Trapped and Free-Wave Propagation in Channels and Harbours

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Abstract

Most of the literature on edge waves is only concerned with edge-wave propagation over a bottom that is uniform in the propagation direction. There is still little knowledge of what happens when the bottom topography changes suddenly, as in the presence of a cape, submarine valley or harbour.

This paper presents some results of our study on the wave motion in such places, where edge waves may be transmitted, reflected or radiated as free waves and thus may have an important role in determining the wave conditions in otherwise sheltered regions.

We send an edge wave along a coast with an alongshore uniform bottom profile and, at a given section change suddenly for another bottom profile. This study is carried out with a very simple model. The linearised shallow water equations are used in a wide channel with the simplest bottom profile that enables the existence of edge waves: a ledge close to the coast and a uniform depth offshore.

Introduction

Edge waves are common in the spectrum of waves at the surf zone, Huntley et. al. (1981) showed that these waves could be responsible for up to 30 % of the energy in the longshore current spectrum measured at Torrey Pines beach, California, and are believed to be responsible by the formation of beach cuspate patterns, (Guza and Inman 1975), and so they can play an important role in coastal morphology.

However, most of the references in the literature on this subject are only concerned with edge-wave propagation over bottom configurations that are uniform

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alongshore. To our knowledge the only exceptions are Smith (1974), where an asymptotic technique is used study the propagation of high frequency edge waves along coasts whose bottom geometry varies gradually, and Evans & Fernyhough (1995), where edge waves are found for a vertical coast with periodic bays at the boundary of a flat ocean.

When the longshore uniformity in the bottom geometry that enables the occurrence of edge waves disappears suddenly, due to the presence of a cape, a submarine valley or a harbour, this obstacle may cause the reflection of that energy, its transmission to the region after the obstacle, if edge waves can propagate there, or its release as free long waves. This is the objective of our study: to understand what happens when such discontinuities appear. We send an edge wave along a coast with an alongshore uniform bottom profile and, at a given section change suddenly for another bottom profile.

We solve these problems by using a very simple model: we have a wide channel, instead of the semi-infinite ocean, and use the simplest bottom profile that enables the occurrence of edge waves, a ledge close to one of the channel walls. The linearised shallow water equations are employed to evaluate the modes (trapped, freely-propagating and evanescent) in each of the regions into which the domain under study can be divided.

In spite of its simplicity the results produced by this model allow us to draw some conclusions on the energy distribution between trapped and free waves when those longshore discontinuities occur and to explain some difficulties in using physical models to model long waves reported by Dodd & Bowers (1995).

Methodology

To study the influence of a sudden variation of the longshore bottom profile on edge wave propagation, we send an edge wave along a straight coast formed by two or more stretches with uniform longshore profile. At a given section, where two stretches meet, a sudden change in the longitudinal bottom profile occurs.

Although the number of edge-wave modes for a given frequency and bottom geometry is limited, the number of the freely-propagating and evanescent modes that can be excited by such a discontinuity is not. For both there is a continuous spectrum in frequency. So, to make analysis easier we use a wide channel instead of the semi-infinite ocean, as shown in figure 1. This gives a discrete spectrum but implies that we must choose a sufficiently wide channel and, as we will show later, special care must be used when interpreting the results so obtained.

Many cross-shore bottom profiles support the existence of edge wave modes. We choose the simplest such bottom profile: a ledge close to the coast. This profile, unlike the plane beach profile of Eckart (1951) or Ursell (1952), makes the establishment of the dispersion relation a straightforward task. In addition to that, it is always possible, by adjusting the ledge dimensions, to get an edge wave mode in this geometry with similar behaviour to a similar mode propagating over other bottom profiles.

