Directional Wave Overtopping Estimation Model and Experimental Verification

Tetsuya Hiraishi and Haruhiro Maruyama [1]

Abstract

The accurate estimation of wave overtopping quantity is of great importance for the design of seawalls to protect artificial islands constructed offshore. The estimation model for overtopping rate in multi-directional waves is proposed and it is experimentally verified. The numerical results demonstrate that the overtopping rate in multi-directional waves becomes smaller than in uni-directional waves when the incident principal wave angles are near to the normal to sea wall face. The result becomes reverse when the wave angle is more than 30°.

Introduction

Artificial islands are widely under construction and plan to create a new area for the human-conscious space like a promenade, marina and recreational parks as well as the residential and commercial area. An artificial island is surrounded by sea walls which need a specified crown height to prevent serious wave overtopping. When the wave overtopping volumes become very large, the human conscious space like a promenade and a green belt are heavily damaged. Meanwhile, a high barrier may cause the disconnection between the water front and inner land. The lower crown height is more beneficial to make a wide human conscious space. Therefore, the crown heights of sea walls in an artificial island to reduce the wave overtopping rate under the appropriate allowable levels should be determined with high accuracy (Franco, et.al.,1994).

Wave overtopping rates have been studied on the basis of the results in the two dimensional channel test (Goda, et.al.,1975). They proposed a practical formula estimating the wave overtopping rate on the vertical and block-armored

---

[1] Wave Laboratory, Port and Harbour Research Institute, Nagase 3-1-1, Yokosuka, Japan 239-0826

sea walls in shallow water rigion with an uniform slope. Uni-directional waves have been employed in the experimental study for wave overtopping mainly because of the lack of the facility generating multi-directional waves in a laboratory. The laboratory works employing multi-directional waves are strongly desired as real sea waves have characteristics of directional random waves of which energy propagates to various directions. In the paper, we discuss the applocability of estimation model of directional wave overtopping rate at vertical sea walls in deep water area.

Numerical Simulation for Directional Wave Overtopping

A new approach to estimate the wave overtopping rate is needed for the design of seawalls in deep water area where the wave directionality appears remarkably. To estimate the wave overtopping rate in multi-directional waves, the evaluation of that for waves propagating obliquely to the sea wall face is needed. Takayama et.al.(1982) suggest that the wave overtopping rate of oblique incident waves may become smaller than that of the normal incident waves. We assume that an infinite vertical sea wall is arranged on the x-axis and that the incident waves are perfectly reflected at the sea wall. **Figure 1** shows the imaginary seawall model for numerical simulation. The water surface elevation at the front of sea wall is expressed in the single summation method (Takayama and Hiraishi,1989) as follows ;

$$\zeta(x,t) = K \sum_{n=1}^{Ns} 2a_n \cos(k_n \sin\theta_n + \sigma_n t + \varepsilon_n) \tag{1}$$

where, $\zeta(x,t)$ is the water surface elevation at the point x and time t. The subscript n denotes the value for the n-th component wave. In the eq.(1), a_n, k_n, θ_n, σ_n and ε_n represents the amplitude, wave number, wave angle, angular frequency and random phase respectively. The wave angle is measured clockwise from the normal line to seawall face. The number Ns is the total number of the synthesized component waves. The indication K is the coefficient for the consideration of wave non-linearity and breaking introduced by Goda et.al.(1975). In this study, assuming that the water depth in front of the sea wall is enough deep, we employ the following expression ;

$$K = \zeta/H = \min\ \{ [1.0 + a_b H_{1/3}/h],\ c_b \} \tag{2}$$

where, 'min' means that the minimum value in { } is adopted. In the vertical sea wall, we employ a_b=1.0 and c_b=10.

Figure 2 shows the image of wave overtopping. When the water surface elevation becomes higher than the crown height, the wave overtopping occurs.

