The form of the mound of rubble dumped by a barge

Prof. Drs. Ir. J.K. Vrijling[1] and Ir. G. van Oord[1]

Introduction

In coastal engineering many structures are (partly) made of rubble or concrete blocks. The usual way of construction is by gradually discharging a specified amount of rubble from a stone dumping barge. The stone dumping barge is steadily repositioned along the structure.To ascertain a geometry as designed in case of a breakwater, a specified evenness in case of a caisson foundation or an acceptable coverage in case of a scour protection a reliable prediction model of the deposition mound of the rubble is needed. Such a model can be helpful in designing and in calculating the cost of the structure. First a model to describe the process af a single stone falling through water, the single stone model (SSM) is given. A mathematical model that describes the dumping from a point source was developed based on the SSM. By integration this model is extended to a description of a line dump as executed by a stationary dumping barge. Finally using the same principle the model is adapted to describe a dump by a laterally translating dumping barge, the area dumping model. In order the verify and to calibrate the SSM a number of experiments were carried out in the Hydualic Laboratory of Delft University of Technology.

The mathematical model

The model divides the gradual deposition of rubble or blocks in three stages. In the first stage the deposition of rubble is governed by a random walk of many independant steps with zero mean. If the rubble is dumped from a single point the diffusion to a cross section of the mound of rubble shaped like a two-dimensional Gaussian p.d.f.. From first physical principles it is derived that the standard deviation should be gouverned by:

$$\sigma_N = \alpha \bullet \sqrt{h \bullet D_{n50}}$$

[1] Delft University of Technology, Faculty of Civil Engineering, P.O. Box 5048, 2600 GA Delft, The Netherlands, E-mail: H.Vrijling@ct.tudelft.nl

in which α = constant depending on the mass density and the shape of the rubble particles
h = water depth
D_{n50} = nominal diameter of the rubble particles.

A detailed derivation of this single stone model (SSM) is given by Vrijling et al. For regularly shaped blocks the random walk consists of one single step and consequently the deposition mound has a circular pattern with a radius proportional to the waterdepth. The cross-section of the ring is gaussian shaped and the standard deviation increases linearly with the waterdepth.

During the first stage the slope of the cross section increases with the amount of rubble dumped, until it exceeds the angle of repose of the rubble at the inflection of the Gaussian shaped mound. This second stage of the build up ends, when in every point of the slopes of the mound the angle of equilibrium is reached. In the third stage the shape of the mound of rubble can then be described by a triangle. If dumping continues the mound maintains its triangular shape. The triangle can also be characterized by its radius of inertia or standard deviation:

$$\sigma_N = \sqrt{\frac{A}{6 \bullet \tan(\varphi)}}$$

in which t_Δ = radius of inertia of the triangle
A = area of the cross section of the mound
φ = angle of repose of the rubble

It should be noted that in the third stage the radius of inertia of the cross section depends only on the dumped volume of rubble and the angle of equilibrium. The second stage, when the Gaussian cross section is reshaped into a triangular form, can be treated as a transition stage of lesser importance..

The deposition from a single point leads for reasons of symmetry to a bi-Gaussian shaped mound: the Point Dumping Model (PDM). To find the theoretical shape of the mound of rubble formed by a side-dumping barge with deck-length L moored at a fixed point with a constant heading is found by the integration of an infinite number of single point dumps along the length-axis Y of the barge. The Line Dumping Model (LDM):

$$z(x, y) = \frac{V}{L\sqrt{2\pi}\,\sigma_G} e^{-0.5(\frac{x}{\sigma_G})^2} \int \frac{1}{\sqrt{2\pi}\,\sigma_G} e^{-0.5(\frac{y-Y}{\sigma_G})^2} dY$$

Solving the integral leads to a slightly simpler form containing the standard normal distribution $\Phi_N(x)$:

$$z(x,y) = \frac{V}{L\sqrt{2\pi}\,\sigma_G} e^{-0.5(\frac{x}{\sigma_G})^2} . [\, \Phi_N(\frac{Y_1 - y}{\sigma_N}) - \Phi_N(\frac{Y_0 - y}{\sigma_N})]$$

It should be noted that the endpoints of the line dump are described by the standard normal distribution, while the cross-section is given by the normal p.d.f. The center-line of the mound shows a offset with reference to the board of the barge. This offsett is not included in the mathematical model.

