

SIMULATION OF NONLINEAR WAVE DEFORMATION BY A SHOAL IN 3D

Paul de Haas ¹ and Maarten Dingemans ¹

Abstract

Three-dimensional propagating nonlinear waves can be simulated with a time-domain numerical model based on a boundary integral equation method. For the simulation of nonlinear wave deformation by a shoal, a domain decomposition method is used to increase the efficiency of the model. Features and efficiency of the domain decomposition method are described and its application to the shoal problem is discussed.

1 Introduction

In the modelling of refraction and shoaling of waves, it is important to describe the nonlinear free-surface boundary conditions properly. It is known, for example, that non-linearity opposes convergence and divergence of wave ray paths. In order to assure the correctness of modelling non-linearity, physical experiments are often used.

We present a three-dimensional time-domain numerical model, based on a boundary integral equation method, which computes the propagation of waves with the exact nonlinear boundary conditions over an arbitrary bottom geometry. This method is therefore able to provide additional material suitable for verification. The results of a computation of the model are compared with the shoal experiment (Berkhoff et al. (1982)) in order to determine the accuracy of the model. Because the study of such problems requires large computational effort, the use of efficient numerical techniques is imperative. Here we present a domain decomposition method which reduces the computational costs of the boundary integral equation method considerably.

This paper is organized as follows. First the numerical model is described in Section 2. In Section 3 the domain decomposition method is described and its

¹Delft Hydraulics, P.O. Box 177, 2600 MH Delft, The Netherlands

efficiency is discussed. In Section 4 the application of the domain decomposition technique to the shoal problem is discussed. Finally some conclusions will be stated in Section 5.

2 Numerical model

2.1 Description

In the mathematical model for nonlinear water waves considered here, the motion of the water is described by the usual potential-flow equations for inviscid irrotational fluid motion with a free surface on water of varying depth. It is described by the field equation for the velocity potential ϕ (Laplace's equation)

$$\Delta\phi = 0, \quad (1)$$

and the boundary conditions on the free surface $\partial\Omega_{FS}$

$$\left. \begin{aligned} \frac{D\phi}{Dt} &= \frac{1}{2}(\nabla\phi)^2 - gz - \frac{p}{\rho} \\ \frac{D\mathbf{x}}{Dt} \cdot \mathbf{n} &= \frac{\partial\phi}{\partial n} \end{aligned} \right\} \mathbf{x} \in \partial\Omega_{FS} \quad (2)$$

and on the bottom $\partial\Omega_B$

$$\frac{\partial\phi}{\partial n} = 0, \quad \mathbf{x} \in \partial\Omega_B. \quad (3)$$

Appropriate in- and outflow boundary conditions are formulated on the lateral boundaries.

The numerical model consists of a time marching scheme for the evolution of the free surface and its boundary conditions. At every time-step, Laplace's equation for the velocity potential is solved by using a boundary integral equation method (BIEM). In the BIEM, only the boundary of the fluid domain is discretized into quadrangular panels represented by a collocation point in the centre of the panel. For each collocation point a Fredholm integral equation of the second kind equivalent with Laplace's equation is formulated (see e.g. Broeze (1993)). These integral equations are discretized and by using the boundary conditions a system of linear equations is built and subsequently solved. Insertion into equations (2) of the solution obtained in this way, provides the time derivatives which are needed for the time marching scheme.

For the time integration of the collocation points, a Lagrangian description is used in combination with (small) tangential correction velocities to control the grid motion. In the present implementation the Lagrangian description is required to obtain stability near the inflow boundaries, see Broeze (1993). The lateral boundaries move along with the free-surface grid with uniform velocities over each vertical. Due to the Lagrangian motion the grid distorts during the time-domain simulation. For this reason a mixed Eulerian-Lagrangian description is preferred which controls the grid in the inside of the domain and retains the

stability. This is however not implemented in the model yet but seems to be of importance to be used in combination with domain decomposition, as will be pointed out in Section 4. See furthermore Broeze (1993) for a more detailed description of the method employed here.