The wave overtopping rate q is calculated as follows ;

$$q = \begin{cases} C(\zeta - h_c{}^*)^{3/2} & (\zeta \geqq h_c{}^*) \\ 0 & (\zeta < h_c{}^*) \end{cases} \tag{3}$$

where C is the wave overtopping coefficient given by $C_o\sqrt{2g}$. The value of Co

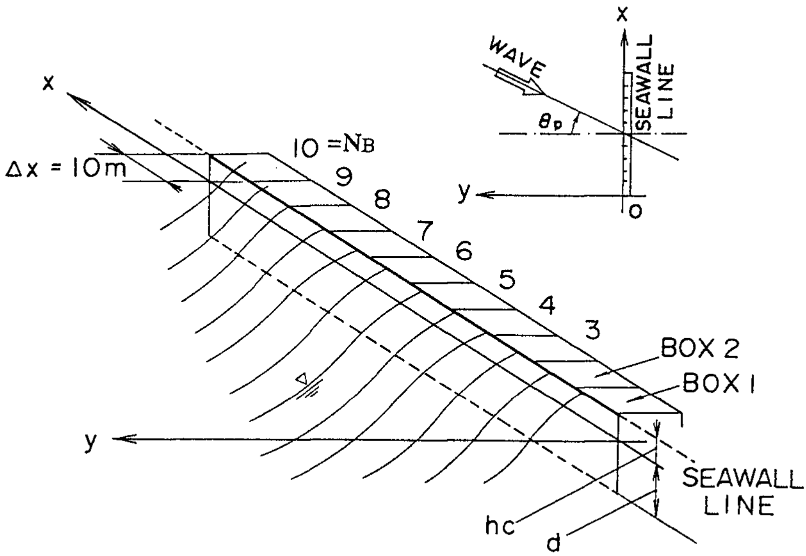


Figure 1 Sketch of sea wall model in numerical simulation

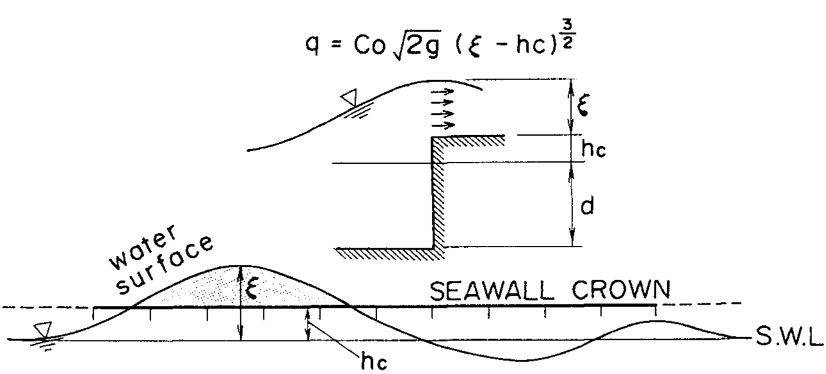


Figure 2 Wave overtopping image at vertical wall

is determined as the estimated q agrees with the measured q. The consideration for the value of C_o is done in the later chapter. The symbol h_c* represents the virtual crown height for oblique component waves. The virtual crown height h_c* is evaluated as the linear superposition of the function λ_n standing for the variation of uni-directional wave overtopping rate for the incident angle.

$$h_c* = \left[\sum_{n=1}^{NS} \frac{S(f_n)\Delta f_n}{m_0}\lambda_n\right] \tag{4}$$

$$\lambda_n = \frac{h_{co}}{\beta_n} \tag{5}$$

where, f, $S(f)$, Δf and m_0 denotes the frequency, spectrum density, interval of representative frequency and total wave energy. The symbol h_{co} represents the original crown height of sea wall.

The function β_n is the dimensionless modification factor of crown height for n–th component waves. It corresponds to the ratio of crown height in oblique incident wave giving a specified overtopping volumes to that gives the same volume in normal incident waves. The factor is determined later according to the variation of overtopping rate in uni–directional waves.

The amplitude of n–th component wave is given by

$$a_n = \sqrt{2S(f_n)\Delta f_n \Delta\theta_n} \tag{6}$$

where $\Delta\theta_n$ represents and the angle band of n–th component. The directional spectrum is the product of the frequency spectrum and directional function. The modified Bretschneider–Mitsuyasu type spectrum (Goda,1987) and Mitsuyasu–type directional function modified for engineering simplification (Goda and Suzuki,1975) were employed as the wave energy spectrum and directional spreading function respectively. The modified frequency spectrum is expressed as,

$$S(f) = 0.205 H_{1/3}^2 T_{1/3}^2 (T_{1/3} f)^{-5} \exp[-0.75(T_{1/3} f)^{-4}] \tag{7}$$

where $H_{1/3}$ and $T_{1/3}$ represents the significant wave height and period. The modified Mitsuyasu–type directional function is given by,

$$G(\theta;f) = G_o \cos^{2S}\left(\frac{\theta-\theta_p}{2}\right) \quad (-90^\circ < \theta < 90^\circ) \tag{8}$$

where θ_p is the principal direction and G_o the normalized coefficient expressed as ;

$$G_0 = \frac{1}{\int_{-\pi/2+\theta_p}^{\pi/2+\theta_p} G(\theta;f)d\theta} \tag{9}$$

