NUMERICAL MODEL OF FULLY-NONLINEAR WAVE REFRACTION AND DIFFRACTION

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Abstract

Nonlinear mild-slope equations are a set of equations which were derived to analyze fully-nonlinear and fully-dispersive wave transformation. In the present study, it is shown that refraction-diffraction equations including the mild-slope equation, nonlinear shallow water equations and Boussinesq equations are derived as special cases of the nonlinear mild-slope equations. Then, a numerical model is developed for fully-nonlinear wave refraction and diffraction on the basis of the nonlinear mild-slope equations. The model is verified through comparison of numerical results with theoretical and experimental results. Finally, effect of nonlinearity on wave diffraction through a breakwater gap is discussed.

1 Introduction

Mild-slope equation derived by Berkhoff (1972) is used to predict transformation of linear waves due to refraction and diffraction. Boussinesq equations were derived for analyzing transformation of weakly-nonlinear and weakly-dispersive waves, and modified versions have been proposed to apply them in deeper water. However, waves are strongly nonlinear especially in very shallow water.

Nonlinear mild-slope equations are among the equations which were derived recently to analyze fully-nonlinear and fully-dispersive wave transformation. In deriving the equations, the velocity potential is expanded into a series in terms of a given set of vertical distribution functions and then substituted into the Lagrangian defined by Luke (1967). The equations are obtained by applying the variational principle to the Lagrangian. No assumptions are made in the derivation so that they are

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applicable even to strongly nonlinear and strongly dispersive waves and more general than any other wave equations derived so far. In the present study, the mild-slope equation, nonlinear shallow water equations, and Boussinesq equations are derived as special cases of the nonlinear mild-slope equations.

Then, a fully nonlinear and fully dispersive numerical model is developed based on the nonlinear mild-slope equations to predict wave refraction and diffraction. Linear dispersion characteristic of the model is first examined by comparing with the small amplitude wave theory. Next, the model is applied to wave transformation due to a circular shoal and diffraction through a breakwater gap. These results show the validity of the present model. Further discussion is made on the effect of nonlinearity on the wave diffraction.

2 Relation Between Nonlinear Mild-Slope Equations and Various Wave Equations

2.1 Nonlinear mild-slope equations

The nonlinear mild-slope equations are derived from the Lagrangian, L, obtained by Luke (1967):

$$\mathrm{L}[\phi,\eta] = \int_{t_1}^{t_2}\iint_A\int_{-h}^{\eta}\left\{\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial z}\right)^2 + gz\right\} dz\, dA\, dt \tag{1}$$

where ϕ is the velocity potential, η the water surface elevation, t_1 and t_2 arbitrary times, A arbitrary area on the horizontal plane, h the water depth, g the gravitational acceleration, z the vertical coordinate and t the time. To terminate the above Lagrangian with respect to ϕ and η is equivalent to satisfy the Laplace equation, kinematic bottom boundary condition, and kinematic and dynamic surface boundary conditions.

The vertical distribution of the velocity potential, ϕ, is expressed as a series in terms of a set of vertical distribution functions, Z_α:

$$\phi(\mathbf{x},z,t) = \sum_{\alpha=1}^{N} f_\alpha(\mathbf{x},t)\, Z_\alpha(z;h(\mathbf{x})) \equiv f_\alpha\, Z_\alpha \tag{2}$$

where f_α are the coefficients to Z_α and therefore independent of z, and $\mathbf{x} = (x, y)$ denotes the position vector on the horizontal plane. As is normally the case, the expression of Z_α can include the local water depth $h(\mathbf{x})$ as a parameter. Substitution of the above expression into the definition of the Lagrangian (1) and analytical integration in the vertical direction yields the following expression:

$$\mathrm{L}[f_\alpha, \eta] = \int_{t_1}^{t_2} \iint_A \chi(f_\alpha, \eta)\, dA\, dt \tag{3}$$

where

$$\begin{aligned} \chi(f_\alpha, \frac{\partial f_\alpha}{\partial t}, \eta, \frac{\partial \eta}{\partial t}) &= \frac{g}{2}(\eta^2 - h^2) + \tilde{Z}_\beta \frac{\partial f_\beta}{\partial t} + \frac{1}{2} A_{\gamma\beta}(\nabla f_\gamma)(\nabla f_\beta) + \frac{1}{2} B_{\gamma\beta} f_\gamma f_\beta \\ &+ C_{\gamma\beta} f_\gamma (\nabla f_\beta)(\nabla h) + \frac{1}{2} D_{\gamma\beta} f_\gamma f_\beta (\nabla h)^2 \end{aligned} \tag{4}$$

