Measurement of Shear Stress on a Moveable Bed

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Abstract

A shear plate has been developed to obtain direct measurements of bottom shear stress under non-breaking surface gravity waves on a movable sand bed. The shear plate provides a direct measurement of the bottom shear stress on a sand bend under complex flow conditions particularly over bottom roughnesses. It was designed for use under laboratory oscillatory flow conditions where the periods of motion range from 1.5 to 6 seconds and the range of shear forces are from 0.08 Nm^{-2} to 84 Nm^{-2}. The spatially averaged bottom shear stress was determined and the quadratic drag law was used to convert measurements of bottom shear stress to values of wave friction factor. The apparent roughness of the bed was specified using four times the sum of the mean and standard deviation of the sediment profile on the shear plate. Values of wave friction factor compared well with both theory and data obtained through previous laboratory studies.

Introduction

The accurate measurement of bottom shear stress over moveable sand beds is critical to our understanding of nearshore processes, including sediment transport and

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shallow water wave attenuation. Although indirect techniques for estimating bottom shear stress including the use of bottom boundary layer velocity profiles, wave energy dissipation, and the measurement of Reynolds stresses have been used successfully in previous experiments, these techniques can be problematic in complex flow regimes particularly in the presence of bedforms. A shear plate, designed using the same principles as those employed in the field of naval architecture for measurement of the resisting force on a towed vessel, avoids these problems. The shear plate provides a non-intrusive, direct measurement of the tangential force on the sand bed and can resolve temporal changes in the bottom stress caused by variable wave-generated bedforms.

As part of a development effort pointed at designing a shear plate for use in the open ocean environment, a shear plate has been constructed for use in a laboratory wave tank. This enabled the evaluation of a number of design issues pertinent to long-term deployments in the field. It also permitted the testing of the apparatus under carefully controlled conditions, and the comparison of results to previous theoretical and experimental investigations of moveable bed dynamics.

Shear Plate Design and Tests

The shear plate design and experimental methodology are outlined in Rankin and Hires (submitted to J. Geophys. Res., 1998). The original experimental design was modified slightly during the present study. These modifications included the use of an aluminum plate mounted on top of an existing drag balance and the reduction of the gaps between the shear plate and the false bottom from 1.5 cm to 0.6 cm.

The shear plate was designed for use under laboratory oscillatory flow conditions where the periods of motion range from 1.5 to 6 seconds and the range of shear forces are from 0.08 Nm^{-2} to 84 Nm^{-2}. The shear plate was 0.875 m long by 0.620 m wide (0.543 m^2 area) and was constructed by mounting a 0.63 cm thick aluminum plate on top of a drag balance (fig. 1). The drag balance was instrumented with a waterproofed linear variable differential transformer (LVDT) which produced linearly increasing voltage with the horizontal deflection of the plate. Resolution was to ± 0.08 Nm^{-2}.

The shear plate was tested for moveable bed conditions under waves in Davidson Laboratory's Large Wave Tank. The wave tank is 95m long, 3.7 m wide and can accommodate water depths up to 1.8 m. It is equipped with a double flap wavemaker that is designed to simulate both regular and irregular waves. It can produce wave heights to 46 cm and wave periods to 6 seconds.

