

## A 2D MODEL OF WAVES AND UNDERTOW IN THE SURF ZONE

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### Abstract

*This paper describes the details of a 2D model of waves and undertow in the surf zone. The model uses an assumed shape of the undertow velocity profile together with the wave energy equation, surface and near-bed shear stresses and the mass flux balance due to the wave drift, surface roller and undertow velocity to compute the cross-shore wave development, surface set-up/set-down and the vertical distribution of the undertow velocity at cross-shore sections. The numerical model has been tested against both laboratory and field data and produces good agreements.*

### INTRODUCTION

The wave period-averaged horizontal cross-shore velocity in the surf zone (known as the *undertow*) is of great importance to hydrodynamic and morphodynamic studies in this area, as it controls the on-off shore sediment transport. The undertow structure is characterized by a two-dimensional circulation in the vertical plane, which has an offshore component near the bed. This flow, which is driven by the shear stresses induced by the waves and water surface slope (set-up/down), and also by the roller in the breaker zone, contributes to the flux balance in conjunction with the wave drift and the roller in the breaker zone. Aspects of the undertow flow have been studied by many researchers, such as Stive and Wind (1982) for radiation stress and surface elevation; Svendsen (1984a) for wave height predictions; Svendsen (1984b) for mass flux and the undertow pattern; Deigaard et al (1991), Fredsøe & Deigaard (1992), Deigaard (1993) for the shear stress and turbulence distributions in the surf zone and Cox and Kobayashi (1997) and Dean (1998) for the undertow velocity profile. However, difficulty has been experienced in computing wave heights, shear stresses, water surface slope and undertow velocity

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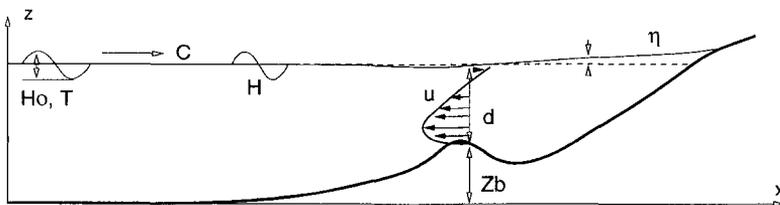
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together using a single simple model. It was also found that the determination of the water surface slope was particularly difficult due to its significant effect on the computation of the undertow velocity profile and the mass flux balance at each cross-shore section.

During project CSTAB (O'Connor, 1996), which was a Liverpool University co-ordinated multi-disciplinary research project involving 10 institutes under the European Community MAST 2 Programme for examining the coastal processes on the Flemish Banks near Middelkerke and the adjacent coastline at Nieuwpoort, a 2D hydrodynamic and sediment transport model was developed to compute the wave decay, undertow flow and sediment transport in the surf zone. The hydrodynamics in the model was based on the solution of the wave energy equation, the force balance due to the shear stresses and the mass flux balance in the water column with an assumed shape of the undertow velocity profile. This paper describes details of the hydrodynamic model as well as the results of model tests against laboratory data (Stive and Wind, 1982; Kraus and Smith, 1994; Dette et al, 1994) and field data (O'Connor, 1996).

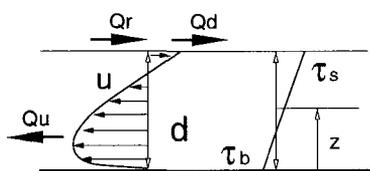
## THEORY AND METHODOLOGY

Figure 1 shows the hydrodynamics associated with a typical beach profile with incident wave characteristics ( $H_0$  and  $T$ ) and initial bed level information ( $Z_b$ ), where  $x =$



**Figure 1** Schematic diagram of cross-shore hydrodynamics

shorewards coordinate;  $z =$  vertical coordinate and  $\eta =$  water surface elevation relative to the mean water level. The computation of the wave height in the cross-shore direction can be carried out using a one-dimensional wave energy equation together with an energy dissipation equation. However, the computed wave height is not correct unless the correct water surface slope is used. Therefore, additional conditions are needed in computing the surface elevation. It was found that by assuming a particular undertow velocity distribution over the flow depth, use could be made of the relations between the undertow



**Figure 2** Undertow velocity and shear stress distributions in the water column

velocity profile, the surface and near-bed shear stresses and the mass flux balance through the water column due to the wave drift, surface roller and undertow velocity to solve the problem (see Figure 2). The following sections describe details of the new approach.