and

$$\begin{aligned} &A_{\alpha\beta} = \int_{-h}^{\eta} Z_\alpha Z_\beta\, dz, \qquad B_{\alpha\beta} = \int_{-h}^{\eta} \frac{\partial Z_\alpha}{\partial z} \frac{\partial Z_\beta}{\partial z}\, dz, \; C_{\alpha\beta} = \int_{-h}^{\eta} \frac{\partial Z_\alpha}{\partial h} Z_\beta\, dz, \\ &D_{\alpha\beta} = \int_{-h}^{\eta} \frac{\partial Z_\alpha}{\partial h} \frac{\partial Z_\beta}{\partial h}\, dz, \; \tilde{Z}_\alpha = \int_{-h}^{\eta} Z_\alpha\, dz \end{aligned} \tag{5}$$

To terminate the Lagrangian with respect to f_α and η, the following Euler equations should be satisfied:

$$\frac{\partial \chi}{\partial f_\alpha} = \frac{\partial}{\partial t}\left[\frac{\partial \chi}{\partial(\partial f_\alpha / \partial t)}\right] + \nabla\left[\frac{\partial \chi}{\partial(\nabla f_\alpha)}\right] \tag{6}$$

$$\frac{\partial \chi}{\partial \eta} = \frac{\partial}{\partial t}\left[\frac{\partial \chi}{\partial(\partial \eta / \partial t)}\right] + \nabla\left[\frac{\partial \chi}{\partial(\nabla \eta)}\right] \tag{7}$$

Substituting the definition of χ, Eq. (4), into the above equations and neglecting the second and higher order terms in the bottom slope, we obtain the following equations:

$$Z_\alpha^\eta \frac{\partial \eta}{\partial t} + \nabla(A_{\alpha\beta} \nabla f_\beta) - B_{\alpha\beta} f_\beta + (C_{\beta\alpha} - C_{\alpha\beta})(\nabla f_\beta)(\nabla h) + \frac{\partial Z_\beta^\eta}{\partial h} Z_\alpha^\eta f_\beta (\nabla \eta)(\nabla h) = 0 \tag{8}$$

$$g\eta + Z_\beta^\eta \frac{\partial f_\beta}{\partial t} + \frac{1}{2} Z_\gamma^\eta Z_\beta^\eta (\nabla f_\gamma)(\nabla f_\beta) + \frac{1}{2} \frac{\partial Z_\gamma^\eta}{\partial z} \frac{\partial Z_\beta^\eta}{\partial z} f_\gamma f_\beta + \frac{\partial Z_\gamma^\eta}{\partial h} Z_\beta^\eta f_\gamma (\nabla f_\beta)(\nabla h) = 0 \tag{9}$$

where

$$Z_\alpha^\eta = Z_\alpha|_{z=\eta}, \qquad \frac{\partial Z_\alpha^\eta}{\partial z} = \left.\frac{\partial Z_\alpha}{\partial z}\right|_{z=\eta} \tag{10}$$

Equation (8) is a vector equation with N components and Eq. (9) is a scalar equation, whereas the unknowns are η and f_α ($\alpha = 1$ to N). Thus the above set of partial differential equations evolutional in the horizontal two dimensions are closed if an appropriate set of initial and boundary conditions are given. We call the set as nonlinear mild-slope equations (Isobe, 1994). No assumptions other than the series expression of the velocity potential are made to derive the nonlinear mild-slope equations; therefore the equations include full nonlinearity and full dispersivity as long as sufficient number of terms are used in the series.

Various wave equations such as mild-slope equation, nonlinear shallow water equations and Boussinesq equations are derived theoretically upon ordering nondimensional parameters. Each ordering results in a specific vertical distribution of wave motion. In a sense, ordering and vertical distribution are equivalent, and the validity of a wave equation depends on the accuracy of the vertical distribution instead of the magnitudes of nondimensional parameters used in the assumption. In the following, it is shown that various wave equations are derived from the nonlinear mild-slope equations by giving a proper set of vertical distribution functions.

