STATISTICAL DISTRIBUTION OF HORIZONTAL WAVE FORCES ON VERTICAL BREAKWATERS

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ABSTRACT

This paper discusses the statistical distribution of wave impact forces on vertical wall structures. It describes new analysis carried out based on the distribution type to identify the parameter combinations that lead to wave impacts.

Comprehensive 2-dimensional random wave model tests were carried out to measure wave pressures on a range of structure types. Initial analysis of these tests showed that the Weibull distribution could be used to describe non-breaking wave forces.

For some structural configurations however the wave forces were found to give a poor fit with the Weibull distribution. These data had been excluded from the initial analysis. The new analysis described in this paper has resulted in a revised parameter map to summarise the risk of wave impact, derived from the full data set and based on the distribution type.

1. INTRODUCTION

Vertical wall structures were widely used in the UK as seawalls and breakwaters before rubble mound breakwaters and rip-rap revetments gained popularity in the 1900's. In Japan and Italy, caissons continue to be favoured for breakwater construction. A thorough understanding of the relationship between wave forces on vertical walls and overall structure geometry is necessary to enable the design of a suitable, cost-effective structure, with an acceptably low risk of failure. It is also desirable to minimise the risk of severe exposure to large breaking wave impact forces, so that future maintenance costs are not unduly high.

Vertical walls in the marine environment are subjected to highly variable wave loads, yet existing design methods are deterministic. The main design method for caissons, developed by Goda (1974, 1985), provides a good estimate of non-breaking wave forces (McKenna, 1997). For wave impacts however, the predicted forces using Goda's

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method are 'effective' values, damped by the response of the structure and foundation, rather than actual impact loads. These wave impact loads were previously thought to be unimportant for the stability of massive structures such as caissons, but Oumeraci et al (1995) have shown that repeated wave impacts can cause incremental displacements.

It is necessary to recognise the wide variation in force type and magnitude, and to incorporate a measure of their probability of occurrence, in order to develop improved design methods. Work within MAST III-PROVERBS has been directed towards this aim.

Identification of a suitable statistical distribution for wave forces on vertical wall structures is particularly important for the development of probabilistic design tools. Previous work on the statistics of wave forces has concentrated on establishing the extreme distribution of a series of (theoretically) regular waves, see for instance Kirkgoz (1995), but these results cannot be applied to real (random) seas.

The Weibull distribution can be used to describe pulsating wave forces measured in model tests on caisson breakwaters using random waves. Allsop et al (1996a) have shown that the onset of wave impacts can be defined as a change in gradient of the probability plot where wave forces start to increase rapidly above those predicted by the simple Weibull distribution.

The design load for many structures will however be due to wave impact forces rather than non-breaking waves. It is therefore important to be able to establish the statistical distribution of wave impact forces, and to know the relative proportions of pulsating and impact forces for a given structural configuration.

2. MODEL TESTS

Comprehensive parametric model tests were conducted in a random wave flume at HR Wallingford during 1994. These tests were designed to investigate the influence of structure geometry on wave pressures and forces, using random waves. Over 200 tests were carried out in all, to explore the effects of varying the following parameters:

- Significant offshore and inshore wave heights, $H_{s0}$ and $H_{si}$;
- Water depth in front of structure, $h_s$; and crest freeboard $R_c$;
- Wave steepness, $s_{m0}$; and peak wavelength at structure toe, $L_{p0}$;
- Water depth over mound in front of wall, $d$; and berm height, $h_b$;
- Berm width, $B_t$;
- Front slope of mound, $\alpha$;
- Depth of embedment of caisson into mound, $h_0-h_c$;

The caisson was instrumented with 8 pressure transducers on the front face and 4 on the underside as shown in Figure 1. Pressure data were acquired on all transducers simultaneously at 400Hz for 500 waves in each test. These tests have been described previously by Allsop et al (1996b).
3. DATA ANALYSIS

A data analysis program was developed to carry out the initial analysis of the pressure data acquired from these model tests. The pressure data were spatially integrated at each time-step to give horizontal and uplift force time histories. In order to study the statistics of the data set, information regarding the force maxima was required for each wave event. An event was defined from the beginning of each rapid pressure rise on the transducer at still water level, and the analysis program was configured to search for the required information within each ‘event’.

