EFFECTS OF WAVE REFLECTION AND DISSIPATION ON WAVE-INDUCED SECOND ORDER MAGNITUDES

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Abstract

The influence of reflection and dissipation on wave-induced mean magnitudes is studied. Starting from linear wave theory the second order quantities are derived in terms of transfer functions considering regular as well as irregular waves. It is shown that any reflective and dissipative natural or artificial structure such as a vegetation field, or an emerged or submerged breakwater, induces spatial variations of the mean quantities such as the mean water level, mass flux, energy flux or radiation stress. The evolution of these magnitudes is analogous to their behavior in the surf zone, showing wave damping and modulation. Compared with the experimental results, the models presented are able to reproduce wave height transformation as well as mean water level variations along the dissipative structures with reasonable accuracy.

Introduction

Wave reflection and dissipation are two important wave transformation processes close to structures or at the shoreline. Through the years, research has been carried out in order to understand the role of reflection and dissipation on coastal engineering problems, especially concentrating on wave height evolution. In this paper attention will be paid to how nonlinear quantities obtained from linear wave theory, such as: mass transport, mean water level, momentum flux, energy flux, radiation stress, etc., are affected by wave reflection and

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Figure 1. Definition Sketch

dissipation. Reflection and dissipation are considered to be induced by natural (vegetation) or artificial emerged or submerged permeable structures. Dissipation by wave breaking is neglected in most of the cases considered since its effects are well known in the surf zone. The detailed examination of these hydrodynamic mean quantities will be a first step to analyze the influence of the presence of the structure on the morphodynamics of its vicinity.

Theory

Starting from linear wave theory the second order quantities are derived assuming complex reflection and transmission coefficients including magnitude and phase information. The coefficients and wave amplitudes can be calculated solving the first order problem. The models used are able to consider several structure geometries, porous material characteristics and incident wave climates. Therefore, it can be established how each of the second order magnitudes is affected by these parameters.

For an emerged vertical permeable breakwater on a horizontal bottom, Fig. 1a, the solution based on an eigenfunction expansion presented in Dalrymple *et al.* (1991) is used to evaluate the first order problem. Regular, oblique incident waves are considered to impinge the structure. As a result the potential inside and outside the structure is known and conse-

quently the free surface evolution, velocity, acceleration and pressure field at any point in the domain considered.

A similar approach is used for a submerged permeable step, Fig. 1*a*, as shown in Losada *et al.* (1996a and b). In these papers the solution also considers irregular incident waves and the model is extended to analyze submerged permeable structures of arbitrary geometry on a sloping bottom, Fig. 1*b*, using a modified mild slope equation. The equation said is suitable to include wave breaking effects via a Dally *et al.* (1985) breaking model or others like the one proposed in Rojakanamthorn *et al.* (1990).

In order to consider a natural reflective, dissipative medium different than a beach a vegetation field is studied, Fig. 1*c*. The wave-induced first order kinematics and dynamics under regular and irregular waves is evaluated extending the work of Kobayashi *et al.* (1993) and Dubi and Torum (1994, 1996) as explained in Méndez (1997).

Once the first order solution is known for any of the structures considered, mean water level, mass flux, energy flux and radiation stresses can be calculated. These magnitudes, time averaged and correct to second order are proportional to the wave height squared. Regular as well as irregular waves are considered.

The expressions of the mean quantities are formulated in a general form in terms of transfer functions which vary depending on the structure considered.

Mass transport

To obtain the total mass flux in the x-direction, M_x , the following integration has to be carried out

$$M_x = \overline{\int_{-h}^{\eta} \rho u dz} = \overline{\int_{-h}^{\eta} \rho u dz} + \overline{\int_{0}^{\eta} \rho u dz} = \overline{\int_{0}^{\eta} \rho u dz}$$
(1)

Expressing the horizontal velocity in terms of a Taylor series, to second order and using transfer functions, the mass flux in the x-direction for a monochromatic wave train is

$$M_x = \frac{1}{2} a^2 \rho \operatorname{Re}[H_\eta H_u^*] \bigg|_{z=0}$$
⁽²⁾

where (*) stands for complex conjugate, ρ is the water density, *a* the incident wave amplitude and Re[] stands for real part of the magnitude in brackets.