2.2 Relation with mild-slope equation

The mild-slope equation (Berkhoff, 1972) is a linear refraction-diffraction equation in which vertical distribution is expressed by the hyperbolic cosine function. Hence we first linearize the nonlinear mild-slope equations (8) and (9):

$$Z^o_\alpha \frac{\partial \eta}{\partial t} + \nabla(A^o_{\alpha\beta}\nabla f_\beta) - B^o_{\alpha\beta} f_\beta + (C^o_{\beta\alpha} - C^o_{\alpha\beta})(\nabla f_\beta)(\nabla h) = 0 \tag{11}$$

$$g\eta + Z^o_\beta \frac{\partial f_\beta}{\partial t} = 0 \tag{12}$$

where the superscript ^{o} denotes the quantity evaluated at the mean water level instead of the water surface. Next we express the velocity potential by only one vertical distribution function of hyperbolic cosine type:

$$\phi(\mathbf{x}, z, t) = f(\mathbf{x}, t)\, Z(z) \tag{13}$$

$$Z(z) = \frac{\cosh k(h+z)}{\cosh kh} \tag{14}$$

where k satisfies the linear dispersion relation. Then, Eqs. (11) and (12) become

$$\frac{\partial \eta}{\partial t} + \nabla\left(\frac{CC_g}{g}\nabla f\right) + \frac{1}{g}\left(k^2 CC_g - \sigma^2\right) f = 0 \tag{15}$$

$$g\eta + \frac{\partial f}{\partial t} = 0 \tag{16}$$

Eliminating η from the above two equations, we obtain the following time-dependent form of the mild-slope equation:

$$\nabla\left(CC_g \nabla f\right) + \left(k^2 CC_g - \sigma^2\right) f - \frac{\partial^2 f}{\partial t^2} = 0 \tag{17}$$

If we assume a sinusoidal oscillation as:

$$f = \hat{f} e^{-i\sigma t} \tag{18}$$

we finally obtain the mild-slope equation:

$$\nabla\left(CC_g\nabla\hat{f}\right)+k^2CC_g\hat{f}=0 \tag{19}$$

Equations for refraction and diffraction of linear random waves are obtained by taking multiple components in Eqs. (11) and (12). The vertical distribution functions are defined as

$$Z_\alpha=\frac{\cosh k_\alpha(h+z)}{\cosh k_\alpha h} \tag{20}$$

$$\sigma_\alpha^2=gk_\alpha\tanh k_\alpha h \tag{21}$$

Then, Eqs. (11) and (12) become

$$\frac{\partial\eta}{\partial t}+A^o_{\alpha\beta}\nabla^2 f_\beta-B^o_{\alpha\beta}f_\beta=0 \tag{22}$$

$$g\eta+\sum_{\beta=1}^{N}\frac{\partial f_\beta}{\partial t}=0 \tag{23}$$

where

$$A^o_{\alpha\beta}=\begin{cases}\dfrac{1}{g}\dfrac{\sigma_\alpha^2-\sigma_\beta^2}{k_\alpha^2-k_\beta^2} & (\alpha\neq\beta)\\ \dfrac{1}{g}c_\alpha^2 n_\alpha & (\alpha=\beta)\end{cases}\qquad B^o_{\alpha\beta}=\begin{cases}\dfrac{1}{g}\dfrac{k_\alpha^2\sigma_\beta^2-k_\beta^2\sigma_\alpha^2}{k_\alpha^2-k_\beta^2} & (\alpha\neq\beta)\\ \dfrac{1}{g}\sigma_\alpha^2(1-n_\alpha) & (\alpha=\beta)\end{cases} \tag{24}$$

From Eqs. (22) and (23), η can be eliminated to yield

$$-\frac{1}{g}\sum_{\beta=1}^{N}\frac{\partial^2 f_\beta}{\partial t^2}+A^o_{\alpha\beta}\nabla^2 f_\beta-B^o_{\alpha\beta}f_\beta=0 \tag{25}$$

By assuming progressive waves with the angular frequency $\hat{\sigma}$ and wave number $\hat{\mathbf{k}}$:

$$f_\alpha=a_\alpha e^{i(\hat{\mathbf{k}}\mathbf{x}-\hat{\sigma}t)} \tag{26}$$

Equation (25) becomes

$$\sum_{\beta=1}^{N}\left(\frac{\hat{\sigma}^2}{g}-B^o_{\alpha\beta}\right)a_\beta=\hat{k}^2\sum_{\beta=1}^{N}A^o_{\alpha\beta}a_\beta \tag{27}$$

To have a nontrivial solution, $\hat{k}^2$ is determined as an eigenvalue for a given $\hat{\sigma}$. It can easily be proved that $\hat{k}=k_\alpha$ for $\hat{\sigma}=\sigma_\alpha$, and therefore the dispersion relation is exactly satisfied at the frequencies σ_α ($\alpha=1$ to N). This suggests that the dispersion relation is accurately satisfied even if the frequency is not equal to either of the selected frequencies. Therefore, transformation of random waves with a wide spectrum can accurately be calculated by Eqs. (22) and (23).

