Numerical Modeling of Long Wave Ship Motions

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Abstract

This paper presents a state-of-the-art numerical model for ship motion analyses along with an application of the model to a seiche problem in the Port of Long Beach, California. The practical uses of the model are emphasized along with a discussion of the efficacy of the modeling approach as a tool for decision-making.

Introduction

The tolerance for delays or inefficient cargo transfer at modern marine terminals due to environmental conditions (i.e., wind, waves or currents) is very low. Competition among shipping lines (and competition among the ports who desire to have the same shipping lines as tenants) demands marine terminals with uninterrupted service. Accordingly, the problem of excessive ship motions has been of particular concern in the development of new marine terminals. Additionally, recent trends in container ship design and terminal development has sparked renewed interest in ship motion problems. Specific developments include: (1) terminals located in areas exposed to harsher environmental conditions; (2) container ships that are larger but do not use commensurately stronger mooring/fender arrangements; and (3) a desire for higher loading/unloading rates which require little or no vessel movement.

The following paragraphs describe a methodology that can be used to simulate the motions of a moored vessel exposed to long waves. The hybrid approach combines the best features of physical and numerical modeling technology. This paper is one in a series on Los Angeles and Long Beach Harbor. The companion papers focus on other aspects of long wave behavior in the Ports (Lee et al, 1998, Poon et al, 1998, Raichlen et al 1998 and Walker et al 1998). Figure 1 shows the overall port complex. The specific application concerns ship motion experienced at the Pier J

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terminal located in the Port of Long Beach. This terminal is presented in Figure 2 and shows protective breakwaters that were constructed to reduce ship motions.

**Modeling Approach**

Overall, the problem to be addressed is that of a moored vessel located in a geometrically complicated harbor exposed to long waves or seiche. From a practical point of view, the modeling approach is normally used to evaluate various design elements (for a new facility) or corrective actions (for an existing facility) that will reduce ship motions to an acceptable minimum.

Physical models have historically served as the most reliable means for assessing wave-induced ship motion problems. In recent years, however, numerical modeling technology has advanced to the point where reliable estimates can be made for relatively short waves (i.e. waves less than 25 seconds) in open water using time domain ship motion models. The predictive fidelity of these numerical models has generally been confirmed through comparisons with physical model tests. In most cases, these comparisons have been made for open water conditions or for conditions with a solid quaywall, i.e., the vessel is exposed directly to waves from the open sea.

In most harbor applications, vessels are moored within protected basins where they are exposed to waves, which have been diffracted through a harbor basin entrance or reflected from the walls of the basin, or both. The complications associated with diffracted/reflected waves make it difficult to apply typical mooring dynamics models without considerable approximation and this is why physical models are often used to confirm the results of mooring models for final design.

Physical models have been used successfully to examine both harbor disturbance and ship motion problems. Distorted physical models are generally required for large harbors and can be used to examine harbor resonance. Unfortunately, distorted physical models cannot be used to examine ship motion problems due to difficulties associated with achieving similitude.

Numerical models can be used to examine both harbor disturbance and ship motion problems. In practice, application of numerical models for short waves is problematic due to requirements for relatively small element sizes which can result in an unwieldy computational effort. Inasmuch as moored ships tend to respond at both short periods (< 25 seconds) and relatively long periods (> 1 minute) any modeling approach must be capable of simulating the full range of wave periods.
Figure 1. Ports of Los Angeles and Long Beach, California

Figure 2. Pier J in the Port of Long Beach
The hybrid approach presented in the present paper involves the combined use of physical and numerical models. A complete description of the physical model used to evaluate long wave problems in the Port of Los Angeles and Long Beach is described in Poon, et al (1998). It suffices to say here that the physical model encompasses the entire harbor complex and was used to evaluate wave amplification factors at various locations throughout the harbor including Pier J. Wave amplification factors derived from the physical model results were used to establish boundary conditions for a local numerical model in the vicinity of Pier J. Additionally, results from the physical model were also used to calibrate amplification factors at interior points in the numerical model mesh. While the physical model results were useful in calibrating the numerical model as regards harbor oscillations, the physical model was of no use in calibrating the mooring model. Reference to Poon, et al (1998) will show that the physical model was used to identify the preferred solution for the Pier J situation. Specifically, the preferred solution was to construct new breakwaters at the entrance to the Pier J basin, see Figure 3. The remainder of this paper focuses on the numerical model and model results regarding the ability of the proposed breakwaters to reduce ship motions.