The parameter s is the angular spreading coefficient determined as follows;

$$s = \begin{cases} (f/f_p)^5 S_{max} & : f \leqq f_p \\ (f/f_p)^{-2.5} S_{max} & : f > f_p \end{cases} \tag{10}$$

where S_{max} denotes the angular spreading parameter representing directionality of wave energy, and f_p the peak frequency of wave spectrum. In the simulation, we assume the wave overtopping volumes are measured in the measurement boxes with the width of Δx behind the sea wall as shown in Fig.1. The total overtopped water volume Q_i in the i-th box is evaluated as,

$$Q_i = q(t)t_0 \Delta x \tag{11}$$

The component value q_i is caluculated in eq.(3). The averaged wave overtopping rate q for a sea wall is estimated as,

$$q = \frac{1}{N} \sum_{i=1}^{NB} \frac{Q_i}{t_o \Delta x} \tag{12}$$

where N_B is the total number of the boxes. Compared with the several computation results with different condition, the following number are adopted in the computation;

$$\Delta x = 10\text{m}, \quad N_B = 10, \quad N_S = 300, \quad t_0 = 20\text{min} \tag{13}$$

Experimental Setup

Figure 3 shows the arrangements of experimental models in a directional wave basin. Along a side, a directional random wave generator with 60 paddles each 50cm wide is installed. The generator is possible to reproduce oblique regular waves, oblique uni-directional waves and directional random waves with the principal wave direction normal to the gerateor face. A continuous vertical wall to represent an offshore vertical breakwater is installed parallel to the generator face with the distance of 6.4m. The total length of the vertical wall is 18m and the both of edge sides are slightly bent to protect the effects of diffracted waves. A measurement box for wave overtopping volumes is attached at the backside of center part of seawall to obtain the total overtopped volume Q. In the

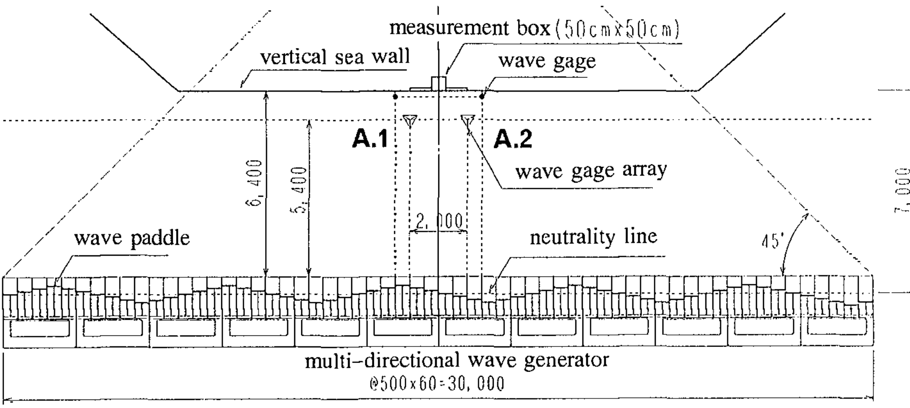


Figure 3 Arrangement of wave generator and sea wall model

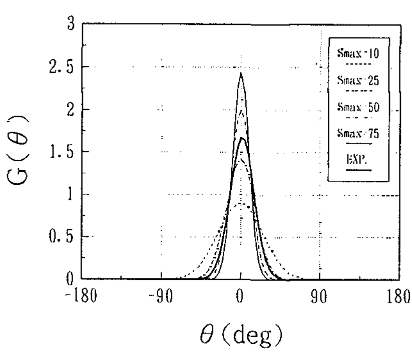


Figure 4 Analysed directional function

experiments, the wave overtopping rates are analysed for the case of (i) uni-directional waves with oblique incident angle and (ii) multi-directional waves with normal principal wave direction. The single summation method is employed to synthesize the wave signal to generator.