The mass flux can be expressed in terms of the incident directional spectrum of the free surface η , $S_{\eta}(f)G(f, \theta)$, by applying the results from the linear theory to individual spectral components, Battjes (1974), where $S_{\eta}(f)$ is a frequency energy spectrum and $G(f, \theta)$ is a directional spreading function. This gives

$$M_x = \rho \int_{-\pi}^{\pi} \int_0^{\infty} \operatorname{Re}[H_{\eta}H_u^*]|_{z=0} S_{\eta}(f) G(f,\theta) df d\theta$$
(3)

For an incident unidirectional spectrum, $S_{\eta}(f)$, the mass flux is

$$M_x = \rho \int_0^\infty \operatorname{Re}[H_\eta H_u^*]|_{z=0} S_\eta(f) df$$
(4)

Assuming a very narrow incident spectrum (one unique component with amplitude a) we

obtain $\int_0^\infty S_\eta(f) df = \frac{1}{2}a^2$ and, therefore, the mass flux can be expressed as eq. (2).

Mean water level

The second order mean water surface displacement, $\bar{\eta}$, can be evaluated averaging the Bernoulli equation over a wave period. Neglecting third order terms and higher, yields:

$$\bar{\eta} = -\frac{1}{2g} \left(\overline{u^2} + \overline{v^2} - \overline{w^2} \right) \Big|_{z=0} + \frac{C(t)}{g}$$
(5)

In terms of the transfer functions the mean water level may be expressed as:

$$\bar{\eta} = -\frac{1}{4g}a^2 \left(|H_u|^2 + |H_v|^2 - |H_w|^2\right)\Big|_{z=0} + \frac{\overline{C(t)}}{g}$$
(6)

where the transfer function expression varies depending on the region where the mean water level has to be evaluated.

Radiation stress

The radiation stress will be affected by the presence of reflected waves and the dissipation induced by the flow through the porous material or breaking. The four components of the radiation tensor in a fluid region are in terms of the transfer functions

$$S_{xx} = \frac{1}{2}a^2 \int_{-h}^{0} \rho\left(|H_u|^2 - |H_w|^2\right) dz + \frac{1}{4}\rho g a^2 |H_\eta|^2 \tag{7}$$

$$S_{yy} = \frac{1}{2}a^2 \int_{-h}^{0} \rho \left(|H_v|^2 - |H_w|^2 \right) dz + \frac{1}{4}\rho g a^2 |H_\eta|^2 \tag{8}$$

$$S_{xy} = \frac{1}{2}a^2 \int_{-h}^{0} \rho \operatorname{Re}[H_u H_v^*] dz$$
⁽⁹⁾

where h is the water depth.

Above and inside a porous layer the components of the radiation stress due to the waveinduced velocities have to be multiplied by the factor εs

$$S_{xx} = \frac{1}{2}a^2 \int_{-h}^{-h+\alpha h} \rho s\varepsilon \left(|H_u|^2 - |H_w|^2 \right) dz + \frac{1}{2}a^2 \int_{-h+\alpha h}^{0} \rho \left(|H_u|^2 - |H_w|^2 \right) dz + \frac{1}{4}\rho g a^2 |H_\eta|^2$$
(10)

$$S_{yy} = \frac{1}{2}a^2 \int_{-h}^{-h+\alpha h} \rho s \varepsilon \left(|H_v|^2 - |H_w|^2 \right) dz + \frac{1}{2}a^2 \int_{-h+\alpha h}^{0} \rho \left(|H_v|^2 - |H_w|^2 \right) dz + \frac{1}{4}\rho g a^2 |H_\eta|^2$$
(11)

$$S_{xy} = \frac{1}{2}a^2 \int_{-h}^{-h+\alpha h} \rho s\varepsilon \operatorname{Re}[H_u H_v^*] dz + \frac{1}{2}a^2 \int_{-h+\alpha h}^{0} \rho \operatorname{Re}[H_u H_v^*] dz$$
(12)

where ε is the porosity of the porous material and s an inertia coefficient, generally taken to be one and αh is the height of the porous material. Note that $\alpha = 1$ corresponds to an emerged vertical permeable structure.

In the Appendix transfer functions, including evanescent modes, in eqs. (2) to (12) for regular incident waves impinging on a submerged permeable step of height αh are given as an example.