If the barge is translated perpendicular on it's length axis along the X-axis during dumping, to cover an area with rubble the theoretical form of the deposition mound is found by a double integration over the area covered by the dumping edge of the deck of the barge. The Area Dumping Model (ADM):

$$z(x,y) = \frac{V}{(X_1 - X_0)(Y_1 - Y_0)} \int \frac{1}{\sqrt{2\pi}\,\sigma_G} e^{-0.5(\frac{x-X}{\sigma_G})^2} dX \int \frac{1}{\sqrt{2\pi}\,\sigma_G} e^{-0.5(\frac{y-Y}{\sigma_G})^2} dY$$

Now the four edges of the dumped area are described by a normal distribution:

$$z(x,y) = \frac{V}{(X_1 - X_0)\,L} . [\, \Phi_N(\frac{X_1 - x}{\sigma_N}) - \Phi_N(\frac{X_0 - x}{\sigma_N})] . [\, \Phi_N(\frac{Y_1 - y}{\sigma_N}) - \Phi_N(\frac{Y_0 - y}{\sigma_N})]$$

The constant thickness of the layer is equal to the volume V divided by the area covered by the edge of the translating barge.

Experimental verification

In order to validate the mathematical model and to find values for the constant α and for the angle of equilibrium of the rubble, model tests (see appendix) were carried out at the laboratories of Delft Hydraulics and the Delft University of Technology in a tank of 2 x 2.5 x 2.5 m. An extra aluminum bottom was installed to facilitate experiments at (four) different waterdepth. Two sides were made of 60 mm thick glass.

To validate the SSM stones were dumped individually from a fixed point in the center of the tank. The dumping of single stones was continued untill the stones started to fall on top of each other thus making the positioning less accurate. After the dumping the tank was drained and digital pictures were taken to produce a top view. From the picture the radius from the center of the tank to every individual stone was determined. Afterwards the cumulative distribution function of the measured radii was compared to the Rayleigh-distribution, which gives the theoretical distribution of radii for bi-normally distributed points (SSM,PDM).

In other tests small amounts of rubble were dumped, taking pictures through the glass sides of the two cross sections of the build up of the rubble mound at regular intervals to follow the forming of the Gaussian profile and the gradual tansition to the triangular profile, when the angle of repose was exceeded.

The test results will be discussed in three categories: natural stone, cubes and spheres and thin shapes.

For broken rubble the agreement between the experiment and the theoretical Rayleigh distribution was quite good (Fig.1). The difference might in part be explained by the fact that the size of the rubble follows a sieve curve.

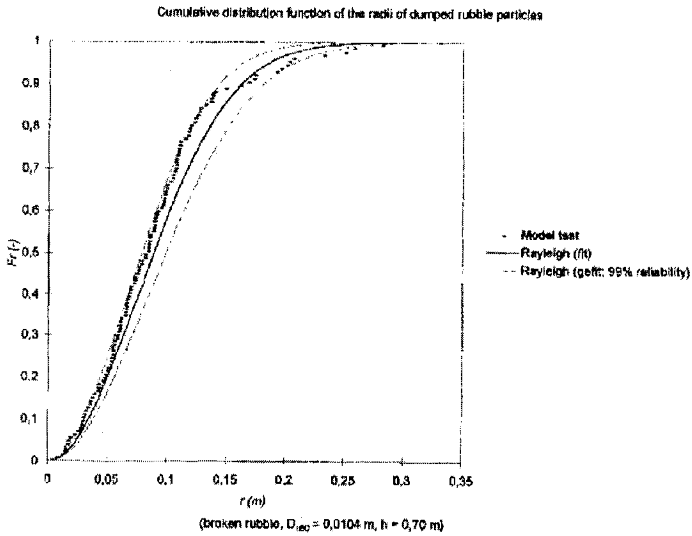
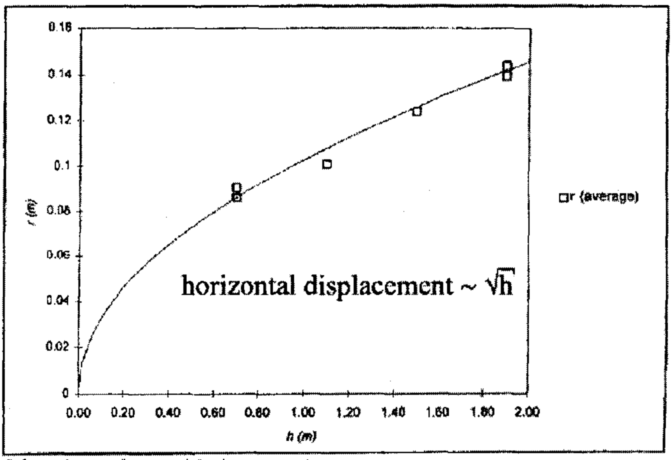


Figure 1.