In the experiment, the scale is 1/100 and the water depth is fixed to be 38m in the prototype. The crown height hc are 6 and 8m. The significant wave height $H_{1/3}$ of incident uni- and multi-directional waves is 6 and 8m. The significant wave periods are changed from 11.3 to 14.1s. In the case (i) uni-directional waves, the incident wave angle varies from 0 to 45°. In the case (ii) multi-directional waves, the principal angle is 0° and the target S_{max} is changed from 10 to 100. **Figure 4** shows the example of wave directional function measured at the wave gage array located at the measurement box without seawall model. The Extende Maximum Entoropy Method (Hashimoto et. al., 1994) was employed to analyze the directional spectrum. The experimental wave represented in the solid line is generated with the target of S_{max}=100. However, the peak height of directional function in the experimental waves is not as tall as the theoretical peak height for case of S_{max}=50. The reduction of angular concentration may be caused by the finite length of generator.

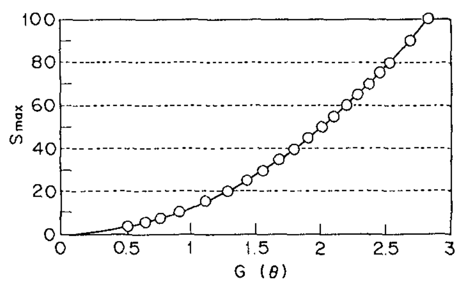


Figure 5 Correlation of S_{max} and $G(\theta_p)$

So, the angular spreading in the experiment is evaluated accoring to the peak value of directional function measured in the wave basin. **Figure 5** shows the correlation of the parameter S_{max} and the measured $G(\theta_p)$. The values of S_{max} are evaluated employing Fig.5 after the values of $G(\theta_p)$ is obtained in the spectral analysis.

Determination of Experimental Coefficient

In the numerical simulation model, the overtopping coeffficient C_o and the crown height modification factor β_n should be evaluated in the experiment. Those value is derived from the comaprison between the measured and estimated results in uni-directional wave condition. **Figure 6** shows the variation of the error between the estimated and measured wave overtopping rates in uni-directional waves with the normal incident angle for the value of C_o. The ratio of wave overtopping rate becomes about 1 for both cases of $T_{1/3}$=11.3 and 14.1sec when C_o becomes 0.3. Therefore, in the simulation model, C_o=0.3 is adopted. This value is smaller than the value proposed by Goda et.al.(1976).

Figure 7 shows the ratio of overtopping rates in oblique uni-directional waves to those in the uni-directional waves with the incident angle normal to wall face. The ratio becomes more than 1.0 in case of oblique incident waves with 7.5° . When the angle becomes larger than 7.5° the overtopping rate decreases as the incident angle increases. When the angle becomes 45° , the rate becomes

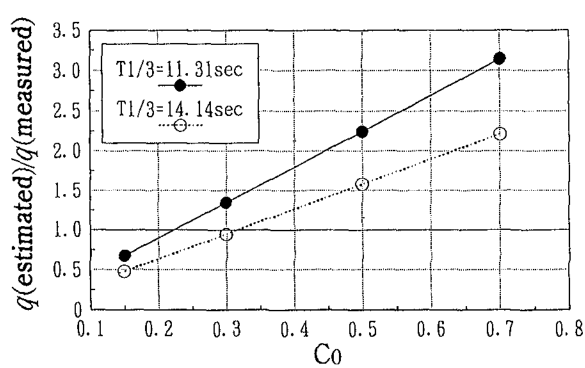


Figure 6 Error ratio of esimated to measured wave overtopping rate

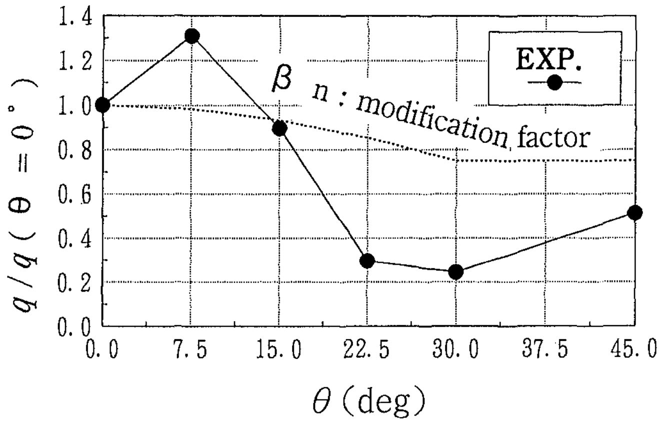


Figure 7 Variation of uni-directional wave overtopping rate and crown height modification factor for incident wave angle