Boundary integral equation methods are very suitable for solving Laplace's equation on such domains because they only require a discretization of the boundary of the domain. Compared with field discretization methods, the advantages of a BIEM are a much smaller amount of grid points and a natural description of the evolution of the free surface. On the other hand, the computational requirements for some specific parts of the solution algorithm depend superlinearly on the number of collocation points and often form the bottleneck for the computation of large-scale wave problems. These parts are the discretization of the boundary integral equations and the solution of the resulting system of linear equations. The domain decomposition technique is used to tackle these problems.

2.2 Applications

There are a number of applications for which the model can be used. For verification the model can be used to check higher-order wavemaker theory for both translating and rotating wavers in two dimensions. In this application the boundary condition related to the moving wavemaker is imposed on the exact position of the wavemaker in time. Because the full nonlinear free surface boundary conditions are solved, the accuracy of the wavemaker theory can be investigated with respect to its order.

The model can also be used for verification of the modelling of the nonlinear free-surface conditions in other wave models, like Boussinesq models. This is especially useful when experimental verification material is hard to get, for example in large scale problems. This also applies to the modelling of wave interaction with bottom topography.

As a simulation model, the numerical model can be useful when interaction with objects is involved, for example with ships. Wave forces and run-up can be determined in order to simulate its motion. A development which has progressed considerably recently, is the simulation of ship and water motion through a simultaneous solution of the equations of motion of the ship and the governing potential flow equations. These are coupled through the boundary conditions on the hull of the ship, which are taken on the exact wet part of the (moving) ship. Development on these kind of computations is still ongoing, see Berkvens (1998).

For most interesting applications however, limited computer capacity often still prohibits the use of the model, especially in three dimensions. Therefore the development of numerical techniques, such as domain decomposition, is needed.

3 Domain decomposition

3.1 Description

The domain decomposition method described here consists of a division of the computational domain into subdomains (see Figure 1) and an iterative procedure which generates a sequence of solutions in the subdomains that converges towards the solution in the original domain.

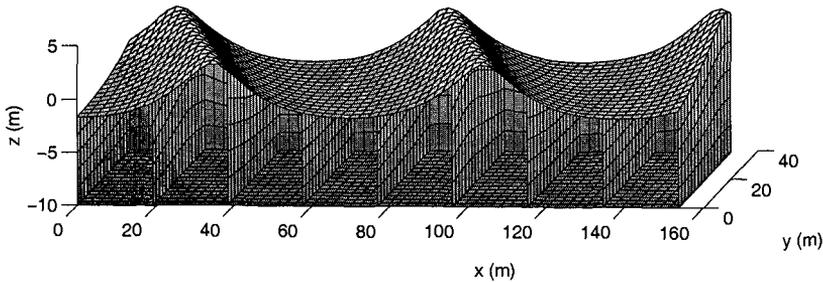


Figure 1: Domain subdivided into 8 subdomains (Panels on the front lateral boundaries are not shown)

Every step of the iterative procedure consists of first solving Laplace's equation for the potential ϕ on the separate subdomains simultaneously and secondly formulating new boundary conditions on the subdomain interfaces. In the latter part the subdomain problems are coupled.

There are many possibilities in the way information can be exchanged between the subdomains. We have chosen here to use the so-called DD/NN-scheme. Every odd step of the iterative procedure Dirichlet conditions are imposed on all interfaces. Neumann conditions are imposed at all even steps. These steps are illustrated in Figure 2 for the first two steps of a two-subdomain problem.

This scheme is also known as a Neumann-Neumann preconditioner in the context of domain decomposition methods for field discretization techniques, see e.g. Le Tallec (1994). In the field of time-domain BIEM's for nonlinear water waves a similar technique was used by Wang et al. (1994) in 2D. In their work interfaces are used to formulate a block-structured matrix which is then solved iteratively. In the present approach the subdivision and the coupling is formulated on the continuous level. It is implemented numerically through separate discretizations for all subdomains leading to matrices to be solved per subdomain and a separate coupling algorithm. The present method has been presented in De Haas et al. (1996) where it was applied to the simulation of propagating nonlinear wave groups in 2D. For a general impression of work being done in the field of domain decomposition the reader can consult for instance Quarteroni (1994).