As horizontal and uplift pressure and force maxima do not necessarily occur at the same time, a number of output files were written, so that each phenomenon could be studied separately.

The next stage of the data analysis was to rank the pressure and force maxima contained in the output files in order of magnitude to enable their statistics to be explored. This paper concentrates on the results of the further analysis of the horizontal forces.

4. STATISTICS OF WAVE FORCES

Wave impact forces on coastal structures are extremely variable, and are therefore better described by their statistics than by any single deterministic value. In selecting an appropriate statistical distribution for wave forces, it is an advantage to maintain any association between wave heights and wave forces. If a narrow band process is assumed, wave heights can be approximated by the Rayleigh distribution, which is a special form of the more generic 3-parameter Weibull distribution.
Analysis of the pressure data from these model tests by Allsop et al (1996a,b) showed that the Weibull distribution could be used to give a good description of wave forces from non-breaking waves (referred to here as pulsating wave forces), as shown in Figure 2. It was also found that the onset of wave impacts could be defined by a change in the gradient of the Weibull plot, where the wave forces start increasing more rapidly with non-exceedance level, Figure 3.

**Figure 2**  Weibull distribution of pulsating wave forces

**Figure 3**  Transition from pulsating forces to impact forces
This method of estimating the percentage of impacts was applied to each test, and was found to give good agreement with observations from the flume testing and with analytical considerations of the physical processes involved. This analysis led to the development of a parameter map, Allsop et al (1996a,b) that could be used to estimate the risk of wave impacts on a particular structure.

Further analysis by McKenna (1997) revealed that the Weibull distribution could also be used to describe wave impact forces in some instances, Figure 4. It was also noted that for other cases the percentage of impacts could be difficult to determine, as the change in the gradient of the distribution was not distinct. These cases were selected for further investigation, and it was found that the data points plotted as a gradual curve on Weibull axes, rather than a straight line with a sharp change in gradient, Figure 5.

The structural configurations corresponding to these curved distributions were investigated and a common link was found in the relationship between the offshore wave conditions and the local water depth at the structure. Limiting values of $H_{ow}/h_s = 0.425$ and $L_{mo}/h_s = 17$ were identified, beyond which wave forces no longer fit the Weibull distribution, Figure 6. The distortion of the distribution is caused by shallow water wave transformations. The smallest waves reach the structure relatively unchanged, but there is a gradual increase in the amount of modification of the wave shape as the wave heights and lengths increase. The incident wave height distribution was found to be non-Rayleigh for these cases, Figure 7.

![Figure 4](image-url)  
**Figure 4**  
Weibull Distribution of horizontal wave impact forces
Figure 5  Non-Weibull distribution

Figure 6  Parameter ranges for Weibull distributed forces
5. DEVELOPMENT OF PARAMETER MAP

Attempts to analyse the full data set by Allsop et al (1996b) had highlighted the need to separate the data into regions of similar response characteristics. The dimensionless parameters that influenced wave forces were identified as:

- Relative mound height, $h_b/h_s$;
- Relative wave height, $H_s/d$;
- Relative berm length, $B_{eq}/L_{pi}$.

The effects of these parameters were summarised in the parameter map presented by Allsop et al (1996a,b). The derivation of that parameter map had concentrated on a central core of data, where the water depth over the rubble mound berm, $d$, was greater than twice the significant inshore waveheight, $H_i$. Cases where the water level was close to or below the top of the rubble mound were excluded.