2.3 Relation with nolinear shallow water equations

In the nonlinear shallow water equations (Stoker, 1957), vertical distribution of the pressure is hydrostatic and that of the horizontal water particle velocity is uniform. Therefore we take one component of the vertical distribution function which is uniform:

$$Z = 1 \tag{28}$$

Then, the matrices $A_{\alpha\beta}$, $B_{\alpha\beta}$ and $C_{\alpha\beta}$ defined by Eq. (5) have one component, respectively, as

$$A = h + \eta, \quad B = 0, \quad C = 0 \tag{29}$$

and the nonlinear mild-slope equations (8) and (9) become

$$\frac{\partial \eta}{\partial t} + \nabla\left[(h+\eta)\nabla f\right] = 0 \tag{30}$$

$$g\eta + \frac{\partial f}{\partial t} + \frac{1}{2}(\nabla f)^2 = 0 \tag{31}$$

By rewriting the above equations in terms of $\mathbf{u}$:

$$\mathbf{u} = \nabla\phi = \nabla f \tag{32}$$

the nonlinear shallow-water equations are obtained:

$$\frac{\partial \eta}{\partial t} + \nabla\left[(h+\eta)\mathbf{u}\right] = 0 \tag{33}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} + g\nabla\eta = 0 \tag{34}$$

2.4 Relation with Boussinesq equations

Since the vertical distribution of the horizontal water particle velocity in the Boussinesq equations is expressed by the linear combination of uniform and parabolic components, the following two vertical distribution functions are employed:

$$Z_1 = 1, \quad Z_2 = \frac{(h+z)^2}{h^2} \tag{35}$$

Then, the nonlinear mild-slope equations (8) and (9) become

$$\begin{aligned}\frac{\partial \eta}{\partial t} &+ \nabla\left[(h+\eta)\nabla f_1 + \frac{(h+\eta)^3}{3h^2}\nabla f_2\right] + \frac{(h+\eta)^2(h-2\eta)}{3h^3}(\nabla f_2)(\nabla h) \\ &- \frac{2(h+\eta)\eta}{h^3} f_2(\nabla\eta)(\nabla h) = 0\end{aligned} \tag{36}$$

$$\frac{(h+\eta)^2}{h^2}\frac{\partial\eta}{\partial t}+\nabla\left[\frac{(h+\eta)^3}{3h^2}\nabla f_1+\frac{(h+\eta)^5}{5h^4}\nabla f_2\right]-\frac{4(h+\eta)^3}{3h^4}f_2$$
$$-\frac{(h+\eta)^2(h-2\eta)}{3h^3}(\nabla f_1)(\nabla h)-\frac{2(h+\eta)^3\eta}{h^5}f_2(\nabla\eta)(\nabla h)=0 \quad (37)$$

$$g\eta+\frac{\partial f_1}{\partial t}+\frac{(h+\eta)^2}{h^2}\frac{\partial f_2}{\partial t}+\frac{1}{2}\left\{\nabla f_1+\frac{(h+\eta)^2}{h^2}\nabla f_2\right\}^2+\frac{1}{2}\left\{\frac{2(h+\eta)}{h^2}f_2\right\}^2$$
$$-\frac{2(h+\eta)\eta}{h^3}f_2\left\{\nabla f_1+\frac{(h+\eta)^2}{h^2}\nabla f_2\right\}(\nabla h)=0 \quad (38)$$

The assumption of $O[H/h]\sim O[(h/L)^2]$ (H: wave height, h: water depth, L: wavelength) leads to the following ordering:

$$\nabla h\sim O\left[\sqrt{\varepsilon}\right],\quad \eta\sim f_1\sim O[\varepsilon],\quad f_2\sim O\left[\varepsilon^2\right] \quad (39)$$

By considering the above ordering, Eqs. (36) to (38) are simplified as

$$\frac{\partial\eta}{\partial t}+\nabla\left[(h+\eta)\nabla f_1+\frac{h}{3}\nabla f_2\right]+\frac{1}{3}(\nabla f_2)(\nabla h)=0 \quad (40)$$

$$\frac{\partial\eta}{\partial t}+\nabla\left[\frac{h}{3}\nabla f_1\right]-\frac{4}{3h}f_2-\frac{1}{3}(\nabla f_1)(\nabla h)=0 \quad (41)$$

$$g\eta+\frac{\partial f_1}{\partial t}+\frac{\partial f_2}{\partial t}+\frac{1}{2}(\nabla f_1)^2=0 \quad (42)$$