\textit{Ship Motion Simulation Models}

A moored ship responds to wave excitation in six degrees of freedom: three translational motions - surge, sway and heave; and three rotational motions - roll, pitch and yaw. An illustration of these motions is shown in Figure 3. The motion response of a container ship at berth is normally dominated by surge motion.

![Figure 3 Definition Sketch of Ship Motion in Six Degrees of Freedom](image)

Ship motion analyses were calculated with two numerical models - HYDRO and TDBERTH. HYDRO applies the finite element method (FEM) to solve the wave-ship-berth interaction, treating the oscillating system of a berthed ship in a harbor as a fully three-dimensional system. Overall, the analysis consists of two successive steps. The HYDRO model calculates harbor basin response, the ship hydrodynamic coefficients (i.e. added mass and damping coefficients) and the first and second order wave forces in the time domain. HYDRO models the hydrodynamics of the ship and the harbor basin simultaneously. This feature of the HYDRO model is critical for long wave problems where interactions occur between the incident long waves, the harbor basin and the moored vessel. TDBERTH computes ship motions in the time domain using results from
the HYDRO model. Specifically, TDBERTH solves for ship motions, mooring line forces, and fender reactions.

**HYDRO Model**

The HYDRO model solves the wave/harbor/ship interaction problem in terms of potential flow theory. The six modes of ship motion are expressed as:

\[ x_j = x_{aj} e^{i\omega t}, \quad j=1,2,\ldots,6 \]

where \( x_j \) is a displacement for \( j = 1,2,3 \) and a rotation for \( j=4,5,6 \) and \( x_{aj} \) is the corresponding complex amplitude, \( t \) is the time, and \( \omega \) is the radian wave frequency.

The total velocity potential describing the flow field is written as:

\[
\Phi = \phi_0 e^{-i\omega t} + \sum_{j=1}^{6} \phi_j x_{aj} e^{-i\omega t} + \phi_s e^{-i\omega t}
\]

\[
= \Phi_w + \Phi_f + \Phi_s
\]

where \( \Phi_w \) = potential of the incident wave

\( \Phi_f \) = potential of the waves generated by the ship motion

\( \Phi_s \) = potential of the scattered waves

Each of these velocity potentials also has to satisfy the Laplace Equation, i.e.

\[ \nabla^2 \phi_j = 0, \quad j = 0,1,\ldots,7 \]

The HYDRO model employs the finite element method to solve the above equations together with the appropriate boundary conditions.

After the velocity potential has been determined, the wave loading on the ship and the hydrodynamic coefficients can be calculated by using the pressure equation. The fluid pressure at any point in the fluid domain can be obtained from the linearized Bernoulli equation:

\[ p = -\rho \frac{\partial}{\partial t} \Phi - \rho g z \]

where \( \rho \) is the fluid density, \( g \) the acceleration due to gravity and \( z \) the depth below water surface.

The total force, \( F_b \), acting on the ship can then be obtained by integrating the pressure over the surface of the ship. Applying Newton's second law of motion, the pressure
equation and the solution of the velocity potentials, the equations of motion of the ship in matrix form can be written as:

\[ m_i x_j + K_{ij} x_j = \zeta_s f_{ai} e^{-i\omega t} + f_{ij} x_j e^{-i\omega t} \]

where \( m_i \) = six-by-six mass matrix of the ship
\( K_{ij} \) = hydrostatic restoring force (moment) matrix
\( x_j \) = motion vector
\( f_{ij} \) = wave exciting forces and moments per unit wave
\( f_{ij} \) = forces and moments per unit ship motion
\( \zeta_s \) = wave amplitude

The added mass and damping coefficients can be found upon separation of the Real and Imaginary parts of the above equation.