Also the relation between the horizontal displacement and the square root of the waterdepth was confirmed by the tests (Fig.2). The value of the constant α was 0.72 on average with a standard deviation of 0.09. If the rubble is dumped simultaneously the average value of α increases by 6%

The gradual transition from the Gaussian profile with a width dependant of the waterdepth to a triangular profile with a width dependant of the volume dumped was experimentally verified. In Fig.3 the depth and the volume dependant models are drawn together with the experimental results.

For rounded rubble the value of the constant α was 0.60 on average with a standard deviation of 0.012. If the rounded rubble is dumped simultaneously the average value of α increases by 22%.



Relation between horizontal displacement and waterdepth (broken rubble, D_{n50} = 0.0104 m)

Figure 2.

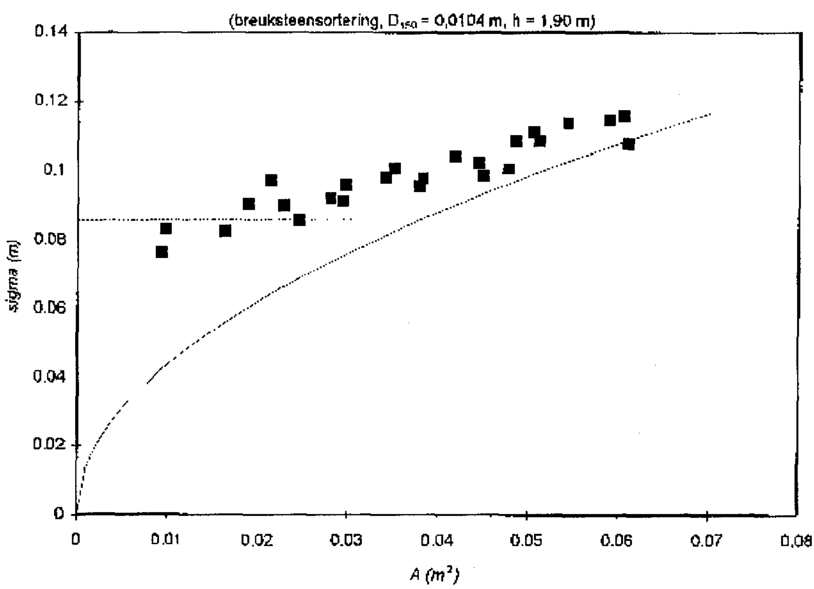


Figure 3.

Surprisingly aluminum cubes formed a ring when dumped. It appeared that the regular sharp edged cubes started rotating after falling over 8 to 15 D. So the formation of the ring may be attributed to the Magnus-effect, which results in an randomly directed, extra and stable horizontal force causing a ring shaped deviation from the center. The cumulative distribution function of the radii of the dumped cubes is not Rayleigh but normal (Fig.4). The total horizontal displacement is not the result of many independant steps with zero mean but of one single with a distinct mean. It appears that the mean radius increases proportionally with the waterdepth irrelevant of the size of the cubes (Fig.5). The previously found relation for the broken rubble is also depicted in this graph.