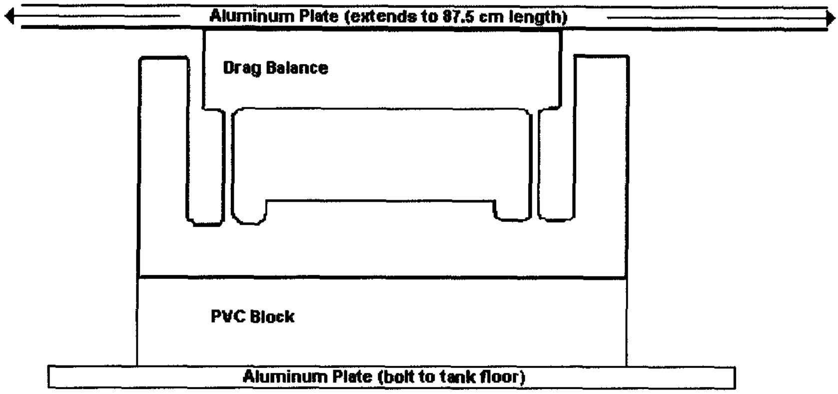

Figure 1. Schematic of Shear Plate Design

The shear plate was bolted to the concrete floor of the wave tank (fig. 2). A false bottom was constructed to surround the plate. The gap between the shear plate and the false bottom was fixed conservatively at 0.6 cm to ensure that sediment grains would not become trapped or wedged in the gap. A 1.0 to 2.5 cm thick layer of cohesionless quartz sand (d_{50} = 0.23mm) was placed on top of the shear plate and the surrounding false bottom. The shear stress on the sand bed was measured under waves ranging in height from 11 to 38 cm and periods from 1.75 to 6 seconds.

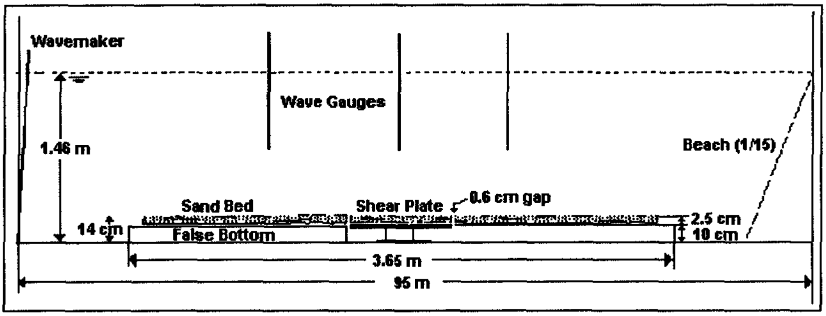

Figure 2. Experimental Set-up (not to scale)

Surface elevation records were obtained using three resistance-type gauges mounted on the sill of the wave tank. The gauges were installed 17.50 m, 28.17 m (over the center of the shear plate) and 38.84 m from the wavemaker, respectively. The gauges resolve ± 1 mm changes in the surface elevation and were calibrated over a range of 15.24 cm.

Profiles of the sediment bed were taken according to the procedure outlined in Rankin and Hires (submitted to J. Geophys. Res., 1998). Table 1 summarizes the test matrix.

Results

Figure 3 represents sample time histories for the resistance on the shear plate with sand, resistance on the shear plate without sand, and the surface elevation.

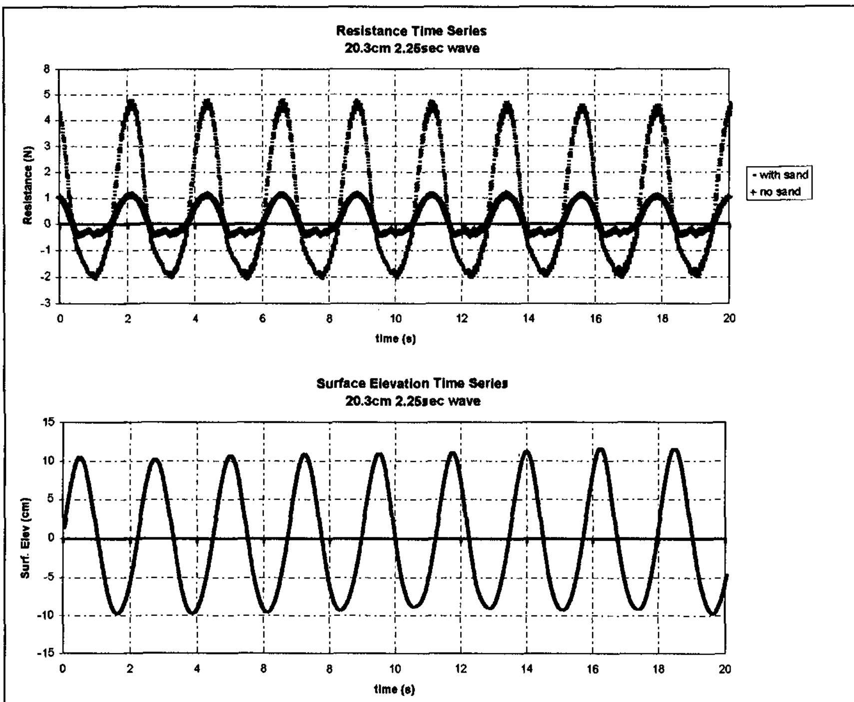

Figure 3. Time History for the Resistance on the Shear Plate and the Water Surface Elevation