Then, because

$$\mathbf{u}=\nabla\phi=\nabla f_1+\frac{(h+z)^2}{h^2}\nabla f_2-\frac{2z(h+z)}{h^3}f_2\nabla h \quad (43)$$

$$\bar{\mathbf{u}}\approx\nabla f_1+\frac{1}{3}\nabla f_2+\frac{1}{3h}f_2\nabla h \quad (44)$$

Equations (40) to (42) are combined to yield the Boussinesq equations (Peregrine, 1967):

$$\frac{\partial\eta}{\partial t}+\nabla\left[(h+\eta)\bar{\mathbf{u}}\right]=0 \quad (45)$$

$$\frac{\partial\bar{\mathbf{u}}}{\partial t}+(\bar{\mathbf{u}}\nabla)\bar{\mathbf{u}}+g\nabla\eta=-\frac{h^2}{6}\frac{\partial}{\partial t}\nabla(\nabla\bar{\mathbf{u}})+\frac{h}{2}\frac{\partial}{\partial t}\nabla\left[\nabla(h\bar{\mathbf{u}})\right] \quad (46)$$

3 Verification of Numerical Model

3.1 Outline of numerical model

A finite difference numerical model is developed based on the nonlinear mild-slope equations, in which the nonlinear equations are solved by simple successive substitution or Newton-Raphson scheme.

Non-reflective boundary conditions are installed along the boundaries by introducing sponge layers in which an energy dissipation term, D, is added to the left side of Eq. (9) (Cruz et al., 1994):

$$D = \epsilon(x) \sum_{\alpha=1}^{N} f_\alpha \tag{47}$$

where

$$\epsilon(x) = \frac{r\epsilon_m}{2(\sinh r - r)} \left[\cosh\left(\frac{rx}{F}\right) - 1\right] \tag{48}$$

$$\epsilon_m = \theta\sqrt{g/h} \tag{49}$$

and F is the width of the sponge layer, $\theta = 1.0$ to 2.0 and $r = 3$. Incident waves are given as a discontinuity at the interface between the actual calculation domain and sponge layer (Ishii et al., 1996). Still water is the initial condition for all calculations.

3.2 Effect of vertical distribution functions

A typical example of sets of vertical distribution functions is a set of polynomial functions:

$$Z_\alpha = \left(1 + \frac{z}{h}\right)^{2(\alpha-1)} \tag{50}$$

The coefficients are easily calculated as

$$Z_\alpha^\eta = \zeta^{2(\alpha-1)} \tag{51}$$

$$A_{\alpha\beta} = \frac{h\ \zeta^{2(\alpha+\beta)-3}}{2(\alpha+\beta)-3} \tag{52}$$

$$B_{\alpha\beta} = \frac{4(\alpha-1)(\beta-1)}{2(\alpha+\beta)-5}\ \frac{\zeta^{2(\alpha+\beta)-5}}{h} \tag{53}$$

$$C_{\alpha\beta} = 2(\alpha-1)\ \zeta^{2(\alpha+\beta)-4}\left[-\frac{\zeta}{2(\alpha+\beta)-3} + \frac{1}{2(\alpha+\beta)-4}\right] \tag{54}$$

$$D_{\alpha\beta} = \frac{4(\alpha-1)(\beta-1)\ \zeta^{2(\alpha+\beta)-5}}{h}\left[\frac{\zeta^2}{2(\alpha+\beta)-3} - \frac{\zeta}{\alpha+\beta-2} + \frac{1}{2(\alpha+\beta)-5}\right] \tag{55}$$

where

$$\zeta = \frac{h+\eta}{h} \tag{56}$$

The above set of polynomial functions gives an accurate linear dispersion relation even in deep waters (Isobe, 1994); however hyperbolic cosine functions are expected to be more effective in deep water. In the present study, two sets are examined for propagation of permanent waves with various water depths and wave heights. In the first set (CASE A), the following two functions are taken as the vertical distribution functions:

$$Z_1(z) = 1, \qquad Z_2(z) = \left(1 + \frac{z}{h}\right)^2 \tag{57}$$

The second set (CASE B) is

$$Z_1(z) = \left(1 + \frac{z}{h}\right)^2, \qquad Z_2(z) = \frac{\cosh k(h+z)}{\cosh kh} \tag{58}$$

in which Z_1 and Z_2 are expected to become effective in shallow and deep waters, respectively.