slightly larger than those in case of θ =30°. The reason why the overtopping rate becomes maximum at the angle slightly apart from the normal is not clear. However, our results demonstrate that the overtopping wave rate has tendency to become large when the incident angle becomes slightly defferent from the normal. The mofification factor for crown height is determined so that it represents the effects of oblique wave components to reduce the wave overtopping rates.

In Fig.7, the overtopping rate decreases rapidly as the angle increases in the range of θ <30°. Meanwhile, the variation of wave overtopping rate becomes small for the range of θ >30°. Therefore, the modification factor is asuumed as follows;

$$\beta_n = \begin{cases} 1-\sin^2\theta_n & (|\theta| \leqq 30^\circ) \\ 1-\sin^2 30^\circ & (|\theta| > 30^\circ) \end{cases} \tag{14}$$

The broken line in Fig.7 indicates the profile of modification factor.

Verification of Numerical Model

Figure 8 shows the comparison between the measured and estimated wave overtopping rate in uni-directional waves. In the following figures, q^* represents the dimensionless overtopping rate given by,

$$q^* = q \;/\; \sqrt{2gH_o'^{\,3}} \tag{15}$$

where Ho' represents the equivalent offshore wave height. The comparison is done for the various cases for different incident angles. The estimated wave overtopping rates agree well with measured ones. The good agreement between the estimated and measured wave overtopping rates demonstrates that the numerical model with the coefficient of C_0=0.3 is suitable for the estimation of uni-directional wave cases.

Figure 9 shows the range of estimation error for uni-directional wave condition. The estimated q are plotted within the range for 50% error of measured values. The estimated error for analytical caluculation model proposed Goda(1975) included the estimation error of 100–200% for wave overtopping rates. Therefore, the error in the present simulation model is small and allowable to be employed in the sea wall design.

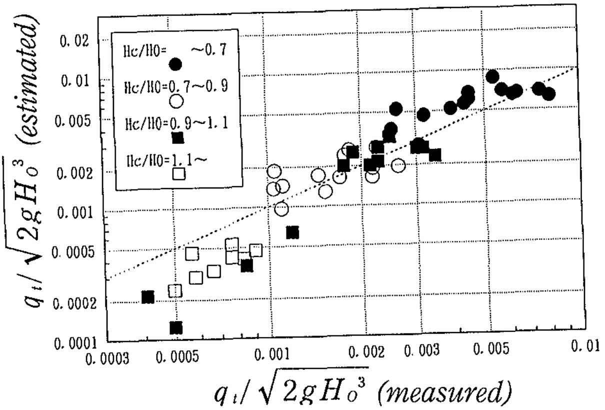


Figure 8 Comparison of estimated and measured ectional overtopping rate in uni-directional wave condition with oblique incident angles

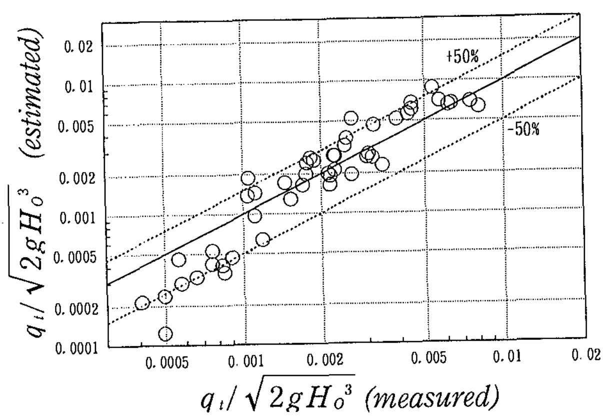
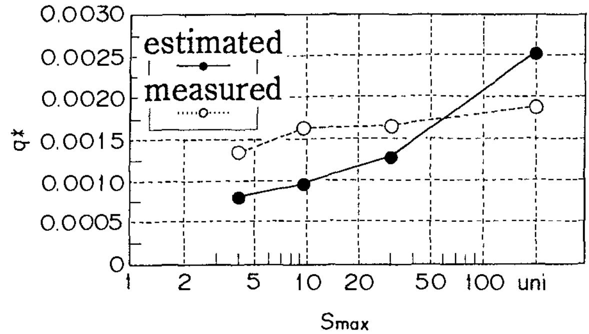
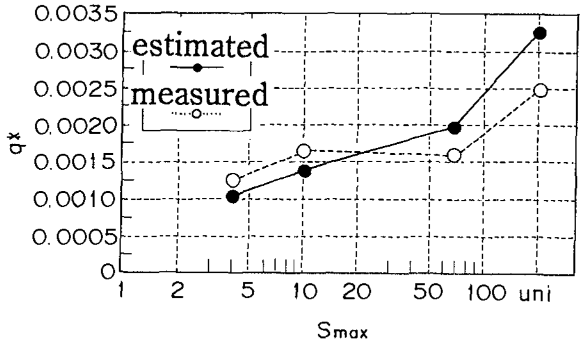