In addition to the first order wave forces, a ship is also subjected to second-order mean and low frequency wave drift forces. These forces are partially attributable to the velocity-square terms in the Bernoulli's equation, partially attributable to the elevation variation of the water surface and partially attributable to first-order ship motions. HYDRO computes these forces based on the perturbation method developed by Pinkster and Van Oortmerssen (1977).

**TDBERTH Model**

Vessel motions and mooring line/fender forces were computed using a six-degree of freedom numerical model TDBERTH. TDBERTH integrates the equations of motion in the time domain by standard numerical methods. The model formulation is similar to that developed by Van Oortmerssen (1975).

The governing equations of motion, which account for motion in six degrees of freedom, are as follows:

\[ \sum_{j=1}^{6} [(M_{kj} + m_{kj}) \ddot{x}_j + \int_{-\infty}^{t} K_{kj} (t - \tau) \dot{x}_j (\tau) d\tau + b_{kj} \dot{x}_j + C_{kj} x_j] = F_{ew} (t) + \sum_{i=1}^{n_l} L_{ik} (t) + \sum_{i=1}^{n_t} N_{ik} (t) \]

\[ k=1,2, \ldots 6 \]

Where:
\( x_j, \dot{x}_j, \ddot{x}_j \) = displacement, velocity, and acceleration in the j-th mode
\( M_{kj} \) = inertia matrix

\( C_{kj} \) = hydrostatic restoring force matrix

\( K_{kj} \) = impulse response function matrix

\( m_{kj} \) = constant added mass matrix

\( F_{kw}(t) \) = wave exciting force in kth mode

\( L_{ik}(t) \) = mooring line force in kth mode from line i

\( N_{ik}(t) \) = fender force in kth mode from fender i

k = denotes mode of motion, (i.e. 1 (surge), 2 (sway), 3 (heave), 4 (roll), 5 (pitch), 6 (yaw))

\( n_i \) = number of mooring lines

\( n_f \) = number of fenders

The inertia matrix, \( M_{kj} \), and hydrostatic restoring force matrix, \( C_{kj} \), are computed using standard methods of naval architecture. The impulse-response function matrix, \( K_{kj} \), and the constant added mass coefficient, \( m_{kj} \), are computed as follows:

\[
K_{kj} = \frac{2}{\pi} \int_0^\infty b_{kj}(\omega) \cos \omega t d\omega
\]

\[
m_{kj} = a_{kj}(\omega *) + \frac{1}{\omega} \int_0^\infty K_{kj}(t) \sin \omega * t dt
\]

Where:

\( a_{kj} \) = frequency-dependent added mass

\( b_{kj} \) = frequency-dependent damping coefficient

The above frequency dependent hydrodynamic coefficients were computed using the HYDRO model.

The wave exciting force, \( F_{kw}(t) \), is computed as follows:

\[
F_{kw}(t) = \sum_{n=0}^{n} f_k^{(1)}(\omega_n) a_n \cos(\omega_n t + \varepsilon_n + \varepsilon_k)
\]

\[
+ \sum_{n=0}^{n} \sum_{m=0}^{m} f_k^{(2)}(\omega_n) a_n a_m \cos((\omega_n t + \varepsilon_n) - (\omega_m t + \varepsilon_m))
\]
Where:

\[ f^{(1)}_{k}(\omega_n) \] = first order wave transfer function in

k-th mode for n-th wave component

\[ f^{(2)}_{k}(\omega_n) \] = second order wave transfer function in

k-th mode for n-th wave component

\[ \varepsilon_k \] = phase of first order wave transfer function

in k-th mode for n-th wave component

\[ a_n \] = wave amplitude of n-th wave component

\[ \varepsilon_n \] = phase of n-th wave component

\[ \omega_n \] = frequency of n-th wave component

The first and second order wave transfer functions were computed using the HYDRO code. The above formula was used to simulate first and second order wave force time-histories resulting from an incident wave spectrum composed of n-waves having amplitudes \( a_n \) and frequencies \( \omega_n \).

*Simulation Conditions*

Several container ships were examined and the characteristics of each are shown in Table 1. The mooring configuration used for each simulation is shown in Figure 4 and consists of 10 lines and 17 fenders. Two mooring configurations were examined, namely: (1) conventional nylon lines and (2) mixture of nylon and steel lines. The lines were assumed to be 20 cm (8 inch) in circumference with a breaking strength of 832 kN (187 kips). The fender spacing was assumed to be 15.2 m (50 ft). Both the lines and fenders had non-linear load-deflection curves.