Figures 1 to **3** show sample results of 1-D propagation of permanent waves. Permanent waves are incident at $x/L = 1$ and a sponge layer is installed from $x/L = 3$ to 9, and thus the actual region is from $x/L = 1$ to 3. The distributions of η/H_I and H/H_I (H: wave height and H_I: incident wave height), are shown in the

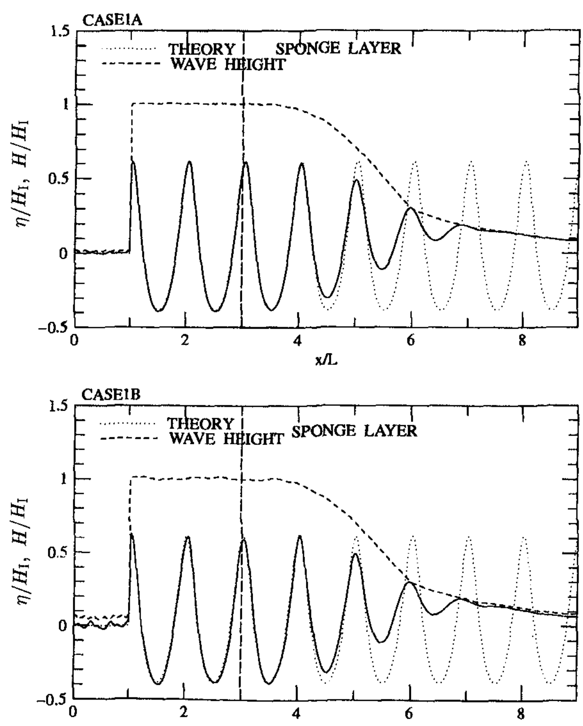

Fig. 1 Comparison between exact and numerical solutions of water surface profile of permanent waves ($h/L_0 = 0.1$, $H_\mathrm{I}/h = 0.3$, $H_\mathrm{I}/L_0 = 0.03$, $U_\mathrm{r} = 16$).

figure. Since **Fig. 1** is for weakly nonlinear waves on an intermediate water depth in which $h/L_o = 0.1$ (h: water depth and L_o: deep water wavelength of linear waves), the agreement with theory is good for the two sets of vertical distribution functions. **Figure 2** is for highly nonlinear waves on a shallow water. Nonlinear effect is well reproduced by the two sets. However, for deep water as shown in **Fig. 3**, difference in wavelength is signigicant in CASE A, indicating that the dispersion effect is not enough. In general, the vertical distribution functions of CASE B are appropriate in deep water, whereas those of CASE A give numerical solution even for near-breaking waves in shallow water.

The numerical model of CASE B is applied to wave diffraction due to a circular shoal for which an experiment is conducted by Ito and Tanimoto (1972). **Figure 4** shows the bottom configuration in the prototype scale and wave height distributions along cross-shore and alongshore directions. The water depth in the uniform region

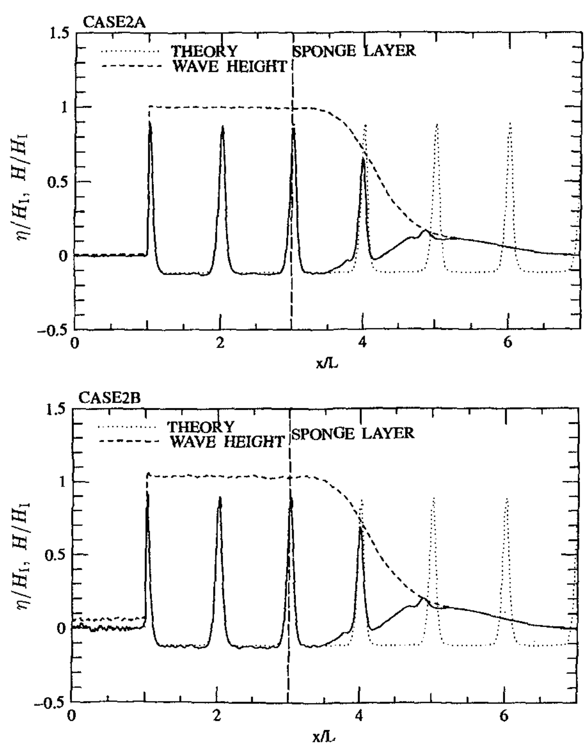

Fig. 2 Comparison between exact and numerical solutions of water surface profile of permanent waves ($h/L_0 = 0.005$, $H_I/h = 0.3$, $H_I/L_0 = 0.0015$, $U_r = 450$).