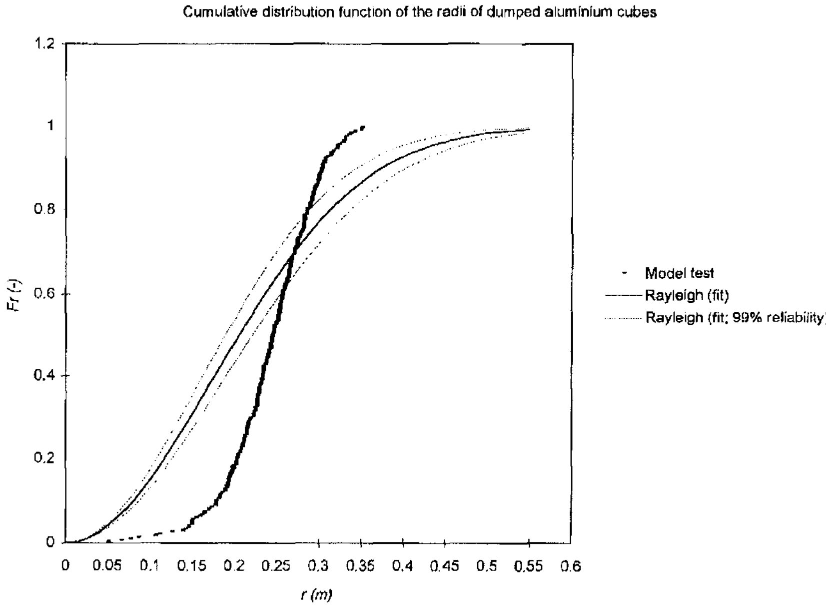
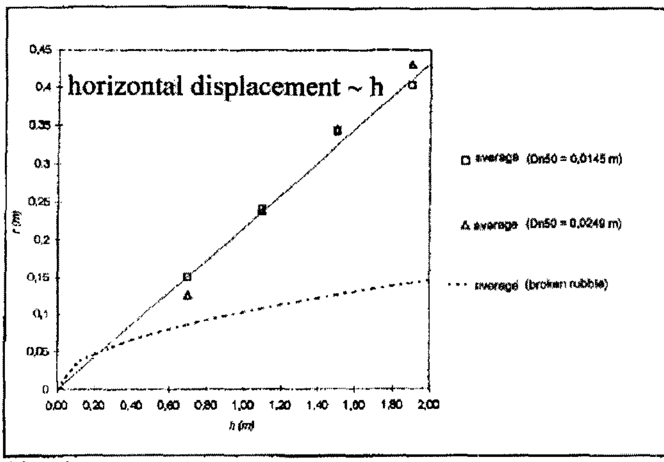


Figure 4: Aluminium cubes with D_{n50}=0.0145m and h=1.10m.

The result for concrete cubes with a diameter of 0.015m was strikingly similar. Also for glass balls (D=0.0156m) the same behaviour was found. The average deviation for the balls was exactly equal to that of the cubes, but the standard deviation around the ring was slightly bigger.

For reasons of comparison also some experiments were performed with square plates 0.05m thick with the same size as the cubes. Here the falling motion was very stable and the horzizontal deviation of the center was almost zero. However experiments with Dutch guilders (D=0.025m), which have approximately the same size, behaved quite differently. A guilder falls with loopings leading to considerable deviations of the center. The c.d.f. of the radii conforms exactly with the Rayleigh distribution (Fig.6). Dutch rijksdaalders (D=0.029m) show exactly the same behaviour but with a bigger standard deviation around the center. The strikingly different behaviour of falling objects depending on form and mass is also reported by Field et al and earlier by Stringham et al.

Tests were carried out to verify the theoretical shape of the mound of rubble formed by a side-dumping barge. An amount of gravel was shoved off a board with a length of 0.70 m, mounted on top of the tank. the form of the mound was analysed and compared with the theory (LDM). A reasonable agreement was found, but not all differences are yet well understood.



Relation between horizontal displacement and waterdepth for aluminium cubes

Figure 5.

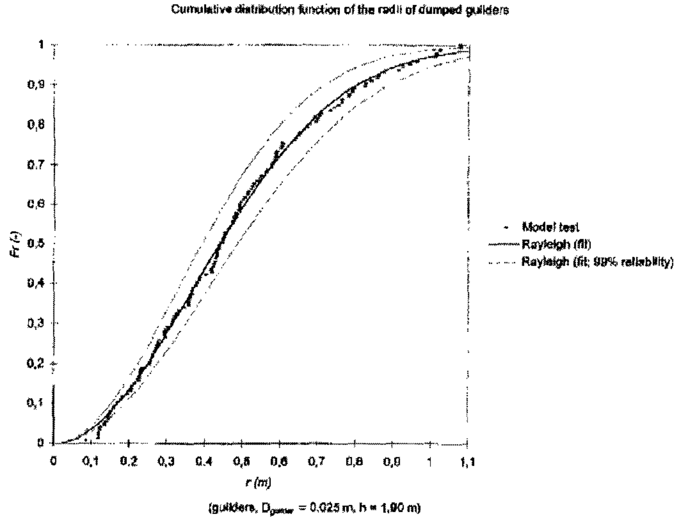


Figure 6.