Potential weaknesses had been recognised in that derivation, both in excluding of a set of data from the analysis, and in using the dimensionless parameter $H_s/d$, which tends to infinity as the water depth over the rubble mound tends to zero. It was concluded that, if possible, this parameter should not be used in further work to extend the parameter map to include the full data set.

The approach used in this further work to include the full data set in the parameter map considered the effects of the non-Weibull distributed forces, and included additional parameters to describe the effects of transformation from the offshore wave climate to the inshore wave climate. The dimensionless parameters used in the new parameter map (Figure 8) are:

- Relative offshore wave height, $H_{so}/h_s$;
- Relative offshore wave length, $L_{mo}/h_s$;
Figure 8  Parameter map showing critical parameter ranges for wave impacts on vertical and composite breakwaters
The significance of each of these parameters, and their contribution to the description of the overall physical processes are described below.

5.1 Degree of Wave Breaking ($H_{so}/h_s$ and $L_{mo}/h_s$)

In order to represent the effects of structural geometry properly, it is first necessary to identify those tests where the seabed causes wave breaking on the approach to the structure. These tests must be identified at the beginning of the analysis, as they would distort any conclusions about the overall response to the structure itself. These tests were easily identified by the curved distribution of data when plotted on Weibull axes, as described previously.

Further work is required to identify the parameter influences in cases for which the data are not Weibull distributed. A suitable statistical distribution must be established for these tests and the response to the structure geometry may then be identified from this new analysis.

It is not possible at this stage to speculate on effects of individual parameters in these cases, but it is certain that structures in shallow water are at risk of exposure to breaking wave forces. The overall level of forces on the structure may however be low, as the incident waves may be substantially broken (and aerated) before reaching the structure. Blackmore and Hewson (1984) have shown that wave forces due to highly aerated waves are significantly lower than corresponding ‘deep water’ waves.

Configurations that fall into this category should be treated as though they will be subjected to high impact forces, and designed accordingly, until further work has been carried out in this region of the parameter map. This is particularly important for structures in areas with high tidal ranges.

5.2 Wave regime at structure ($H_d/h_s$)

The most significant parameter affecting the onset of wave impacts for Weibull distributed data was found to be the relative incident wave height, $H_{si}/h_s$. This parameter represents the likelihood of wave breaking at the toe of the structure, before any significant interaction with the structure has taken place. The critical value of $H_{si}/h_s$ to cause the onset of impacts was investigated by plotting the percentage of impacts, $P_i\%$, for each test against $H_{si}/h_s$.

Initial investigations using Weibull distributed data from structures 0, 1, and 2 suggested that if the value of $H_{si}/h_s$ was less than a critical value of 0.2, there would be no wave breaking, and the resulting forces would therefore be pulsating, see Figure 9.

Extension of this analysis to include all structures showed that impacts did occur for some configurations where $H_d/h_s < 0.2$ (Figure 10), but that all these structures had very high rubble mounds in comparison to the local water depth. This effect is described in section 5.3. The limit of $H_d/h_s = 0.2$ was therefore accepted as the impact indicator for
vertical wall structures and structures with low rubble mounds. A second limit of $H_{si}/h_s = 0.3$ was identified as the point beyond which some impacts were recorded in all tests, also illustrated in Figure 10.

**Figure 9**  Variation in percentage of impacts with $H_{si}/h_s$ (structures 0, 1 and 2)

**Figure 10**  Variation in percentage of impacts with $H_{si}/h_s$ (all structures)
In the zone between these limits, ie $0.2 < \frac{H_{si}}{h_s} < 0.3$, a mixture of pulsating and impact conditions was observed. In this region, the occurrence of wave impact forces was later found to be dependent on the relative berm length, $B_{eq}/L_{pi}$, as described in section 5.4.