The time history for the resistance on the shear plate without sand contains a conspicuous departure from a sine wave due to a relatively large second order harmonic signal. What is not immediately apparent is that this signal is also present in the time history for the resistance on the shear plate with sand; in this case the amplitude of the first order harmonic is over two orders of magnitude larger than the amplitude for the second order harmonic and therefore the non-linearity is not easily discerned.

Harmonic analysis was performed on the resistance and surface elevation time histories to determine the amplitude and phase of the harmonic components of the signal. The analysis program reads the input file and determines the fundamental period by counting the number of zero crossings. Next, it estimates the amplitude and phase of the signal by using a least squares method. The analysis assumes that the signal is of the form

$$y_i(t) = A_{i0} + \sum_{n=1}^{NH} A_{in} \cos(n\frac{2\pi}{T}t) + \sum_{n=1}^{NH} B_{in} \sin(n\frac{2\pi}{T}t) \qquad (1)$$

where

$n = 1,2,3, \ldots$
A_{i0} : mean of the time series
A_{in} : harmonic coefficient
B_{in} : harmonic coefficient
T : fundamental period
NH : order of harmonics

The program calculates the harmonic coefficients, A_{in} and B_{in} to minimize the sum of the squares of the difference between the recorded value and the estimate. The cosine and sine coefficients, A_{in} and B_{in}, were changed to the amplitude and phase as,

$$y_i(t) = C_{i0} + \sum_{n=1}^{NH} C_{in} \cos(n\frac{2\pi}{T}t - \phi_{in}) \qquad (2)$$

where

C_{i0} : mean of the time series
C_{in} : amplitude of the nth harmonic of the time series
ϕ_{in} : phase of the nth harmonic of the time series

Amplitudes of the first order harmonics are reported in Table 1. The amplitudes of the second order harmonics were nearly identical for runs with sand and without sand and had relatively small magnitudes that ranged from 0.05 to 0.20 N. It is believed that the second harmonics are related to the mechanical properties of the instrumentation.

The elimination of secondary forces to obtain the force on the shear plate due to the resistance associated with the sand bed alone, F_{res}, through the subtraction of the first order amplitude of the resistance on the shear plate without sand from the first order amplitude of the resistance on the shear plate with sand, and the calculation of the spatially averaged bottom shear stress, $\overline{\tau_{bm}}$, are reviewed in Rankin and Hires (submitted to J. Geophys. Res., 1998). These data are summarized in Table 1.

Analysis

apparent roughness

The apparent roughness, k_n, was specified initially as $k_n = 4 * \eta$ as suggested by Wikramanayake and Madsen (1994). After their extensive review and analysis of field and laboratory studies, they suggest that the ripple length does not appear to have a significant influence on the apparent bottom roughness. It has been concluded by Wiberg and Harris (1994) that for wave dominated environments, the ripple length scales with wave excursion amplitude ($\lambda \cong 0.75A_b$) where A_b is the bottom excursion amplitude and the ripple steepness, η / λ, is nearly constant between 0.15 and 0.17 for equilibrium conditions. The results presented here are in close agreement with Wiberg and Harris (1994) with ripple steepnesses between 0.11 and 0.15 and an average wave steepness of 0.13 indicating that equilibrium conditions were achieved (Table 1).

Figure 4 illustrates the profile of the rippled sand bed and represents a general example of the bottom roughness.

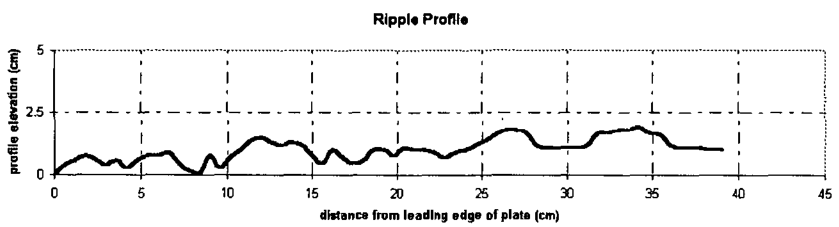

Figure 4. Sample of a Sediment Profile on the Shear Plate

Since the excursion amplitude of a section of the flow extended from the false bottom, across the gap and onto the shear plate, the ripple height alone was not sufficient to specify the apparent roughness (fig. 5).