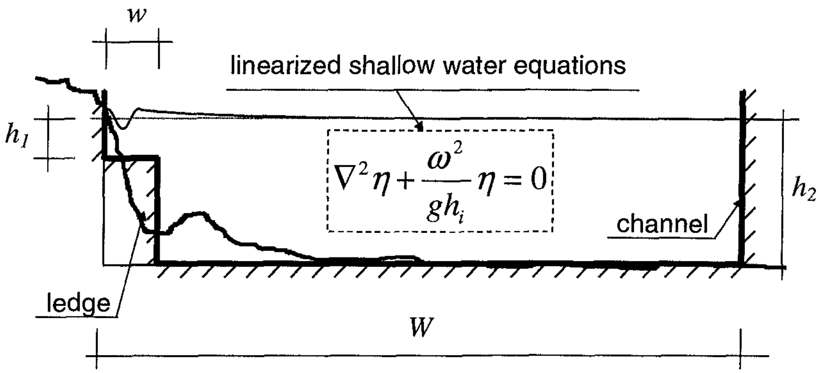


Figure 1. Scheme of an edge wave with the simplifications adopted in this study

Because the region we are interested in is quite close to the shoreline and the waves long enough to have negligible vertical variation of the horizontal velocity, we use the linearised shallow water equations. Assuming a time harmonic variation of the free--surface elevation, η, gives the following equation:

$$\nabla^2\eta + \frac{\omega^2}{gh}\eta = 0 \qquad (1)$$

where h is the water depth, ω the angular frequency of the wave and g the acceleration of gravity. Using this equation the dispersion relations for both the propagating (either trapped or free) and the evanescent modes in the channel with the ledge, figure 2, are evaluated.

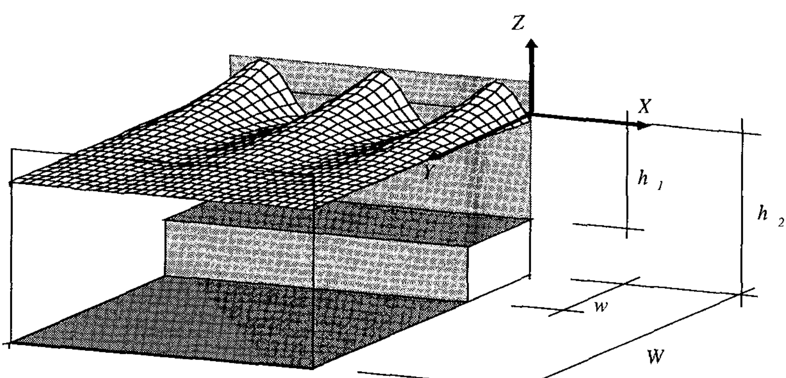


Figure 2. Channel with ledge

We scale the space quantities by the maximum channel depth, h_2, and the time quantities by $\sqrt{h_2 / g}$ and the following dispersion relation is obtained:

$$\gamma\sqrt{\frac{\Omega^2}{\gamma} - K^2}\tan\left(\alpha\sqrt{\frac{\Omega^2}{\gamma} - K^2}\right) = \sqrt{\Omega^2 - K^2}\tan\left[(\alpha - \beta)\sqrt{\Omega^2 - K^2}\right] \qquad (2)$$

where $\alpha = w/h_2$ is the ledge width ratio, $\beta = W/h_2$ is the channel width ratio, γ is the depth ratio, $\Omega = \omega\sqrt{h_2/g}$ is the dimensionless frequency (can be interpreted also as the ratio between the total water depth and the free-wave wavelength) and $K = kh_2$ is the dimensionless longshore wavenumber (or the ratio between the total water depth and the longshore wavelength). See figure 2 for definitions of depths and widths.

Each mode is defined by the number of zero-crossings in the y-variation of its free-surface profile. For a given frequency, the character of the modes produced by equation (2) depends on their dimensionless longshore wavenumbers, K. For $K^2 < 0$ we have evanescent modes or, being more precise, we have non-propagating modes with an alongshore exponential variation of the free-surface elevation. We chose the sign of that variation such that it decays exponentially inside our domain. For $0 \le K^2 \le \Omega^2$ we have freely propagating modes and for $\Omega^2 \le K^2 \le \Omega^2 / \gamma$ we have trapped or edge-wave modes.

If the ledge occupies the whole width of the channel then $\alpha = \beta$, $\gamma = 1$ and equation (2) gives the dispersion relation for a flat channel. In this case no trapped modes are possible in the channel.

In what follows, we are going to designate by "leaky modes" the free-propagating modes in the channel with ledge. Although this does not make sense for a ledge along a channel wall, since no energy can escape the channel walls, these freely propagating modes do constitute a sample (albeit not a evenly spaced one) of the leaky modes continuum that would be obtained for the same ledge at the coast of a semi-infinite ocean.