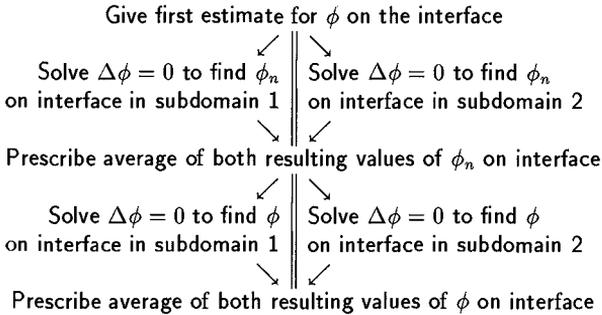


Figure 2: Schematic representation of the DD/NN-scheme

3.2 Implementation and application

In the application of the domain decomposition technique to the time-domain numerical model described here, the domain is divided in one direction only. It is possible to change the subdivision of the domain during the computation which may for example be useful when initially having large subdomains in parts where hardly no waves are present and required CPU time per node is small, and small subdomains in parts where a lot of wave action takes place. We have chosen to use a fixed initial subdivision of the computational domain with subdomains of equal size so that no reorganization of data over the subdomains is necessary and the number of nodes in all subdomains is (approximately) the same.

In the present implementation, each subdomain is treated numerically as a one-domain problem with respect to both the solution of Laplace's equation and the time integration. This implies that for each subdomain the interfaces move as if they were in- and outflow boundaries of this subdomain, moving along with the free-surface grid in a Lagrangian fashion. The subdomain problems are coupled to ensure a unique description of the interface position (the motion of each interface is determined from solutions of the neighbouring subdomains) and they are coupled through the averaging of the interface boundary conditions as required in the iterative process.

3.3 Convergence characteristics

The performance of the domain decomposition method is determined by the convergence of the iterative process. It can be monitored on each interface by considering the jump across the interface between the solutions on both sides. The convergence on the interfaces depends on the geometrical form of the subdomains and on the coupling scheme used. Convergence characteristics for 2D-problems have been outlined in De Haas *e.a.* (1996) and are repeated here with inclusion

of typical three-dimensional aspects.

- The convergence of the iterative procedure deteriorates if there is more asymmetry near the interfaces due to a disturbed free surface or an uneven bottom. In numerical experiments it was shown that it is mainly determined by the slope in the direction perpendicular to the interface, see De Haas and Zandbergen (1996).
- If either the length-to-height ratios or the length-to-width ratios are not too small (typically larger than 1) then convergence on each interface is not influenced by iterative processes on other interfaces.

Implications of these features with respect to the division of a computational domain in more than two subdomains can be considered in two ways.

- For a fixed length-to-height ratio L/h of the computational domain, the convergence of the iterative procedure deteriorates as the number of subdomains N increases. For problems with an even bottom a specific number \tilde{N} (and corresponding length $\tilde{l} = L/\tilde{N}$) exists for which the iterative process on one interface is influenced by the iterative processes on neighbouring interfaces.
- For a fixed length-to-height ratio l/h of the subdomains, the convergence rate does not change as the number of subdomains increases, in the case of rectangular subdomains of equal size $l > \tilde{l}$. In applications with a disturbed free surface over an even bottom it is seen that convergence is determined by the interface with the worst convergence. The number of iterations has an upper bound which is independent of N .

Examples of the first type of computation are shown in Section 3.4. The shoal problem treated in Section 4 represents a problem with an uneven bottom.

3.4 Efficiency

The efficiency of the domain decomposition technique is of course related to the convergence of the iterative method. It is considered for the case of a computational domain with a fixed length-to-height ratio and illustrated with an example.

- If for a computational domain with a fixed length the number of subdomains is increased, on the one hand the number of required iterations will increase. On the other hand, the CPU-time to solve Laplace's equation per subdomain decreases, since the subdomains become smaller. In general a certain optimal number of subdomains exists with respect to required computer capacity to solve a given water-wave problem.
- If the computational domain is built from subdomains with a fixed length-to-height ratio and the number of subdomains is increased, the number of iterations increases but remains below a upper bound independent of

the number of subdomains used. Therefore the computational costs per subdomain have an upper bound independent of the number of subdomains and the maximum total computational cost per time step can be given by a function linearly dependent on the size of the computational domain.