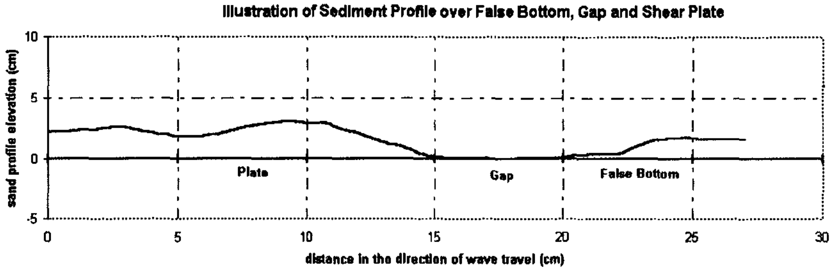

Figure 5. Sediment Profile from the False Bottom, across Gap and onto Shear Plate

The thickness of the sand bed itself made a significant contribution to the bottom roughness. The sand bed on the plate was therefore treated as a large ripple ($\eta \cong 2.5$cm and $\lambda \cong 91.5$cm where η is the ripple height and λ is the ripple length) with smaller surface ripples superimposed ($\eta \cong 0.3$cm and $\lambda \cong 10.0$cm). The apparent roughness was finally specified using the mean and standard deviation of the sand bed profile by $k_n = 4 * (mean + s.d.)$ (Table 1; fig. 6)

<u>friction factor</u>

The quadratic drag law was used to convert measurements of bottom shear stress to values of wave friction factor using a spatially integrated version of Jonsson's (1966) equation

$$\overline{\tau_{bm}} = 1/2 f_w \rho \overline{u_{bm}^2} \qquad (3)$$

where $\overline{\tau_{bm}}$ is the maximum spatially integrated shear stress on the bed over the area of the shear plate, f_w is a wave friction factor to be experimentally determined, ρ is the density of the water and $\overline{u_{bm}^2}$ is the spatially integrated maximum of the squared near-bottom velocities on the plate at one instant in time.

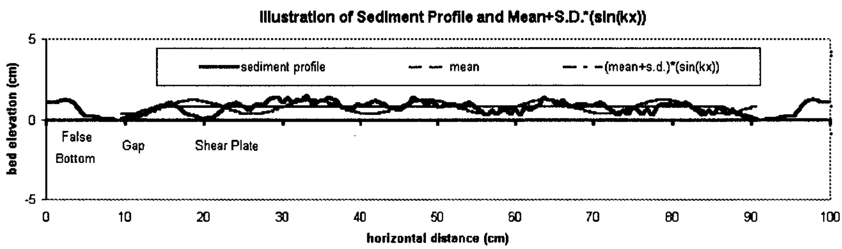

Figure 6. Specification of the Apparent Roughness of the Bed using the Mean and Std. Dev. of the sediment profile

Our values of wave friction factor were compared with Jonsson's (1966) semi-empirical theory,

$$\frac{1}{4\sqrt{f_w}} + \log_{10}\left[\frac{1}{4\sqrt{f_w}}\right] = \log_{10}\left[\frac{A_b}{k_n}\right] - 0.08 \tag{4}$$

(where k_n / A_b is a measure of relative roughness), and Grant and Madsen's (1979) theory,

$$f_w = \frac{0.08}{\ker^2 2\sqrt{\varsigma_o} + i\, kei^2\, 2\sqrt{\varsigma_o}} \tag{5}$$

(where ζ_o is a non-dimensional distance from the boundary) (figs. 7 and 8). Our values were also compared to values of wave energy dissipation factor, f_e, from previous studies using moveable beds, assuming that $f_w = f_e$.