### 1) Wave energy equation

To compute the wave height distribution in the cross-shore direction (see Figure 1), the following one-dimensional wave energy equation (Fredsoe & Deigaard, 1992) was used:

$$\frac{dE_f}{dx} = -\check{D} \quad (1)$$

where  $E_f$  = wave energy flux;  $x$  = cross-shore distance and  $\check{D}$  = energy dissipation rate. In the breaker zone, the energy dissipation rate is calculated based on the energy loss in a hydraulic bore, which can be expressed as follows:

$$\check{D} = \frac{1}{T} \rho g d \left( \frac{H^3}{4d^2 - H^2} \right) \quad (2)$$

where  $d$  = water depth;  $\rho$  = water density;  $g$  = acceleration of the gravity;  $H$  = wave height;  $T$  = wave period. For non-breaking waves, the wave energy dissipation due to the bed shear was taken into consideration. However the energy dissipation for non-breaking waves is rather small by comparison with the conditions produced by breaking waves.

### 2) Surface and near-bed shear stresses

The wave period-averaged shear stresses at the water surface and near the sea bed in the surf zone contain contributions from the wave motion, the surface roller and the water surface slope. In the present model, expressions for the surface and near-bed shear stresses given by Deigaard (1993) were used:

$$\frac{\tau_s}{\rho} = -\frac{g}{8C} \frac{\partial(H^2C)}{\partial x} - \frac{\partial}{\partial x} \left[ \frac{AC}{T} \right] \quad (3)$$

$$\frac{\tau_b}{\rho} = -\frac{3g}{16} \frac{\partial(H^2)}{\partial x} - \frac{\partial}{\partial x} \left[ \frac{AC}{T} \right] - gds \quad (4)$$

where  $\tau_s$  = wave period-averaged shear stress at the water surface;  $\tau_b$  = wave period-averaged shear stress near the sea bed;  $s$  = wave period-averaged surface slope;  $A$  = roller area given by  $A=0.9H^2$ .

### 3) Mass fluxes

In the surf zone, mass transport occurs due to the wave drift, streaming and the surface roller if the waves are broken. By neglecting the flux due to the streaming, the fluxes due to wave drift and the surface roller can be expressed respectively as follows (Fredsoe & Deigaard, 1992):

$$Q_d = \frac{\pi H^2}{4T} \frac{1}{\tanh(kd)} \quad (5)$$

$$Q_r = \frac{A}{T} \quad (6)$$

where  $k$  = wave number.

#### 4) The vertical distribution of undertow velocity

Theoretical and experimental results suggest that the undertow velocity profile in the vertical direction can be described by the following equation:

$$u = a \left( \frac{z-z_0}{d} \right)^2 + b \ln \left( \frac{z}{z_0} \right) \quad (7)$$

where  $u$  = horizontal undertow velocity;  $z$  = vertical coordinate measured upwards from the bed;  $z_0$  = height where the velocity is zero;  $a$  and  $b$  are arbitrary constants. It can be seen that the undertow velocity consists of two parts: a parabolic distribution representing the upper part of the flow and a logarithmic distribution representing the lower part of the flow. The assumed undertow profile was also confirmed by recent laboratory work of Cox and Kobayashi (1997).

By relating the assumed undertow velocity profile (Eq. 7) gradient to the surface and near-bed shear stresses and balancing the mass flux due to undertow with the mass fluxes due to the wave drift and the surface roller, the following expressions can be obtained:

$$\tau_s / \rho = v_s \frac{\partial u}{\partial z} \Big|_{z=d} \Rightarrow f_1(a,b,s)=0 \quad (8)$$

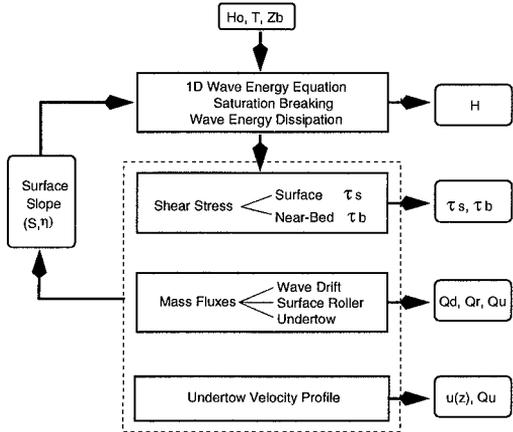
$$\tau_b / \rho = v_b \frac{\partial u}{\partial z} \Big|_{z=z_0} \Rightarrow f_2(a,b,s)=0 \quad (9)$$

$$Q_u = \int_{z_0}^d u dz = Q_d + Q_r \Rightarrow f_3(a,b,s)=0 \quad (10)$$

Solving Equations 8, 9 and 10 gives the constants  $a$  and  $b$  and the water surface slope  $s$ . The mean water depth determined by the surface slope is then returned to the wave energy equation (Eq. 1) to give a better estimate of the wave height. The whole process is then repeated until a converged water surface is obtained. The flow diagram of the model is shown in Figure 3.