<u>Results</u>

In Fig. 2 the wave height, H_{rms} and the mean water level, $\bar{\eta}$ evolution is presented, under non-breaking conditions, along two different submerged steps of identical geometry and different material (impermeable and permeable with $D_{50} = 2.09 \text{ cm}$ and $\varepsilon = 0.521$). The step geometry is given by $\alpha h = 0.385 \text{ m}$ and b = 0.8 m. Different incident wave conditions have been also considered in the experiments described in Losada *et al.* (1997). The mean water level variation has been obtained using eq. (6) and (14) to (14).

Fig. 3 shows the experimental results in Rivero *et al.* (1998) versus the numerical results obtained using the mild slope model, Losada *et al.* (1996a) for a submerged breakwater, with 1:1.5 slopes on both sides, a crown width of 0.61 m and constructed with an impermeable core and an armour layer of quarrystones with mean weight of 25 Kg. The water depth at the toe of the structure was 1.50 m. The mean water level variation is solved using the time-averaged and depth-integrated momentum equation

$$\frac{\partial S_{xx}}{\partial x} = -\rho g(h + \bar{\eta}) \frac{\partial \bar{\eta}}{\partial x}$$
(13)

where S_{xx} is expressed as in eq. (10) and the corresponding transfer functions. Wave breaking takes place along the crest and therefore, it is considered in the modelling using Rojanakamthorn *et al.* (1990), Méndez *et al.* (1998).

In general, there is a good agreement between the experimental and numerical wave height transformation for both the rectangular and the trapezoidal breakwaters under breaking and non-breaking conditions. The modulation of wave height induced by the reflection in front and above the breakwater is very well reproduced by the theory. However, behind the breakwater only the trapezoidal breakwater model does reproduce the modulation well since it considers reflection induced by the slope. This is due to the fact that the eigenfunction model used for the rectangular breakwater assumes the region leewards the structure to be semi-infinite.

Results show that the mean water level presents a set-up induced by the reduction in wave height due to dissipation even without breaking. However, results have shown that the portion of the total set-up induced by wave breaking is more important than the part induced by dissipation inside the pores.

In Fig. 4 the theoretical results for wave height variation and nondimensional mean water



Figure 2. Wave height and mean water level evolution along two different submerged steps (impermeable and permeable)



Figure 3. Wave height and mean water level evolution in a submerged breakwater. Comparison between experimental data (Rivero et al., 1998) and numerical model results.

level are compared with the experimental results for artificial seaweed included in Kobayashi *et al.* (1991). Unfortunately, experimental mean water level data are not available. The experiment was carried out in a 27 m long, 0.5 m wide and 0.7 m high wave flume. The artificial seaweed was made of polypropylene strips with a specific gravity $s = \frac{\rho_w}{\rho} = 0.9$. The length, width and thickness of each strip was: $d_v = 0.25 m$, $b_v = 5.2 cm$ and $t_v = 0.03 mm$, respectively. Each of the strips was fixed to a wire net at the bottom of the flume and placed to produce maximum resistance to the incident flow. The vegetation field, located at the center of the flume had a total width of b = 8 m, (Kobayashi *et al.*, 1993, fig. 1). The number of uniformly distributed strips per unit horizontal area was N = 1110 and 1490 units/m².

The wave height evolution is well reproduced by the model including the modulation in front and above the vegetation field induced by reflection. Associated with the wave height decay along the vegetation field there is a clear wave set-up showing similar modulations as the wave height.

Fig. 5 shows the wave height evolution and mean water level variation on an artificial *laminaria hyperborea* field for regular and irregular wave conditions (a: JONSWAP spectrum, $H_{rms,i} = 1.41 \text{ m}, k_p h = 0.65, \gamma = 3.3, h = 6 \text{ m}$) and (b: JONSWAP spectrum, $H_{rms,i} = 1.51 \text{ m}, k_p h = 0.51, \gamma = 3.3, h = 6 \text{ m}$). $H_{rms,i}$, is the incident root-mean-square wave height. The theoretical results are compared with the experimental data in Dubi (1995). C_D is the drag coefficient used for calibration.