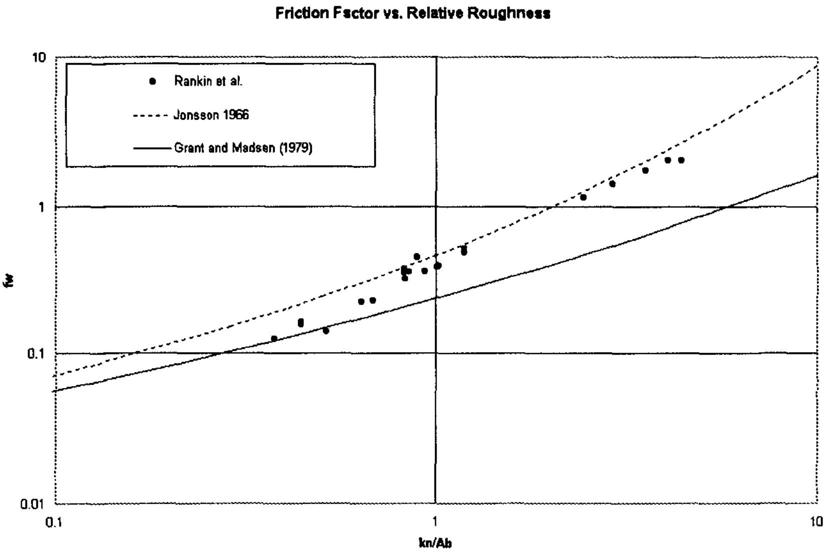

Figure 7. Comparison of f_w with Jonsson's (1966) and Grant and Madsen's (1979) theories (u_b is specified by linear wave theory)

These data compared well with those obtained from previous studies; values of the wave friction factor derived from the shear plate were as accurate as those derived from the energy dissipation method. The data also suggest that the value for the wave friction factor may be predicted using Jonsson's (1966) theory (fig. 8) for values of relative roughness greater than unity. Simons et al. (1996) also found agreement for this range of relative roughness.

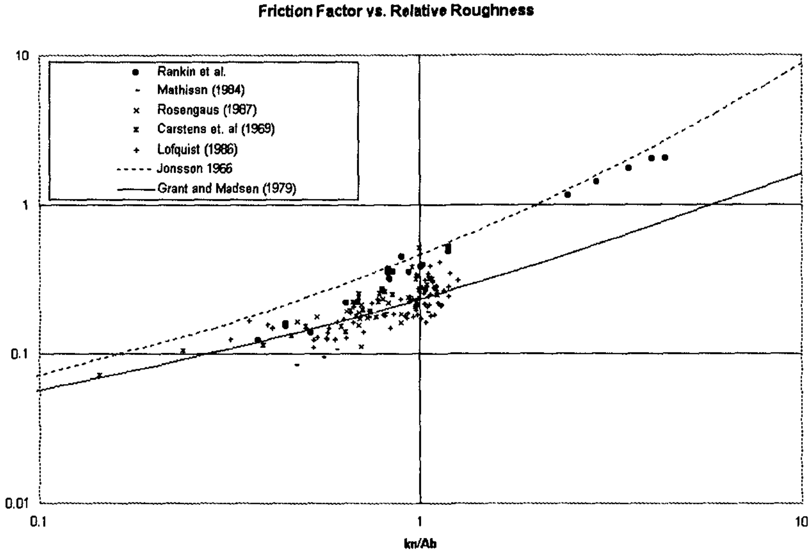

Figure 8. Comparison of f_w with previous studies

Conclusions

A shear plate has been designed and constructed for use in the laboratory measurement of bottom shear stress over moveable beds. The device has been tested in a large wave tank under a range of wave conditions that offers the opportunity of comparing the results to the findings of previous investigators. The values of wave friction factor obtained here compare well with both theory and the data obtained through former laboratory experiments. These comparisons have proven the utility of the device. Discretion must be exercised in the specification of the apparent bottom roughness; for this study, the bed thickness as well as the ripple height significantly contributed to the bottom roughness. The wave friction factor does not reach a constant value for values of relative roughness exceeding unity. Instead, data in this region were in reasonable agreement with Jonsson's (1966) theory. A new configuration of the shear plate will include an attempt to eliminate the gap between the plate and the false bottom by introducing a layer of latex between the plate and the sand bed. In addition, the dimensions of the shear plate will be greatly reduced.