For a given mode, as the dimensionless frequency increases, its character changes from evanescent to freely-propagating and then to trapped. In the channel with ledge this change in character implies the concentration over the ledge of the zero crossings in that y-variation. So, the sinusoidal variation over both the ledge and the deep part of the channel, when the mode is evanescent, changes to sinusoidal both across and along the channel, for the "leaky" modes, and ends up as a sinusoidal variation over the ledge with an exponential decay away for the ledge in the deep part of the channel, when the mode is trapped.

The transition between the "leaky" and the trapped character occurs when $K = \Omega$, or when the dimensionless frequency is such that

$$\Omega = n\pi\sqrt{\gamma}/\left(\alpha\sqrt{1-\gamma}\right) \qquad (3)$$

where n is the mode number. Since this expression is independent of the channel width it is also valid for a ledge along the cost of a semi-infinite ocean. In fact, when a mode becomes trapped the y-variation of the free-surface profile after the ledge is horizontal, this can be seen at Ω = 5.236 in figure 3 which shows how mode 2 changes with frequency. In this case we have a ledge with $\alpha = 0.4$ and $\gamma = 0.1$, in a channel with $\alpha / \beta =$ 0.05, but it is clear that, for $\Omega = 5.236$, the shape of this mode, and the associated value of K, does not change when other channel widths, or even a semi-infinite ocean, are considered.

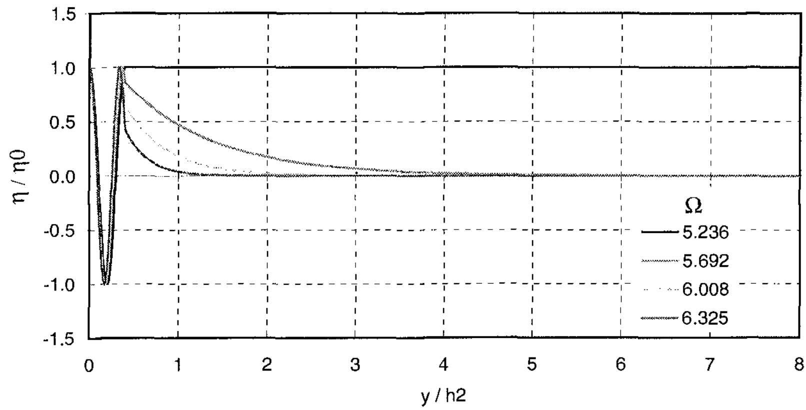


Figure 3. Free-surface elevation for mode 2 trapped waves in channel with ledge. $\alpha = 0.4$, $\alpha / \beta = 0.05$ and $\gamma = 0.1$.

The figure also indicates that for larger values of Ω the same insensitivity to channel width is likely. In this case the free surface after the ledge approaches the undisturbed mean water level and the y-coordinate of this coincidence varies with Ω.

The superposition of all the modes in each of the uniform regions into which we can divide our domain and the enforcement of the continuity of both the free-surface elevation and of the mass flux across the boundaries between those regions allow us to define a set of equations whose solution provides the amplitudes for all the modes considered. This mode-matching technique is used to get all the results presented here.

The number of modes in each region into which the domain of the problem was divided was always 200. Since this is the total number of modes (trapped + "leaky" + evanescent, in channel with ledge, or freely-propagating + evanescent, in flat channel) and the number of propagating modes increases with the frequency, this means that high frequencies have less evanescent modes than low frequencies. However, in the frequency range considered, the number of evanescent modes was always larger than 100.

The Beginning of the Ledge

We started by studying the excitation of trapped waves by external incident waves at the beginning of the ledge. Our domain is made of a flat channel, for $x > 0$, and a channel with ledge, for $x < 0$. A flat channel mode comes from $+\infty$ and meets the beginning of the ledge at $x = 0$.

Since the incident energy is uniformly spread across the channel and the ledge occupies only a small fraction of the channel width, one would expect no significant influence of the ledge on the wave propagation in this problem, being most of the energy transmitted to $-\infty$ in the "leaky" modes of the channel with ledge.

