected for slope effects on an inclined bottom. Madsen (1993) presents a relationship,

![Figure 2. Discriminating erosive and accretive shoreline conditions on the basis of wave steepness and velocity ratios.](image-url)
$q_p = q \frac{1}{1 + \frac{\tan \beta}{\tan \theta_m}} = q k_p \quad (14)$

where $\beta$ = bottom slope (positive for upward slope in transport direction), $\theta_m$ = angle of moving friction (here set to $30^\circ$ according to King (1991)), and $k_p$ = slope coefficient.

**Actual Transport Rate**

With the transport potential $q_o$, the net direction and magnitude of transport $\xi$, and the slope coefficient $k_p$ all determined, the actual transport rate $q$ is given by:

$q = q_o \xi k_p \quad (15)$

**Shoreline Change**

Following the one-line theory, the shoreline location $y$ is calculated based on the continuity equation:

$$\frac{\partial y}{\partial t} + \frac{1}{D_c} \left( \frac{\partial Q}{\partial x} - q \right) = 0 \quad (16)$$

where $y$ = shoreline position, $t$ = time, $D_c$ = vertical extension of the active profile, $Q$ = the longshore sediment transport rate, and $x$ = the alongshore coordinate. By assuming no longshore transport gradients ($\partial Q/\partial x = 0$), the shoreline change $\Delta y$ during a time step $\Delta t$ is given by:

$$\Delta y = \pm q \frac{\Delta t}{D_c} \quad (17)$$

where a positive sign corresponds to onshore transport.

**FIELD APPLICATION**

**Field Data**

As an application of the proposed procedure, simultaneously collected data on waves and beach profiles from the US Army Field Research Facility at Duck, North Carolina (Figure 3) (Howd and Birkemeier 1987, Lee and Birkemeier 1993) were analyzed to investigate the relationship between the incident waves and the seasonal shoreline variations over a longer
time period. The data set comprised shoreline positions extracted from surveys taken bi-weekly in profile line 188 during 11 years (Figure 4). Spectral wave properties (significant height and peak period), recorded at least every 6 hours, were available for the same period.

The beach at Duck is one of the most well-documented field sites in the world with collected time series on waves and profiles that are unique in their length and quality. Thus, it was a natural choice to employ data from Duck in the development and validation of the cross-shore transport algorithm. However, coarse material is often found around the shoreline at Duck (Larson 1991) creating a steep beach face and an armoring effect that could significantly reduce the shoreline response to changes in the wave conditions. Because of this the beach response at Duck might not be as well-behaved as on other beaches. In spite of this, the Duck data was chosen. The long-term variation of different contour lines was extracted by linear interpolation from the measured profiles. For this study, the MSL (+0.08 m NGVD) of profile line 188 was analyzed.

In order to investigate the temporal scale in the shoreline (defined as MSL) variation, an FFT analysis was performed on the data series after a linear trend was removed. Figure 5 (solid line) shows that there is a strong annual variation with a frequency of 1 year, that most likely is associated with seasonal variations in the wave climate. The long-term trend in the shoreline signal was assumed to be associated with alongshore processes and was, therefore, removed from the continued analysis. For simplicity, the long-term trend was assumed to be linear, although the FFT analysis indicates that there are other low-frequency oscillations in the signal.

Model Simulation using Fall Speed

Based on the wave time series, the cross-shore sediment transport rate was calculated at each time step according to Eq. (15). In the calibration procedure, a best fit value of the discriminator coefficient $M_s$, that separates onshore from offshore transport, was determined to minimize the difference between measured and calculated shoreline positions on the basis of visual evaluation. The shoreline change and the corresponding shoreline location associated with the cross-shore sediment transport was calculated based on Eq. (17). A best fit
Figure 4: Temporal variation of MSL in profile line 188.

Figure 5: FFT analysis of periodicity in shoreline fluctuations.
value of $M_1 = 0.00056$ was obtained (Figure 6), which is only marginally greater than the value obtained from the comparison with laboratory model tests (Figure 1). As for the actual shoreline change, the calculated shoreline variation was analyzed by FFT. The temporal behavior of the oscillations is shown in Figure 5 (dashed line). Similar to the measured variations the calculated ones show a strong annual signal indicating that there is a pronounced seasonal variation in addition to a low-frequency variation associated with interannual changes in the wave climate. It is also worth noticing that there are virtually no higher frequencies represented. Thus, the calculations show that, although the seasonal behavior of the shoreline is clearly expressed, the impact of individual storms are not represented to any significant degree. In addition, a comparison between measured and calculated shoreline changes (Figure 6) indicates that he excursion around the mean value is of the right order of magnitude. Hence, the overall temporal and spatial properties seem to be well represented in the model. However, there does not seem to be a coherent instantaneous behavior of the two signals. The phase difference is generally quite significant.

**Model Simulation using Relative Velocities**

Using the same data as in the above section, the shoreline change was determined using the criterion based on the velocity ratio. Figure 7 shows the result of a simulation with a best fit value of $M_2 = 8.0$, which is somewhat smaller than the value obtained from the comparison with the LWT and field data sets (Figure 2), but still with a skill of 0.85, i.e., as

![Figure 6. Comparison between calculated and measured temporal variation of MSL.](image-url)
good as the proposed value of 9.85. A visual evaluation seems to suggest that this method reproduces the actual shoreline change slightly better than the previous method based on fall speed. Like in the previous application, the seasonal behavior is quite well represented.

CONCLUSIONS

The present analysis showed that it is possible to reproduce the overall spatial and temporal behavior of seasonal cross-shore transport related shoreline variation using two simple one-line model compatible approaches. Similar to measured shoreline changes at Duck, N.C., model calculations over 20 years showed clear seasonal variations. FFT analysis of measured and calculated shoreline changes also showed strong similarities for more long-term variations. As expected, the impact of individual storms that are present in the measurements, are not represented in the simulated shoreline behavior to any significant degree. More work is still needed to represent the instantaneous changes correctly with the proposed methods.

The reason for the limited success of the proposed methods could be that the selected parameters are more related to bar behavior than to shoreline behavior and that the lag represents an intrinsic phase shift between the shoreline response and the bar movement as documented in Hanson et al. (1997). Another, more pessimistic, explanation could be that
the approach is too simplified to be able to represent such a complex process as cross-shore transport under breaking waves. It is not totally clear, at present, which of the two hypothetical reasons is true even though the former is our working hypothesis.

The predictive capability of the proposed method is still not demonstrated. In this respect, it will have to be shown that the criteria cannot only be used to determine the direction of the transport but also the magnitude. For this reason, further applications will be made with field data from other well-documented sites.

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