**NUMERICAL MODELLING**

A finite difference method was used to solve wave energy equation (Eq. 1). The wave height was computed at each node point in the cross-shore direction, while the undertow velocity profile was computed mid-way between the node points (in the cell). Saturation wave breaking criteria were used in the computation without considering the wave breaking transition length. In order to increase the resolution of the undertow velocity near the bed, a non-uniform vertical grid system was adopted with a finer grid size near the bed and a coarser grid size near the water surface.



**Figure 3** Flow diagram of the model

The eddy viscosities at the water surface and near the sea bed, which are needed in Eq. 8 and Eq. 9 were computed by a mixing length approach so that:

$$v = l_{mix} u_* \tag{11}$$

where,  $l_{mix}$  = mixing length;  $u_* = \sqrt{\tau/\rho}$ . The mixing length increases linearly from zero at the bed up a maximum value of  $0.19d$  at a particular level and remains constant at the higher level (Fredsoe & Deigaard, 1992).

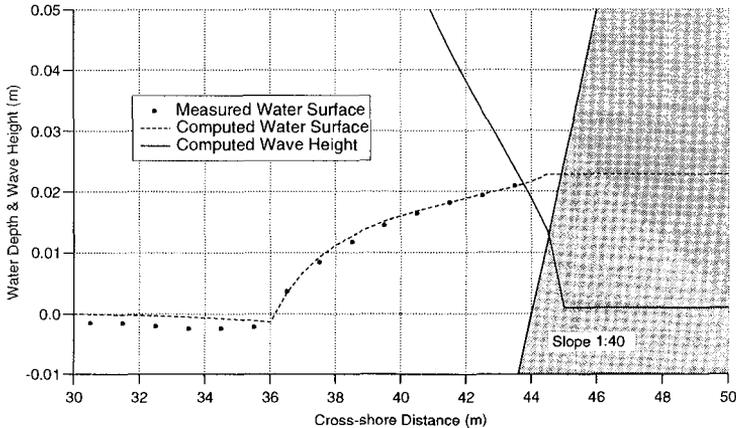
**RESULTS**

The model was tested against prototype-scale laboratory and field data for the water surface set-up/down, wave height and undertow velocity. The test results involved laboratory data collected by Stive and Wind (1982), Kraus and Smith (1994) in Supertank and Dette et al (1992) in the Large Wave Flume at Hannover University, as well as field data measured at Nieuwpoort beach on Belgian coast (O'Connor, 1996). It should be noted that the laboratory test cases involved monochromatic waves with normal incidence.

**1) Stive and Wind (1982) data**

The experimental data used in the model were obtained from tests conducted in a wave flume at the Delft Hydraulics Laboratory. The wave flume was 55 m long, 1 m wide and 1 m deep. The bed slope was 1:40 and the water surface elevation was measured by conductivity-type wave gauges. The wave height was 0.159 m with 1.79 s period and the offshore water depth was 0.7 m.

Figure 4 shows a comparison of the computed and measured surface elevations with an exaggerated scale of the water depth for the purpose of clarity. The model results agree well with measurements and the breaking point was also predicted satisfactorily.



**Figure 4** Computed and measured surface elevation for Stive and Wind (1982) data

## 2) Supertank data (Kraus and Smith, 1994)

The Supertank Data Collection Project (Kraus and Smith, 1994) was conducted in a large wave flume 104 m long, 3.7 m wide and 4.6 m deep at Oregon State University. Among 20 major data collection runs, ST\_G0 was a case with monochromatic waves (normal incidence). The incident wave height was 0.8 m, the wave period was 3.0 s and offshore water depth was 4.0 m.

The numerical model was run with 146 grid points and a step size of 0.61 m (2 feet), which covered a total length of 91.38 m (300 feet). Five data sets measured at various stages of this run were used to validate the model, two of which are presented in this paper; the other data sets are in similar accuracy to those presented herein.

Figures 5 and 6 show comparisons between computed and measured wave heights and undertow velocities respectively for Case S0414a. In this case, a nearshore bar was in an early stage of its development. The results given in Figure 5 show that the model produces good agreement for the wave height; the breaking point is also well predicted. However, only few measurement points were available for use in the comparison of the undertow velocity. Figure 6 shows general agreement of the computed undertow with the

measurements, but discrepancies can be seen, particularly in the areas near the bar and break point.