However, figure 4 shows that this is not true when the incident wave is a zero "flat channel" mode. In that figure the energy transmitted in the trapped modes of a channel with ledge, $\alpha = 0.4$, $\gamma = 0.1$ and $\alpha / \beta = 0.05$ (the same used in figure 3), is plotted for several values of the dimensionless parameter $\Omega\,\alpha$ and for three different "flat channel" modes meeting the beginning of the ledge: zero, five and ten. The curve for mode zero has jumps at some frequencies and the transmitted energy in trapped modes reaches, at those frequencies, quite high values. In fact, the important property is how close the "channel with ledge" modes are to the incident "flat channel" modes.

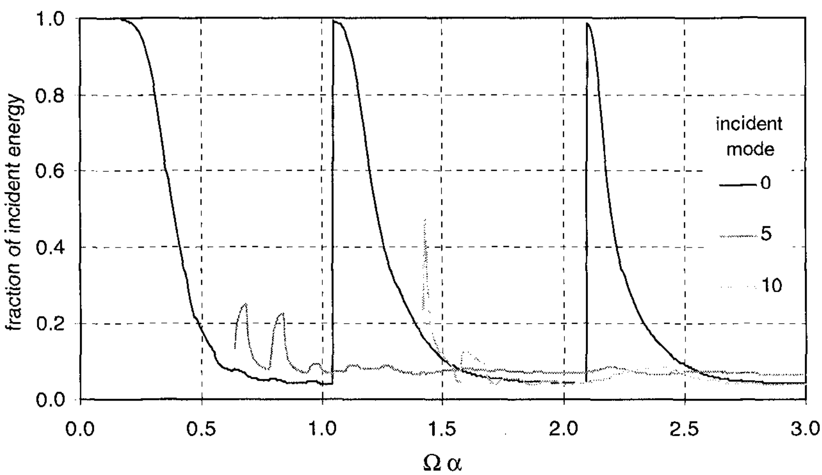


Figure 4. Total energy transmitted in trapped modes for different incident modes meeting the beginning of ledge in channel. $\alpha = 0.4$, $\alpha / \beta = 0.05$ and $\gamma = 0.1$.

We saw before, figure 3, that whenever a new trapped mode appears, the free surface elevation in the deep part of the channel is horizontal. For this frequency the free-surface elevation of the trapped mode almost coincides with the zero-mode incident wave over the deep part of the channel and so most of the incident energy is transmitted in it towards $-\infty$. Then a jump appears in the associated curve for the transmitted energy in trapped modes.

Figure 3 shows also that, as frequency increases, the slope of the free-surface elevation in the deep part of the channel becomes more pronounced and the disturbance in the free-surface elevation narrows down to the region close to the ledge. Then the free-

-surface elevation of this mode is quite different from the zero-mode incident wave and the amount of energy transmitted in trapped modes decreases.

If, instead of trapped modes only, we look to all of the propagating modes in the "channel with ledge" domain, we observe a more gradual variation in the transmitted energy, figure 5. In fact, the sudden increase in the energy in trapped modes happens because the "leaky" mode that was carrying most energy becomes a trapped mode at the frequency where the jump occurs. As the frequency increases the energy in this trapped mode goes to zero (the only exception occurs for the zero mode) and the energy carried in the next leaky mode increases. Another interesting feature in figure 5 is the absence of energy transported in the other leaky modes when a leaky mode becomes trapped.

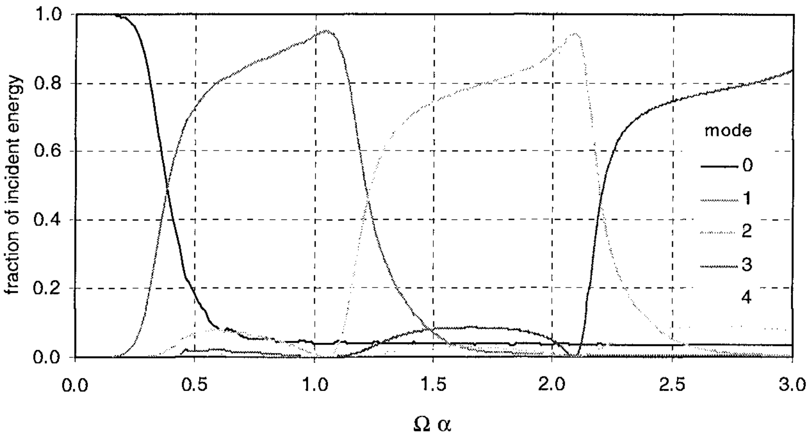


Figure 5. Distribution of transmitted energy by several modes when zero-mode incident wave meets the beginning of ledge in channel. $\alpha = 0.4$, $\alpha / \beta = 0.05$ and $\gamma = 0.1$.