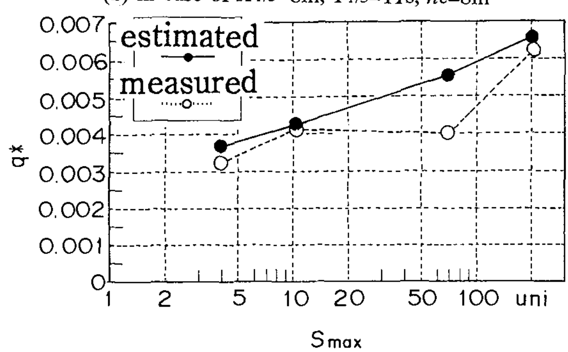
Figure 9 estimation error range for uni-directional wave overtopping rate



(1) in case of $H_{1/3}$=6m, $T_{1/3}$=11s, h_c=6m



(2) in case of $H_{1/3}$=8m, $T_{1/3}$=11s, h_c=8m



(3) in case of $H_{1/3}$=8m, $T_{1/3}$=14s, h_c=6m

Figure 10 Comparison of estimated and measured q^* in multi-directional waves

Figure 10 shows the variation of computed and measured wave overtopping rate in multi-directional waves with different angular spreading parameters. The figures represent the comparison between the estimated and measured wave overtopping rate for the various wave height and period conditions. The indication "uni" in the figures shows the case for uni-directional waves. In Fig.10(1), the measured wave overtopping rate gradually increases as the value of S_{max} increases. The estimated wave overtopping ratea are smaller than the measured ones by about 50%. In case of uni-directional waves, the estimated wave overtopping rate becomes larger than the measured one by 25%. The error is small and allowable for the sea wall design metod. The numerical model gives good agreemnet between the estimated and measured overtopping rate.

In Fig.10(2), the differences between the estimated and measured wave overtopping rates are small at the various angular spreading parameter. The estimated values agree well with the measured ones. In Fig.10(3) for the case of $T_{1/3}$=14.1s, the agreement between the estimated and measured wave overtopping rate is good except the case of S_{max}=75. For the case of S_{max}=75, the error between the estimated values is about 30%. Therefore, the simulation model is applicable to estimate the wave overtopping rate of directional random waves with different angular spreading parameter.

The good agreemnet in the cases for multi-directional waves shown in Fig.10 demonstrates that the simulation model is applicable to estimate the wave overtopping characteristics in directional sea conditions.

Wave Overtopping Characteristics in Directional Seas

Figure 11 shows the numerical results for the variation of directional wave overtopping rate for angular spreading parameter in the various crown height. The principal wave direction is normal in the cases. For any crown height cases, the wave overtopping rate becomes large as the S_{max} becomes large. In the normal principal wave dierection, the case of uni-directional waves becomes critical. If the wave directionality is obtained in a construction site, the employment of multi-directional waves may give reduction of the design crown height of sea walls.