Results are shown for the fully nonlinear water wave problem illustrated in Figure 1. This problem involves a Fourier series solution which propagates undisturbed in water of constant depth. It is computed using the method by Rienecker and Fenton for waterdepth $h = 10.0$ m and wavelength $\lambda = 60.0$ m. The computations are started from an initial state prescribed by the solution with the wave direction under an angle of 30° with the positive x -direction, see Figure 1.

In the first example a computational domain of length $L = 153.8$ m is considered with subdivisions into 2, 4 and 8 subdomains. Time domain computations are done over the time interval $[0, 10]$ s. Required CPU times are measured for computations using Gaussian elimination and for computations using a conjugate gradient type of solver for the system of linear equations. To reduce CPU times these computations are performed for a wave height $H = 2.5$ m instead of the wave height $H = 5.0$ m shown in Figure 1. For the investigation of the number of iterations the waveheight $H = 5.0$ m is only used for the 8-subdomain problem. The number of required iterations is shown in Figure 3. The number of required

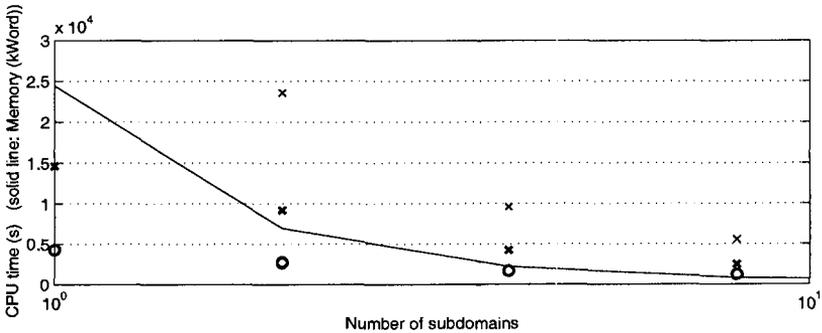


Figure 3: Number of iterations k required per timestep during the computation over the time interval $[0, 10]$ using 2, 4 and 8 subdomains indicated by the thick, medium thick and thin line respectively, for $H = 2.5$ m. The dashed line corresponds to the 8-subdomain computation with $H = 5.0$ m.

iterations for the 2- and 4-subdomain problem is approximately equal. For the 8-subdomain problem the number of required iterations is significantly larger. The variation during the considered time interval is clearly visible for the 8-subdomain problem with $H = 5.0$ m. This variation is related to the varying distortion of the free-surface grid and the connected interfaces. After approximately one wave period the number of required iterations is on the initial level again because all

the free surface collocation points have made the same excursion through space. The initial domain is recovered apart from a Lagrangian drift in the wave propagation direction. See Figure 4 in which the domain at $t = 10.0 \text{ s} \approx 1.53T$ is shown.

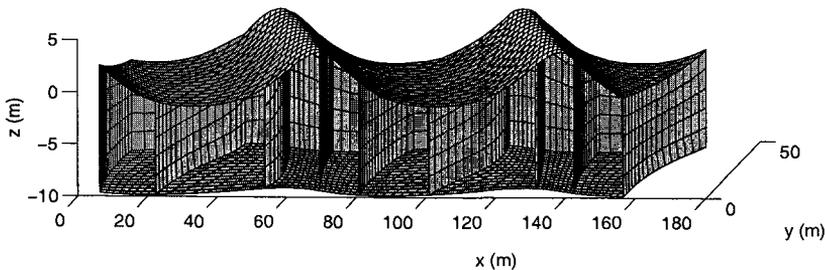


Figure 4: Domain at time level $t = 10 \text{ s}$ using 8 subdomains. Again the networks of the front lateral boundary are not shown.