A similar behaviour is observed when the incident wave is a flat channel mode different from zero. In this case the energy is transmitted mainly in "leaky" modes, because no trapped mode has a free-surface elevation that oscillates in the deep part of the channel. Assuming that the ledge does not occupy a significant fraction of the channel width, the mode that carries most energy is the one that has the same number of zero-crossings in the deep part of the channel as the incident mode. When the frequency increases, the zero-crossings of any given mode tend to concentrate over the ledge. This phenomenon makes the "leaky" mode that was carrying most energy increasingly different from the incident wave and the next "leaky" mode more similar to it. Then, even before becoming a trapped mode, the "leaky" mode that was carrying most energy loses it to the next leaky mode.

The End of the Ledge

We now look at what happens when a trapped wave meets the abrupt end of the cross-shore depth variation that enabled its existence. This problem is also used to illustrate the influence of the channel width on the results obtained.

We use the same geometry, i.e. a channel with ledge for $x < 0$ and no ledge for $x > 0$, to study what happens when a zero-mode trapped wave coming from $-\infty$ meets a discontinuity in the ledge at $x = 0$.

Plotting the total energy reflected back in trapped modes for several values of the parameter $\Omega\,\alpha$ we get figure 6. In all the curves there, each corresponding to a given value of the depth ratio, γ, three regions with distinct behaviour can always be identified: a region in the low frequency range where most of the incident energy is transmitted to the flat channel; a region in the high frequency range where the reflected energy in trapped modes tends to the value that would be obtained in a channel where the ledge occupies the whole channel width, this means that at $x = 0$ we would have a depth discontinuity of γ; and an intermediate region of high oscillations.

In the low frequency range the free surface elevation of the zero mode trapped wave, the incident wave, is almost horizontal and the phase velocity of this wave is quite close to the phase velocity of the zero mode in the "flat channel" part of the domain. Then all the energy can be transmitted to the region after the end of the ledge.

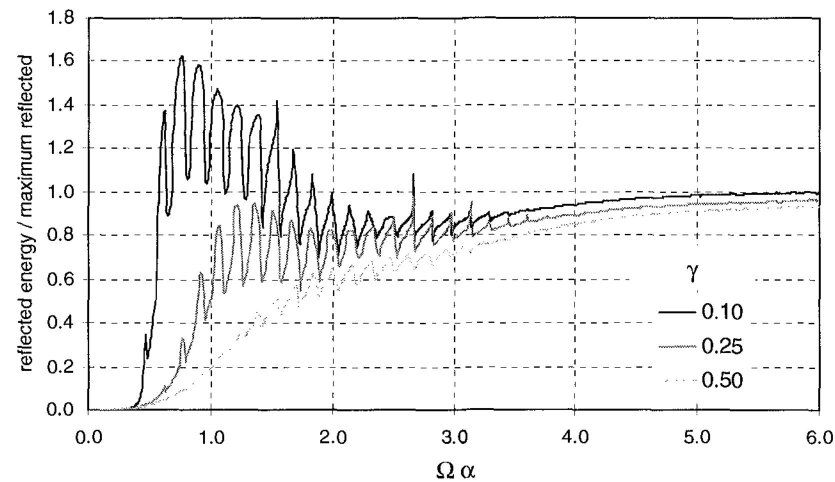


Figure 6. Influence of ledge depth ratio on reflected energy carried in trapped modes when zero-mode incident wave meets the end of ledge in channel. $\alpha = 0.4$, $\alpha / \beta = 0.05$.