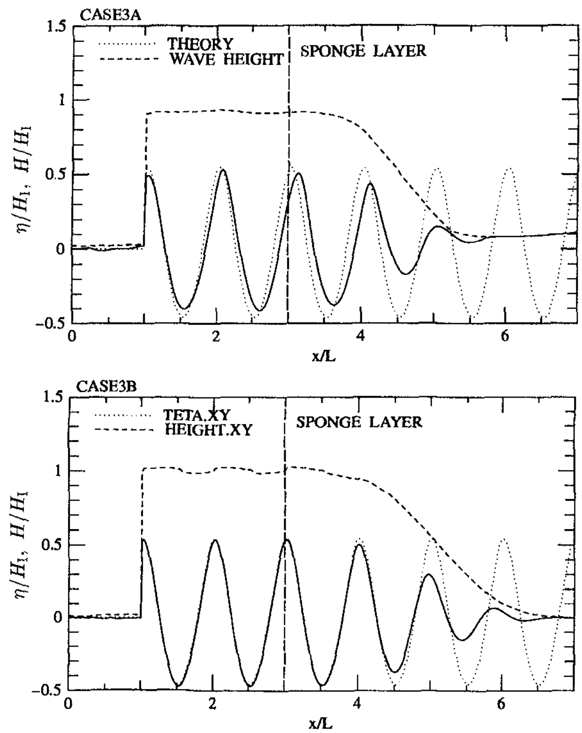

Fig. 3 Comparison between exact and numerical solutions of water surface profile of permanent waves ($h/L_0 = 0.1$, $H_\mathrm{I}/h = 0.5$, $H_\mathrm{I}/L_0 = 0.05$, $U_\mathrm{r} = 0.41$).

is 15m, the wave period 5.1s and the incident wave height 1m, resulting in fairy strong nonlinearity. Comparison of wave height distribution between calculation and measurement indicates the validity of the present model.

3.3 Effect of nonlinearity on wave diffraction

Diffraction of waves through a breakwater gap is calculated to elucidate the effect of nonlinearity on diffraction. **Figure 5** shows an example in which $h/L_0 = 0.05$, $B/L = 2$ (B: gap width and L: wavelength of linear waves), and $H_\mathrm{I}/L_0 = 0.0002$, 0.024, and 0.032. As can be seen from comparison among the three figures, nonlinearity accelerates diffraction, causing smaller wave height in direct wave incidence region along $y = 0$ and larger wave height in the shadow region. This implies that the function of breakwater to make a calm region behind it is less effective in

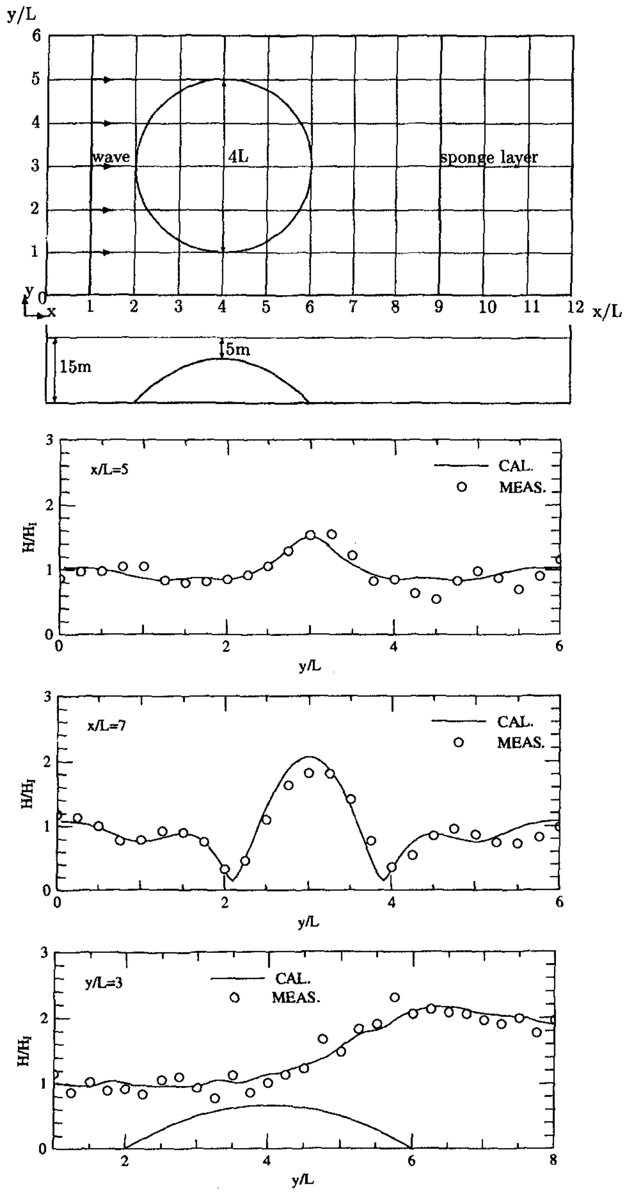

Fig. 4 Comparison of calculated wave height distribution around a circular shoal with measurement by Ito and Tanimoto (1972).