With respect to the computational requirements results are shown in Figure 5. The following observations are made:

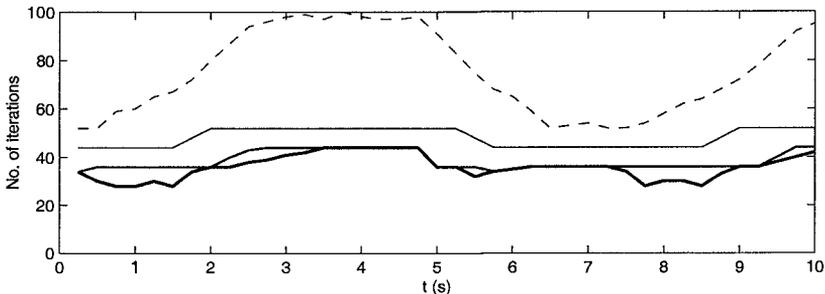


Figure 5: Measured CPU-times using 1, 2, 4 and 8 subdomains with $H = 2.5 \text{ m}$. Crosses indicate computations using Gaussian elimination and circles indicate computations using CGS. Symbols printed thin indicate CPU-times that are corrected for the performance of the computing system by normalizing with the performance of the 1-domain computation. The solid line indicates the required memory. In cases in which the thin symbols can not be observed, the thin and the thick symbols coincide.

- The CPU-time for the one-domain problem is much smaller using CGS compared with using Gaussian elimination. This is due to the small efficiency of the latter method for large numbers of panels.

- For the computations using Gaussian elimination, the CPU-times for the subdivisions are smaller than the CPU-time for the original one-domain problem. If the results are corrected for the performance of the system still less CPU-time is required except for the two-subdomain problem.
- For the computations using CGS, reductions in CPU-time are obtained in all cases up to a factor 3 when using 8 subdomains. The performance is hardly affected so that it can be concluded that the number of floating point operations is decreased.
- The memory required for these computations is largely decreased when using domain decomposition.

4 Shoal problem

For the intercomparison of some wave propagation models a laboratory experiment was set up at Delft Hydraulics in which a coastal area was schematized, see Berkhoff et al. (1982). The bottom geometry consists of a sloping bottom with slope 1/50 and a shoal. A number of tests were done with different wave conditions. Wave heights were measured along a number of sections. See also Dingemans (1997), sections 4.6 and 5.6.6. Figure 6 shows the geometry of the shoal experiment.

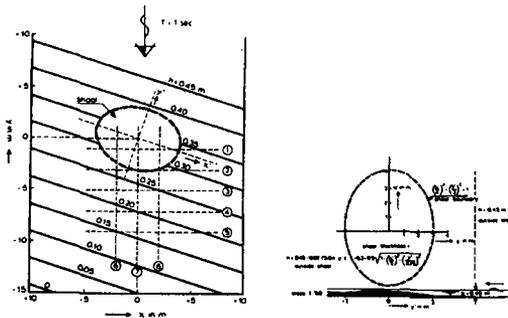


Figure 6: The geometry of the shoal experiment.

The experiment has been used by Broeze (1993) to test the accuracy of the present numerical model. Due to the relatively large domain and the limited number of panels given the available memory at the time, computations were restricted to only a part of the domain and using coarse resolutions. Nevertheless

the results agreed fairly well with the measured data. Figure 7 shows computed and measured wave heights along four sections.

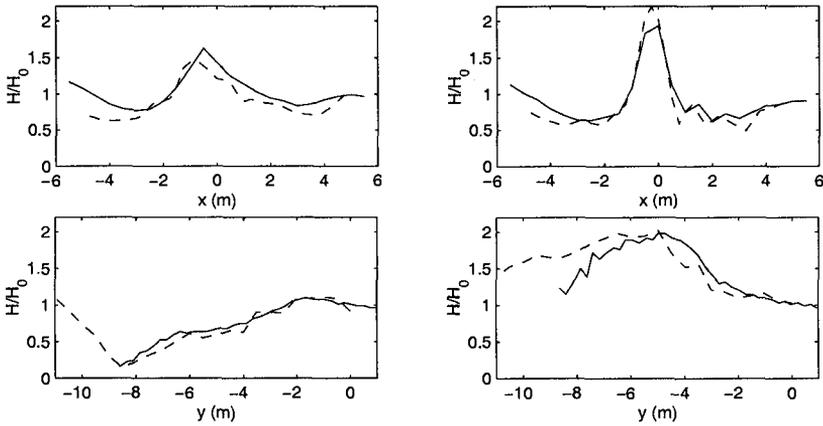


Figure 7: Wave heights relative to height of incoming wave. Computed (solid line) and measured (dashed line) wave heights along sections 2 and 3 (upper two plots) and 6 and 7 (lower two plots).