The theoretical and experimental results show the modulation induced by the reflection



Figure 4. Wave height and mean water level evolution in a submerged vegetation field. Comparison between experimental data (Kobayashi et al., 1991) and numerical model results.

at the front and back face of the vegetation field. This modulation is not present in the theoretical models presented in Dubi and Torum (1994, 1996) since they did not consider the regions offshore and leewards the vegetation field in their first order solution. Comparing with Fig. 4, it can be seen that for irregular waves, the modulation of H_{rms} along the vegetation field is less pronounced than for regular waves. Even if the agreement between theoretical and experimental results is good it has to be pointed out that the artificial seaweed used in Dubi (1995) is characterized by, at least, two degrees of freedom and not only one as assumed by the model presented in this paper. In fact, the kelp plant consists of a stipe, which can be regarded as a slender vertical cylinder with uniformly distributed mass and a frond. Therefore, the obtained C_D represents a depth-averaged value.

The *x*-components of the radiation stress and energy flux are presented in Fig. 6 for a submerged vegetation field. The second order magnitudes are nondimensionalized using the average energy and the energy flux, respectively. For irregular waves and considering a TMA spectrum with the given characteristics, results are shown for two different μ values. The nondimensional friction coefficient μ , is equivalent to the friction coefficient, *f*, for porous media and can be calculated using Lorentz hypothesis of equivalent work, Méndez



Figure 5. Root-mean-square wave height and mean water level evolution in a submerged vegetation field. Comparison between experimental data (Dubi, 1995) and numerical model results.

(1997). As it can be observed, in front of the vegetation field the energy flux is not very much affected by its presence taking an almost identical value for both μ values considered. However, as the waves propagate along the vegetation field the energy flux strongly decays especially for the higher friction coefficient. In the leeward region the energy flux magnitude is constant since it has been assumed this region to be semiinfinite.

The radiation stress shows a similar pattern only varying in front of the vegetation field, where, for $\mu = 0.5$, the higher reflection induces a modulation of S_{xx} . The strong decay in the radiation stress will result in a set-up along the vegetation field.

For oblique incident regular waves, Fig. 7 presents the evolution of the nondimensional radiation stress component S_{xy} along an emerged permeable vertical structure. Different angles of incidence are considered. From the results it can be concluded that S_{xy} decreases towards the lee face of the vertical structure due to the dissipation induced by the structure. Furthermore, increasing oblique incidence results in higher S_{xy} values until a maximum close to 45^o is reached. For higher angle values S_{xy} decreases due to the fact that S_{xy} is proportional to $\sin 2\theta$.



Figure 6. Radiation stress S_{xx} and energy flux evolution in a submerged vegetation field.



Figure 7. Radiation stress S_{xy} evolution in an emerged permeable breakwater.

Conclusions and further applications

In this paper the influence of reflection and dissipation on wave induced mean quantities is analyzed. The obtained results show that any reflective and dissipative natural, such as vegetation fields, or artificial structure, like emerged or submerged breakwaters induce spatial variations of the mean quantities, water level, mass flux, momentum flux and energy flux, among others, similarly to the surf zone. Computed values are compared to experimental results from different sources. Generally speaking, the agreement between theory and experimentation is very good. From these results the following conclusions may be drawn.

- 1. The transfer functions are a very convenient and useful tool for computing easy and efficiently the mean quantities in the vicinity of natural or artificial coastal structures.
- 2. In front of and above the reflective structures considered second order magnitudes are modulated due to the reflection induced by the structure.
- 3. Over the structure mean momentum flux and mean energy flux are attenuated even if no breaking is present. The dissipation rate is dependent upon wave conditions, breakwater geometry and porous material or vegetation characteristics.
- 4. The mean water level variations, due to the radiation stress gradients induced by the dissipation associated with both breaking and friction, show a maximum set-down at the beginning of the structure and a progressive increase along the crest reaching its maximum value at the crest or at the leeside of the structure depending on the reflecting conditions. However, it is noticed that the set-up due to breaking is much more important than the one due to dissipation induced by the porous material.
- 5. The present method can be used to analyze further coastal problems such as: the influence of the mean water level variations on the functionality and stability of breakwaters, the analysis of water table dynamics in shingle beaches, the influence of the presence of dissipative structures on beach profile dynamic models or the evaluation of longshore currents induced by rubble-mound breakwaters, Baquerizo and Losada (1998).