In the high frequency range the free-surface elevation of the incident wave is concentrated over the ledge, being its phase velocity similar to the phase velocity of a zero mode wave in a flat channel whose the water depth equals the water depth over the ledge. Since most of the energy in the "flat channel" part of the domain is transmitted in the zero mode, when the incident wave meets the end of the ledge this is similar to a free zero mode in a flat channel meeting a depth discontinuity γ in such a channel. So, in the high frequency range, the amount of reflected energy in trapped modes depends on γ only.

In the intermediate frequency range, where the response varies rapidly with frequency, the oscillations in the curves are found to be dependent on the channel width. This dependence is illustrated figure 7 where the reflected energy in trapped modes is presented for two different values of channel width, $\alpha / \beta = 0.05$ and $\alpha / \beta = 0.1$, for a channel with ledge with $\alpha = 0.4$ and $\gamma = 0.1$. When the channel width increases, for the same frequency range, more "leaky" modes appear in the channel and so more oscillations in the reflected energy curve can be observed. Actually whenever a "leaky" mode appears a sudden increase in that curve occurs while a new mode in the "flat channel" part of the domain is responsible for a sudden decrease in that curve. The reduction in the amplitude of those oscillations can be explained by wave radiation from the end of the ledge: the farther the channel wall the lower the amplitude of the radiated waves when reaching the far channel wall and so the smaller their influence on the reflected energy.

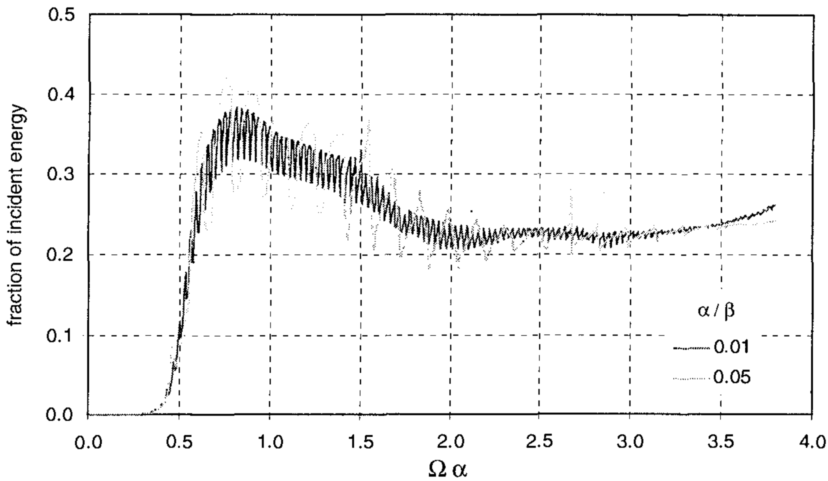


Figure 7. Influence of channel width on reflected energy carried in trapped modes when zero-mode incident wave meets the end of ledge in channel. $\alpha = 0.4$, $\gamma = 0.1$.

This strong dependence of the results on the channel width, in this frequency range, is not the best result for our initial assumption that a semi-infinite ocean could be replaced by a wide channel.

However, this does show how careful one must be when interpreting results from scale model tests where long waves are to be produced. Since long waves are difficult to attenuate at the boundaries of the model, results from this kind of model may have a strong variability with the frequency of the incident wave.

In addition to that, this geometry is also appropriate for modelling wave propagation in narrow or enclosed seas.

A Square Bay

The last geometry presented is an interruption in the ledge produced by a square bay that opens perpendicularly to the coast and whose bottom is at the same level of the deep part of the channel. The channel has a ledge along the whole length except in the bay region.

We investigate what happens when a zero-mode incident trapped wave coming from $-\infty$ meets that discontinuity. It is expected that part of the energy shall be reflected back, part shall be radiated through the gap, part shall be transmitted to the region after the gap and that waves are excited in the bay.

The behaviour of the whole system is controlled by the ratio between the bay dimensions (width, a, and length, b) and the wavelength of the zero mode in the "flat channel" part of the domain since resonance may occur in the bay. So, the relevant non--dimensional parameters are $\Omega\mu$ and $\Omega\nu$, where μ is the non-dimensional bay width, $\mu = a / h_2$ and ν is the non-dimensional bay length, $\nu = b / h_2$.

Of course the dimensionless parameter $\Omega\,\alpha$ also plays a role in this behaviour because it controls the energy spreading across the channel and this does influence energy transmission across the interruption in the ledge.