The effects of relative incident wave height, $\frac{H_{si}}{h_s}$, may be summarised as follows:

- $\frac{H_{si}}{h_s} < 0.2$ pulsating wave forces except for structures with high mounds and long berms;
- $0.2 < \frac{H_{si}}{h_s} < 0.3$ pulsating wave forces except for structures with long berms;
- $\frac{H_{si}}{h_s} > 0.3$ high probability of impacts on all structure types.

### 5.3 Height of rubble mound berm ($h_b/h_s$)

The relative height of the rubble mound berm was found to be significant in cases where $\frac{H_{si}}{h_s} < 0.2$, ie the relative incident wave height was low. In this region, no impacts were observed for cases where $h_b/h_s < 0.7$, defined here as 'low mounds', as shown in Figure 11. A mixture of pulsating and impact forces was observed in tests on structures with $h_b/h_s \geq 0.7$, defined here as 'high mounds'. The relative berm length, $B_{eq}/L_{pi}$, was found to be important in these cases, as described in section 5.4.

![Data for tests in range $H_{si}/h_s < 0.2$](image)

**Figure 11** Influence of relative berm height, $h_b/h_s$ (structures with small relative incident wave height)

The relative berm height, $h_b/h_s$, is a much less significant parameter than the relative incident wave height, $H_{si}/h_s$. It plays an important role however in causing impacts for those configurations with small values of $H_{si}/h_s$ and may therefore be particularly important for structures in areas with high tidal ranges. The parameter $h_b/h_s$ describes the effect of a small water depth on top of the rubble mound on the breaking process, identified as important by Allsop et al (1996a,b). The relative berm height replaces $H_{si}/d$ to describe this effect. It is a much more stable parameter, as it tends to infinity only if
the water depth at the toe of the structure is zero, and may therefore be used over the range of practical structures.

The effect of $h_b/h_s$ is markedly less significant than the effect of $H_i/h_s$ over the parameter ranges considered here. It is however an important part of the overall physical processes involved, and further investigations concentrated in the region $H_s/h_s < 0.2$ with various berm configurations may well uncover potentially damaging cases related to the berm height. In addition, further work covering the whole region of the parameter space might indicate that $h_b/h_s$ is important in other instances, for example when combined with the relative berm length.

5.4 Length of rubble mound berm ($B_{eq}/L_{pi}$)

The relative berm length, $B_{eq}/L_{pi}$, was found to be important in the regions $H_i/h_s < 0.2$ and $0.2 < H_i/h_s < 0.3$ (small and intermediate relative wave heights).

A mixture of impacting and non-impacting cases occurred for high mounds ($h_b/h_s > 0.7$) in the region $H_s/h_s < 0.2$, as stated previously in 5.2. Configurations with relative berm lengths, $B_{eq}/L_{pi}$, of at least 0.25 were found to experience wave impacts, as shown in Figure 12.

In the region $0.2 < H_i/h_s < 0.3$, a similar analysis showed that impacts occurred in those tests with relative berm lengths, $B_{eq}/L_{pi} \geq 0.14$ as shown in Figure 13.

Figure 12 Influence of relative berm length $B_{eq}/L_{pi}$ (Structures with high mounds and small relative incident wave height)
Figure 13  Influence of relative berm length, $B_{eq}/L_{pi}$ (Structures in the range $0.2 < H_{si}/h_s < 0.3$)

6. CONCLUSIONS

Random wave forces on vertical walls may be described well by the Weibull distribution, both in the pulsating and impact zones. In these cases, the onset of impacts can be identified by a sharp change in the gradient of the probability plot.

If significant shallow water wave transformations have occurred, the resulting wave forces plot as a curve on Weibull axes. Critical values of relative offshore wave height and wave length have been identified, to determine whether the Weibull distribution may be used to describe wave forces.

The parameter map presented by Allsop et al (1996a,b) has been improved following new analysis and the identification of non-Weibull distributed data.

A revised parameter map for determining the risk of wave impacts is presented, which includes data for structures where the water depth over the rubble mound berm is small.

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