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Appendix

In this Appendix the transfer functions, including evanescent modes, in eqs. (2) to (12) for regular incident waves impinging on a submerged permeable step of height αh are given as an example.

$$H_{\eta}(x) = \begin{cases} \sum_{n=0}^{\infty} [e^{-iq_n x} + R_n e^{iq_n x}] & x \le 0 \\ \sum_{n=0}^{\infty} [A_n e^{-iQ_n x} + B_n e^{iQ_n(x-b)}] & 0 \le x \le b \end{cases}$$
(A.1)

$$H_{\eta}(x) = \begin{cases} \sum_{n=0}^{\infty} iq_n I_n(z) \left[-e^{-iq_n x} + R_n e^{iq_n x} \right] \\ \sum_{n=0}^{\infty} iQ_n M_n(z) \left[-A_n e^{-iQ_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} iQ_n P_n(z) \left[-A_n e^{-iq_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} -iq_n I_n(z) T_n e^{-iq_n (x-b)} \end{cases} \end{cases} \begin{cases} x \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \end{cases}$$
(A.2)

$$H_{v}(x, z) = \begin{cases} \sum_{n=0}^{\infty} -i\lambda I_n(z) \left[e^{-iq_n x} + R_n e^{iq_n x} \right] \\ \sum_{n=0}^{\infty} -i\lambda I_n(z) \left[A_n e^{-iQ_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} -i\lambda I_n(z) \left[A_n e^{-iq_n x} + B_n e^{iQ_n(x-b)} \right] \end{cases} \end{cases} \begin{cases} x \le 0 \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \end{cases}$$
(A.2)

$$H_{w}(x, z) = \begin{cases} \sum_{n=0}^{\infty} -i\lambda I_n(z) \left[A_n e^{-iq_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} -i\lambda I_n(z) T_n e^{-iq_n x} + B_n e^{iQ_n(x-b)} \right] \end{cases} \end{cases} \end{cases} \begin{cases} x \le 0 \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \\ 0 \le x \le b \\ -h \le z \le 0 \end{cases}$$
(A.3)

$$H_{w}(x, z) = \begin{cases} \sum_{n=0}^{\infty} \frac{dI_n(z)}{dz} \left[A_n e^{-iQ_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} \frac{dI_n(z)}{dz} \left[A_n e^{-iQ_n x} + B_n e^{iQ_n(x-b)} \right] \\ \sum_{n=0}^{\infty} \frac{dI_n(z)}{dz} T_n e^{-iq_n(x-b)} \end{cases} \end{cases} \end{cases}$$

where $q_n = \sqrt{k_n^2 - \lambda^2}$, $\lambda = k_o \sin \theta$, θ is the wave incidence angle and k_o the progressive wavenumber in the offshore region. $I_n(z)$, $M_n(z)$ and $P_n(z)$ are depth functions in the different regions defined associated to the *nth* evanescent mode, Losada *et al.* (1996a).

 R_o and T_o are the reflection and transmission coefficients, R_n and T_n are the nondimensional coefficients of the evanescent mode n, and k_n is the eigenvalue (wave number) that satisfies the standard dispersion relationship

$$\sigma^2 = gk_n \tanh k_n h \tag{A.5}$$

where σ is the wave frequency.

 aA_n and aB_n are the complex amplitudes of the waves propagating above the breakwater and $Q_n = \sqrt{K_n^2 - \lambda^2}$. For normal incidence $Q_n = K_n$.

The complex wavenumber K_n can be determined using the complex dispersion equation derived by Losada *et al.* (1996a) for wave propagation over a porous medium,

$$\sigma^2 - gK_n \tanh K_n h = F_n[\sigma^2 \tanh K_n h - gK_n]$$
(A.6)

where

$$F_n = \left(1 - \frac{\varepsilon}{(s - if)}\right) \frac{\tanh K_n \alpha h}{1 - \frac{\varepsilon}{(s - if)} \tanh^2 K_n \alpha h}$$

and f is a linearized friction coefficient that can be obtained using the Lorenz equivalent work hypothesis, Sollitt and Cross (1972). This coefficient depends on the intrinsic characteristics of the permeable material K_{p} , intrinsic permeability, C_f turbulent friction coefficient and porosity ε . The evaluation of f is flow dependent, therefore the problem has to be solved by iterations, Losada *et al.* (1996a).

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