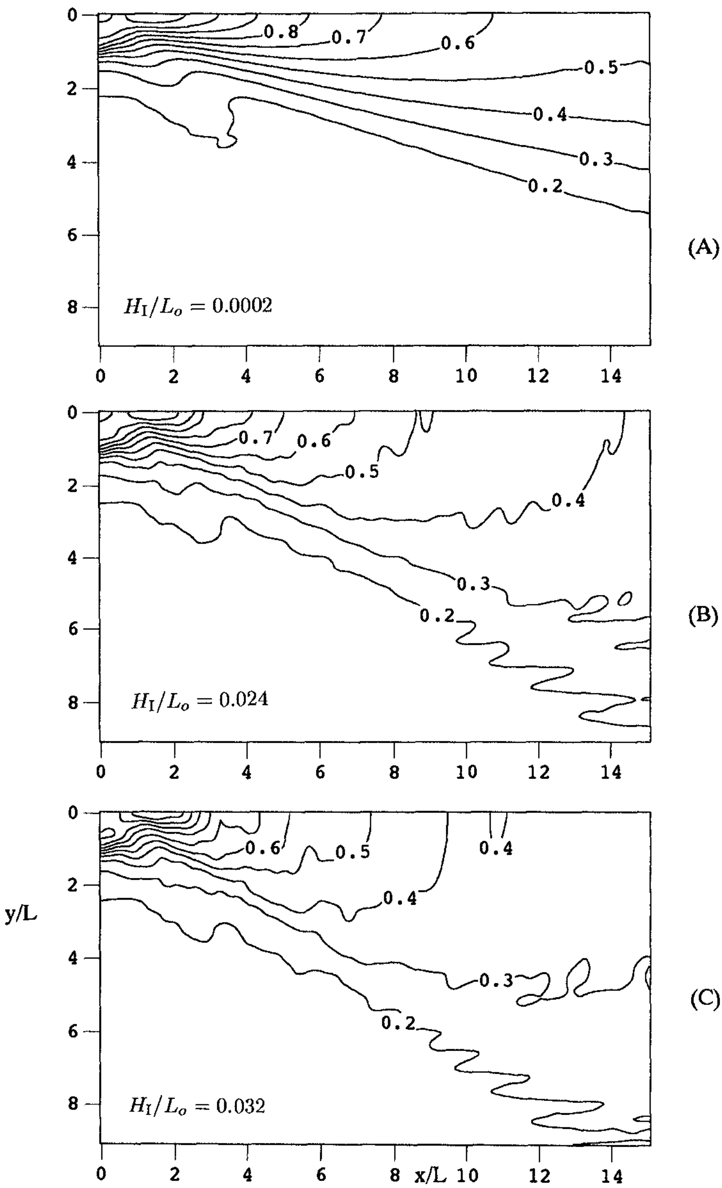

Fig. 5 Effect of wave nonlinearity on diffraction behind a breakwater gap ($B/L = 2$, $h/L_o = 0.05$).

rough wave condition.

4 Conclusion

Nonlinear mild-slope equations are derived only by expanding the velocity potential into a series in terms of a given set of vertical distribution functions and hence include full nonlinearity and full dispersivity. In the former part of the present paper, it was shown that the mild-slope equation, nonlinear shallow water equations and Boussinesq equations can be derived as special cases of the nonlinear mild-slope equations. It was also shown that the dispersion relation is accurately expressed by linearized forms of the nonlinear mild-slope equations.

A numerical model is developed based on the nonlinear mild-slope equations. The validity of the model is verified through calculations of waves of permanent type and wave transformation around a circular shoal. The result for diffraction of waves through a breakwater gap showed the larger diffraction effect for the more nonlinear waves. This suggests the importance to consider nonlinear effect in the diffraction diagrams since they are used to predict tranquility in the shadow region for rough incident waves. Systematic calculations will be made in the future.

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