Figure 12 shows the variation of multi-directional wave overtopping rates for principal wave direction θ_p. In both cases of uni- and multi-directional waves, the wave overtopping rate decreases as the the incident angle increases. The variation ratio, however, is different between the uni- and multi-directional waves. The wave overtopping rate changes more gradually in case of multi-directional waves. Therefore, the wave overtopping rate in multi-directional waves is smaller than in uni-directional wave conditions for the case of θ_p=0 and 15°. Meanwhile, for the case of θ_p=30°, the wave overtopping rate in multi-directional waves is larger than in the uni-directional wave conditions. The

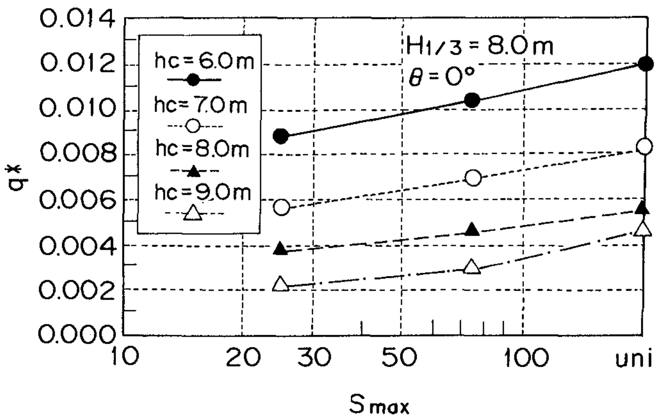


Figure 11 Esimated variation of q^* for sea wall crown height

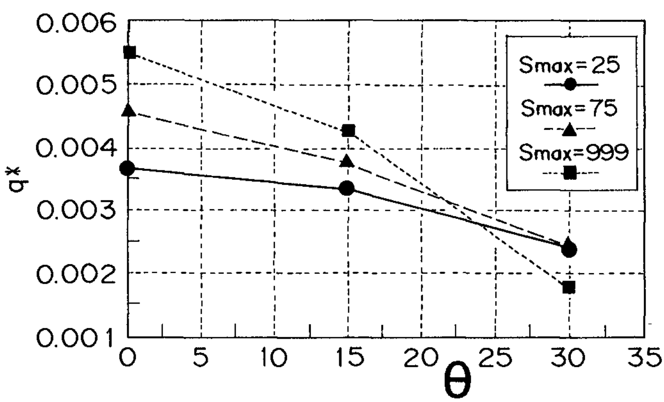


Figure 12 Variation of q^* for principale wave angle θ_p

multi-directional wave condition gives the critical condition to the design. Therefore, in case that the principal wave direction θ_p becomes larger than 30°, the multi-directional waves should be considered in the design of seawall.

Conclusions

A numerical model for directional wave overtopping is proposed. The following conclusions are derived from the comparison of numerical and experimental results;

(1) The numerical model is applicable to estimate the wave overtopping rates in the both cases of oblique uni-directional waves and of multi-directional waves with the normal principal direction when the overtopping flow coefficients and modified crown height coefficient.

(2) The wave overtopping rates in multi-directional waves become larger than in uni-directional waves in case of the principal wave direction θ p more than 30° meanwhile the former ones become smaller than the later one in case of normal and small incident principal angle.

REFERENCES

Franco, L., M. de Gerloni, J.W. van der Meer (1994) : Wave overtopping on vertical and composite breakwaters, *24th ICCE*, pp.1030-1045.

Goda,Y., Y.Kishira and Y.Kamiyama (1975) : Laboratory investigation on the overtopping rate of seawalls by irregular waves, *Rept.Port and Harbour Res. Inst.*, Vol.14, No.4, pp.3-44.

Goda,Y. and Y.Suzuki (1975) : Computation of refraction and diffraction of sea waves with Mitsuyasu's directional spectrum, *Tech.Note Port and Harbour Res. Inst.,* No.230, 45p.

Goda,Y. (1987) : Standard spectra and statistics of sea waves derived by numerical simulation, *Proc.34th Japanese Conf. Coastal Engi.*,pp.131-134.

Hashimoto, N., N. Nagai and T. Asai (1994) : Extension of the maximum entoropy principle method for directional wave spectrum estimation, *24th ICCE*, pp.232-246.

Takayama,T., T.Nagai, K.Nishida and T.Sekiguchi (1984) : Experiments on oblique random wave overtopping rates over sea walls, *31st Japanese Conf. Coastal Engi.*, pp.542-546.

Takayama,T. and T.Hiraishi (1989) : Reproducibility of directional random waves in laboratory wave simulation, *Rept. of Port and Harbour Res. Inst.*,Vol.28, No.4, pp.3-24.