Appendix

TABLE 1: Measured Wave Parameters, Ripple Geometry and Residual Force on the Shear Plate

Runs with Sand									Runs with no Sand							
H	T	F	k_n		η	λ	η/λ	Re_b	H	T	F	F_{res}	τ_b	u_b	k_n/A_b	f_w
cm	s	N	cm	cm	cm	cm			cm	s	N	N	N/m^2	cm/s		
			4*mean	4*s.d.	mean	mean										
27.9	3.00	3.83	5.2	1.6	1.2	11.0	0.11	3.7E+04	26.9	3.00	0.93	2.89	5.33	27.9	0.5	0.14
27.9	3.00	3.51	4.4	0.8	no ripples			3.7E+04	26.9	3.00	0.93	2.58	4.76	27.9	0.4	0.13
35.1	2.25	4.23	2.8	1.2	0.7	6.7	0.11	2.6E+04	35.1	2.25	1.16	3.07	5.46	27.1	0.4	0.16
35.1	2.25	4.09	4.0	0.4	no ripples			2.6E+04	35.1	2.25	1.16	2.94	5.41	27.1	0.4	0.16
21.8	3.00	3.51	4.8	2.4	1.6	13.0	0.12	2.2E+04	21.6	3.00	0.71	2.80	5.17	21.7	0.7	0.23
20.3	3.00	3.11	5.6	0.4	no ripples			1.9E+04	21.6	3.00	0.71	2.40	4.43	20.1	0.6	0.22
21.1	2.25	3.25	4.4	0.4	no ripples			9.2E+03	21.6	2.25	0.76	2.49	4.59	16.0	0.8	0.38
20.8	2.25	3.25	4.0	1.6	1.0	7.5	0.13	8.9E+03	21.6	2.25	0.76	2.49	4.59	15.8	1.0	0.38
29.0	1.75	3.20	3.6	1.2	0.6	4.8	0.13	5.5E+03	28.7	1.75	0.67	2.54	4.67	14.0	1.2	0.53
29.0	1.75	2.85	2.8	0.8	no ripples			5.5E+03	28.7	1.75	0.67	2.18	4.02	14.0	0.9	0.46
40.7	2.25	12.32	9.2	1.6	1.3	10.0	0.13	3.6E+04	41.6	2.25	2.76	9.56	17.63	31.7	0.9	0.37
27.3	3.00	8.45	9.2	1.6	1.3	10.7	0.12	3.5E+04	27.8	3.00	2.18	6.27	11.56	27.2	0.8	0.32
22.3	3.00	6.85	9.2	1.6	1.2	10.7	0.11	2.3E+04	22.8	3.00	1.69	5.16	9.51	22.1	1.0	0.40
32.5	2.25	10.14	9.2	1.6	0.9	6.0	0.15	2.2E+04	31.9	2.25	2.31	7.83	14.43	25.0	1.2	0.49
13.9	5.02	3.42	9.2	1.6	1.3	10.0	0.13	2.1E+04	14.4	5.02	0.85	2.58	4.76	16.4	0.8	0.36
11.4	5.98	2.45	9.2	1.6	0.8	7.0	0.11	1.7E+04	11.1	5.98	0.67	1.78	3.28	13.5	0.8	0.36
32.0	1.75	8.50	9.2	1.6	0.7	5.0	0.13	6.8E+03	31.5	1.75	1.51	6.98	12.87	15.7	2.5	1.16
27.4	1.75	7.29	9.2	1.6	0.7	5.0	0.13	4.8E+03	27.2	1.75	1.29	6.00	11.07	13.2	2.9	1.42
22.9	1.75	6.05	9.2	1.6	0.7	5.0	0.13	3.3E+03	22.4	1.75	1.11	4.94	9.10	10.8	3.6	1.74
20.1	1.75	5.34	9.2	1.6	0.7	5.0	0.13	2.5E+03	20.5	1.75	0.98	4.36	8.04	9.4	4.1	2.03
18.6	1.75	4.67	9.2	1.6	0.7	5.0	0.13	2.1E+03	18.3	1.75	0.93	3.74	6.88	8.7	4.4	2.04

Acknowledgements

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