Conclusion

A practical model to predict the deposition of rubble dumped from a stone dumping barge was developed and validated in principal form. The model shows that three phases have to be discerned in which different variables gouvern the width and the form of the deposition.

The model is extended to describe the dumping process of a side dumping barge. With this model the form of the mound and the evenness of the resulting surface can be predicted.

The single stone model (SSM) gives a good description of the falling motion of rubble in water. Also the further build up when the angle of repose is exceeded is confirmed.

The horizontal displacement of the falling motion measured at the bottom is proportional to the quare root of the waterdepth as found in the SSM. The influence of the diameter D_{n50} of the rubble needs however further verification.

The development of the Gaussian profile into a triangle after the angle of repose has been exceeded was also confirmed for rubble.

Due to rotation the SSM does not give a good description of the falling of cubes and spheres. Here a ring is formed with a radius proportional to the waterdepth irrespective of the size of the particles.

The SSM gives a good description of platelike objects. However due to stronger lift-forces the horizontal displacement is far greater than for rubble.

In view of the good agreement for rubble, the model has shown to be of great practical value in the construction of rubble mound structures.

References

Oda K. (1990): Studies on deposition-mound configurations of rubble discharged from a split-hull barge, *Abstracts 22nd Inter. Conf. on Coastal Engg.* (ASCE), pp. 481-482, Paper no. 241.

Matsumi Y., Kimura A. (1992): Hydraulic approach to determining optimum interval of discharge sites of barge in constructing rubble foundation of deep water breakwater, *23rd Inter. Conf. on Coastal Engg.* (ASCE), pp. 3149-3162.

Cregten,H.J., The use of probabilistic methods in judging the dumped mound of a side dumping barge (in dutch),*MSc thesis*, Delft 1995

J.K.Vrijling et alt.,Prediction of the deposition-mound of dumped rubble, *Proceedings of COPEDEC IV*, Rio de Janeiro, 1995

Oord, G.van, The spreading of stone in the dumping process of side dumping barges (in dutch),*MSc thesis*, Delft 1996

Stuckrath,T.,Volker,G. en Meng,J., *Untersuchungen zum Fallverhalten würfelförmiger Einzelkörper in Wasser*, T.U.Berlin,1996

Field,S.B. et al.,Chaotic dynamics of falling disks, *Nature*,Vol.388, 17 july 1997

Stringham,G.E. et al.,The behaviour of falling particles in quiesquent liquids, *Prof. Pap. US Geol. Surv.* 562-c,1969

Appendix

Dumping material	Mass density	Characteristic dimension	Description of the form	Water depth
Crushed basalt	3000 kg / m^3	D_{n50} = 10.4 mm	Irregular, sharply edged	0.70 m 1.10 m 1.50 m 1.90 m
River pebbles	2500 kg / m^3	D_{n50} = 12.7 mm	Irregular, rounded	1.90 m
Aluminium cubes	2700 kg / m^3	D_{n50} = 14.5 mm D_{n50} = 24.9 mm	Regular, sharply edged	0.70 m 1.10 m 1.50 m 1.90 m 0.70 m 1.10 m 1.50 m 1.90 m
Concrete cubes	2400 kg / m^3	D_{n50} = 15.0 mm	Regular, rounded edges	0.70 m 1.10 m 1.50 m 1.90 m
Glass marbles	2500 kg / ^{M3}	D_{n50} = 12.6 mm D_s = 15.6 mm	Regular, spherical	0.70 m 1.10 m 1.50 m 1.90 m
Al. plates, d_p = 2,5 mm Guilders, d_d = 1,5 mm Rijksdaalders, d_d = 2,0 mm	2700 kg / m^3 ≈ 7000 kg / m^3 ≈ 7000 kg / m^3	D_p = 25.0 mm D_d = 25.0 mm D_d = 29.0 mm	Regular, square, flat Regular, circular, flat Regular, circular, flat	1.90 m 1.90 m 1.90 m