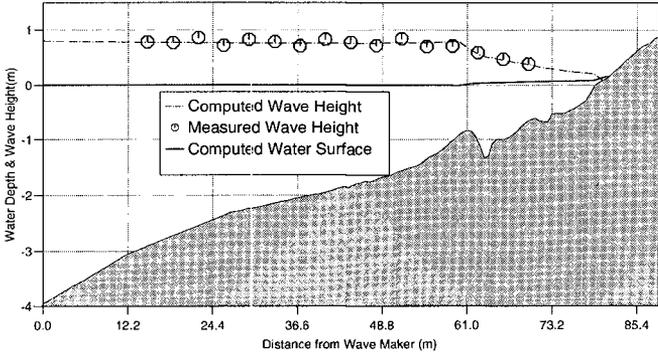


Figure 5 Computed and measured wave heights for Supertank data (Case S0414a)

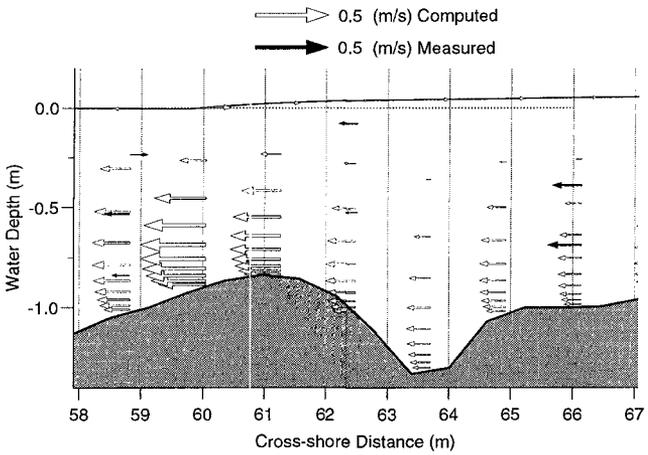


Figure 6 Computed and measured undertow velocities for Supertank data (Case S0414a)

Figures 7 and 8 show comparisons between the computed and measured wave heights and undertow velocities respectively for Case S0418a. In this case, the bar was further developed compared with Case S0414a. Again, the computed wave heights shown in Figure 7 are in good agreement with the measurements and the position of the break point

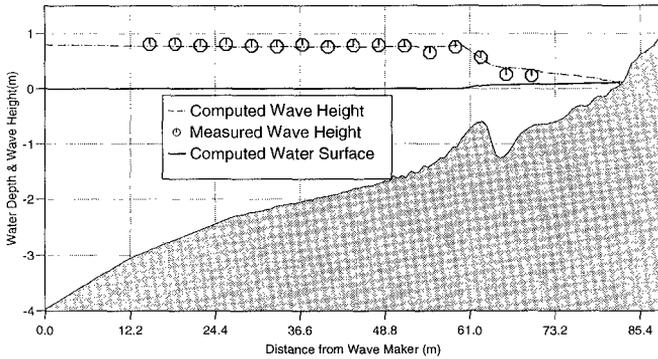


Figure 7 Computed and measured wave heights for Supertank data (Case S0418a)