The domain decomposition technique has the potential to decrease the restrictions mentioned above with respect to the limited available memory. Using domain decomposition more panels can be used taking up the same amount of memory. The computational domain can be increased or a finer resolution can be used.

In application to the shoal problem, however, we found that stability problems occurred due to the Lagrangian motion of the grid and a subsequent deterioration of the convergence of the iterative process. These problems were not solved in the limited time available. A representative computation for these problems is shown in Figure 8.

At the time level shown in this figure, the iterative process diverges on the interfaces located near $y = 13.1$ m and $y = 16.5$ m. Most probably it is related to the distortion of the grid on the interfaces, especially on the interface near $y = 13.1$ m. Due to the large variation of wave height in the region behind the shoal, the variation in horizontal velocities of the free-surface grid is large which causes the grid to distort. As a consequence the interface located behind the shoal distorts as well.

Also subdivisions in the direction perpendicular to the incoming wave direction were applied and showed to be much more stable. But still simulation over the time interval $[0, 18]$ s (required to obtain periodic wave height measurements) did not succeed. As pointed out in Section 2, the use of a mixed

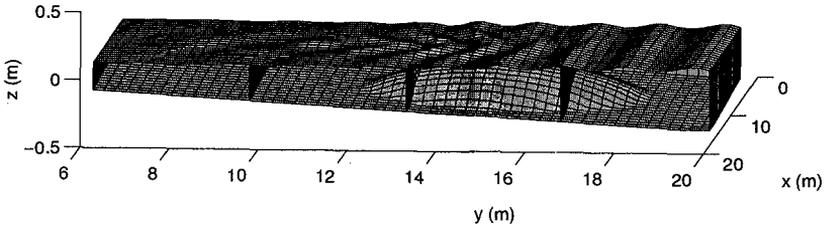


Figure 8: Domain of the shoal-problem at $t = 9.26$ s in the computation using 4 subdomains. The networks in the front lateral boundary are not shown. Waves enter the domain from the right.

Eulerian-Lagrangian method seems to be required to successfully apply the domain decomposition technique.

5 Conclusion

Domain decomposition is a suitable technique to improve the efficiency of boundary integral equation methods, especially in domains originating from water wave problems. In the application to time domain simulations of propagating nonlinear water waves over even bottoms, it leads to large reductions of required computer capacity. In the application to simulations of waves propagating over uneven bottoms, the method still fails. This is probably related to the Lagrangian motion of the free-surface grid and the consequent deterioration of the convergence process.

Acknowledgements

The use of supercomputer facilities was provided by NCF.

References

- [1] Berkhoff, J.W.C., (1982). Refraction and diffraction of water waves, wave deformation by a shoal; comparison between computations and measurements, Delft Hydraulics, report W154-VIII, The Netherlands.
- [2] Berkvens, P.J.F., (1998). *Floating bodies interacting with water waves; Developments of a time-domain panel method*, Ph.D. thesis, University of Twente, Enschede, The Netherlands.

- [3] Broeze, J., (1993). *Numerical modelling of nonlinear free surface waves with a 3D panel method*, Ph.D. thesis, University of Twente, Enschede, The Netherlands.
- [4] De Haas, P.C.A., M.W. Dingemans, and G. Klopman, (1996). Simulation of free long-wave generation due to uneven bottoms, in: *Proc. 25th Int. Conf. on Coastal Engng.*, Orlando, **1**, pp. 165-174.
- [5] De Haas, P.C.A., and P.J. Zandbergen, (1996). The application of domain decomposition to time-domain computations of nonlinear water waves with a panel method, *J. Comp. Phys.*, **129** (2), pp. 332-344.
- [6] Le Tallec, P., (1994). Domain decomposition methods in computational mechanics, *Comp. Mech. Adv.*, **1**, pp. 121-220.
- [7] Quarteroni, A., Ed., (1994). *Proc. Sixth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations*, Como, June 1992 (AMS, Providence).
- [8] Rienecker, M.M. and J.D. Fenton, (1981). A Fourier approximation method for steady water waves. *J. Fluid Mech.* **104**, pp. 119-137.
- [9] Wang, P., Y. Yao and M.P. Tulin, (1994). *Int. Journ. of Offshore and Polar Eng.*, **4**, pp. 200-205.