**Table 1: Characteristics of Container Ships**

<table>
<thead>
<tr>
<th></th>
<th>LBP* (ft)</th>
<th>Beam Width (ft)</th>
<th>Draft (ft)</th>
<th>Dead Wt. (ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-Class</td>
<td>294</td>
<td>32.2</td>
<td>12.2</td>
<td>60,350</td>
</tr>
<tr>
<td>A-Class</td>
<td>201</td>
<td>30.5</td>
<td>10.0</td>
<td>30,950</td>
</tr>
<tr>
<td>F-Class</td>
<td>317</td>
<td>42.8</td>
<td>12.5</td>
<td>85,350</td>
</tr>
</tbody>
</table>

* Length-Between-Perpendiculars
Figure 4. Mooring Geometry

Figure 5. Finite Element Mesh
The container ship was moored at either the eastern or western berth of the nearly rectangular harbor (366 m by 975 m) with uniform water depth of 15.2 m (50 ft). The finite element grid used for numerical modeling is shown in Figure 5. The grid consisted mainly of a two-dimensional (2D) region with an embedded three-dimensional (3D) region (hatched area) to properly schematize the modeled ship. The 3D grid consisted of five layers, with the top three layers constructed to fit the ship, and two bottom layers representing the water between the ship and the basin floor.

The wave condition at the harbor basin entrance was chosen such that there were substantial surge motions under base conditions. The wave spectrum at the harbor entrance has the same spectral shape as the average spectrum measured by the US Army Corp of Engineers (1993) at Platform Edith, the latter is located about eight miles south of the Ports of Los Angeles and Long Beach.

**Simulation Results**

Ship motion response for each mooring/fender configuration was computed in the time domain with the TDBERTH model for a two-hour simulation period. Example time series of motions, line forces and fender forces are shown in Figures 6, 7 and 8.

The above figures show the moored container ship response to be a mixture of 1st order and 2nd order wave responses at low frequency. Overall, the vessel response in surge is dominated by motions having periods near the natural period of the moored ship system (i.e., about 1.5-2 minutes). Results of the simulations are presented in Table 3 in terms of normalized peak-to-peak surge motions for a representative wave condition.

<table>
<thead>
<tr>
<th></th>
<th>Western Berth</th>
<th>Eastern Berth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Synthetic Lines</td>
<td>Steel/Synthetic Lines</td>
</tr>
<tr>
<td>Existing</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Breakwater</td>
<td>51%</td>
<td>96%</td>
</tr>
</tbody>
</table>
Figure 6. Motion Time Histories

Figure 7. Mooring Line Time Histories
Table 3 shows that the breakwater will substantially reduce surge motions for most of the simulations. For example, the existing surge motions are reduced to 51% of existing conditions with the breakwater for a vessel moored at the western end of the basin using all synthetic lines. Interestingly, the motions are not significantly reduced for a vessel moored by a combination of synthetic and steel lines at the western berth. The above results show that the proposed breakwaters would significantly reduce problematic vessel motions for conventional moorings at either berth. The results also show that it is important to consider both the mooring configuration and the location of the vessel within the harbor.

Summary

A methodology for examining ship motions excited by long waves has been presented. The hybrid approach combines the strengths of physical and numerical modeling. Physical model results are used to define boundary conditions for, and calibration of, the numerical model. The numerical model described in this paper provides a powerful and flexible means for examining complicated mooring problems. The model was recently used to examine the efficacy of constructing breakwaters to reduce ship motions at a berth in Long Beach Harbor, California. The model showed the proposed breakwaters would significantly reduce vessel
motions under most conditions. The breakwaters were constructed and preliminary experience indicates that the breakwaters have been successful.

The state-of-the-art in modeling of ship motions is mathematically advanced, however, additional model testing and field data collection efforts are necessary to gain as much confidence in ship/basin models as there presently exists in open sea models. A systematic series of physical model tests involving a vessel moored in simple basin geometries is recommended to advance the currently available modeling technologies.

References