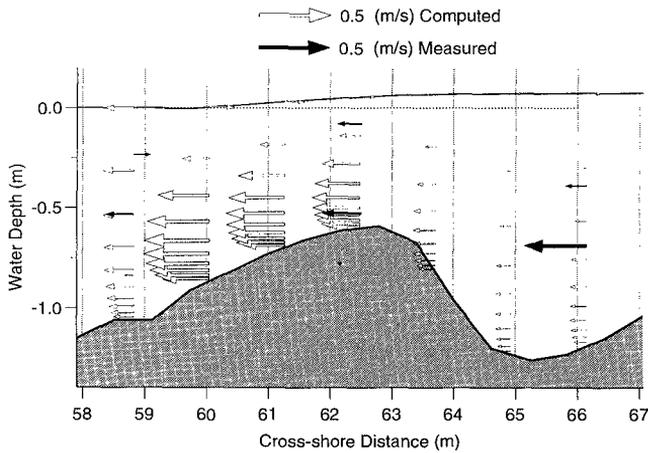


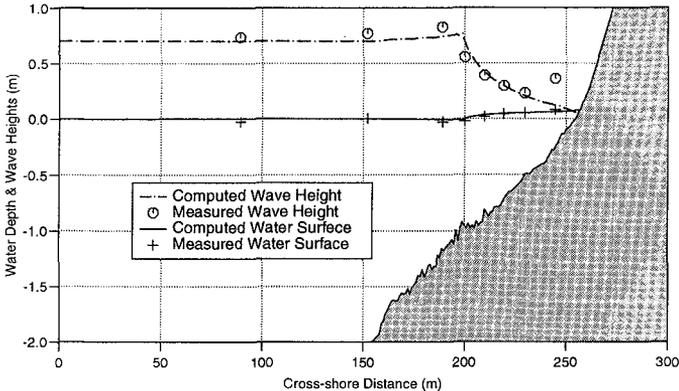
Figure 8 Computed and measured undertow velocities for Supertank data (Case S0418a)

was predicted well. Similarly to the previously mentioned case, the undertow velocity computed by the model shown in Figure 8 agrees reasonably with the experimental data. Discrepancies between the computed and measured undertow velocities for both cases landwards of the bar are believed to be due to a vortex generated by plunging breakers. The present model is not capable of predicting undertow velocities in such detail. In Figures 6 and 8, the computed undertow is found to be in the opposite direction from that of the measurements at the measuring points near the surface offshore of the bar. This may be due to these measuring points being partially exposed to the air within the wave period leading to spurious wave period-averaged velocity measurements.

### 3) Large Wave Flume data (Dette et al, 1992)

Prototype-scale experiments were carried out in the Large Wave Flume in Hannover University in 1990 (Test series I) and 1991 (Test series II) (Dette et al, 1992) in a flume 320 m long, 5 m wide and 7m deep. One case (16109001) in Test Series I (1990) with monochromatic waves was used in the model test. The incident wave height was 0.7 m with a 4.0 s wave period. The offshore water depth was 2.0 m. The computations were carried out with 1 m grid size.

Figure 9 shows comparisons of the computed and measured wave heights and surface elevations. Good agreement is obtained for the surface elevation and the wave break point is also well predicted. The computed wave heights also agree well with the measured results, except the wave height at the point nearest to the shore. The reason for this discrepancy is unclear, but it could be due to downwash effects. It may also be due to long wave effects as described in recent investigations by Kamphuis (1998).

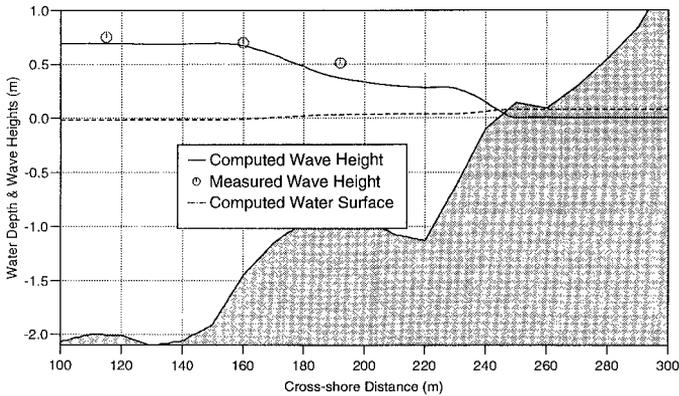


**Figure 9** Computed and measured wave heights and surface elevations for Large Wave Flume data (16109001)

#### 4) Field data - Nieuwpoort Beach (O'Connor, 1996)

Field data had been obtained during the CSTAB Project which took place on Nieuwpoort Beach, off the Belgian coast (O'Connor, 1996). As the present model is limited to dealing with monochromatic waves, the root-mean square value of wave height was used as a first approximation to the random waves measured on site. The following wave parameters were used in the model test for the field data: root-mean square wave height was 0.7 m, mean wave period was 4.0 s and offshore wave depth was 4.33 m. The computation was carried out with 1.0 m grid size.

Figure 10 shows a comparison of computed and measured wave heights. The results once again demonstrate that the model predicts the wave height satisfactorily. Realistic undertow velocities have also been obtained, but are not shown here, see O'Connor (1996).



**Figure 10** Computed and measured wave heights at Nieuwpoort beach

## CONCLUSIONS

A 2D wave and current model has been developed to compute the wave decay, water surface slope and the undertow velocity distribution in the surf zone. Results produced by the model show good agreement as regards both wave height and water surface elevation against the laboratory and field data. The undertow flow structure was also reasonably reproduced by the model. This model provides a simple and practical method for engineers to predict the wave characteristics and hydrodynamics in the surf zone.

## ACKNOWLEDGEMENTS

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