This was clearly seen in one special case of the bay geometry we studied: a gap in the ledge (the bay length, b, is nil). We looked at the transmitted energy for several values of the gap width, μ, for two values of the parameter $\Omega\,\alpha$, 5.06 and 0.63.

For $\Omega\,\alpha = 5.06$ the incident energy is concentrated over the ledge and the curve with the transmitted energy in trapped modes versus the gap width oscillates as if the ledge occupied the whole channel width. The amplitude of those oscillations decreases slightly as the gap width increases.

Figure 8 shows the contour lines of the free surface elevation for $\Omega\,\alpha = 5.06$ when the gap width equals 4.5 times the wavelength of the zero mode in the "flat channel" part of the domain. There we can see the oscillating mode excited inside the gap region and the energy being radiated from the end of the ledge. Some of this energy misses the ledge after the gap and is transferred into "leaky" modes at that part of the domain.

Something different happens for $\Omega\mu = 0.63$. Then the incident energy is not concentrated over the ledge and the amount of energy transmitted to the region after the gap decreases sharply with the gap width. It is difficult to identify any pattern in the oscillations of that curve.

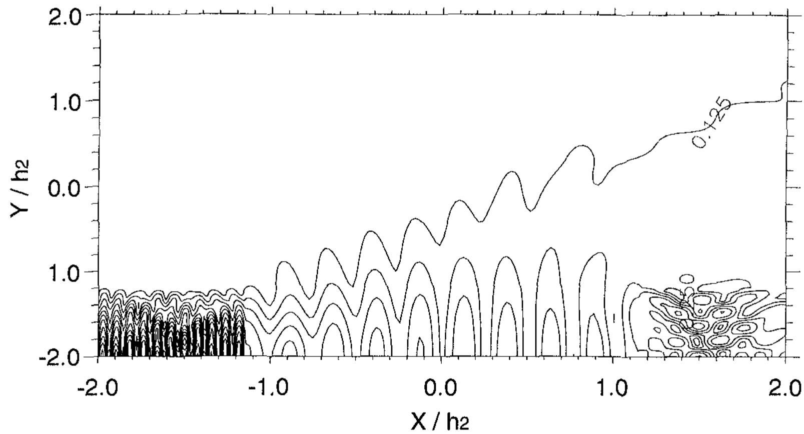


Figure 8. Amplitude of free surface elevation when zero-mode incident wave, with $\Omega\ \alpha = 5.06$, in channel with ledge, $\alpha = 0.4$, $\gamma = 0.1$ and $\alpha / \beta = 0.05$, meets gap in the ledge, $\mu = 2.28$.

For the square bay we keep the dimensions of the domain constant, $\alpha = 0.4$, $\gamma = 0.1$, $\alpha / \beta = 0.05$, $\mu = \nu = 1.0$, and vary the dimensionless frequency of the incident wave, Ω, such that the ratio between the bay width and wavelength of the zero mode in the "flat channel" part of the domain varied between zero and 1.2.

This variation of the incident wave frequency implies that $\Omega\ \alpha$ varies with the dimensionless parameter $\Omega\ \mu$ and so does the concentration over the ledge of the incident energy. The evolution, with the dimensionless parameter $\Omega\ \mu$, of the fraction of the incident energy transmitted in all the modes as well as the fraction transmitted in trapped modes is not smooth. There are many jumps and depressions related to the appearance of the several trapped and freely-propagating modes, as the frequency increases. That is why we focus only at the evolution, with $\Omega\ \mu$, of the free-surface elevation amplitude at three points at the far end of the bay:

- left : at the bay corner close to the end of the ledge, $x / h_2 = -0.5$ and $y / h_2 = -1.0$;
- center: at the middle at bay far end, $x / h_2 = 0.0$ and $y / h_2 = -1.0$;
- right: at the bay corner close to the beginning of the ledge, $x / h_2 = 0.5$ and $y / h_2 = -1.0$.

Figure 9, shows the amplitude of the free surface elevation at those points, with jumps and depressions due to the appearance of new modes in the channel. However, they cannot be mistaken for the sharp peaks on the same curves around $\Omega\ \mu = \pi$ and $\Omega\ \mu = 2\pi$ that are related to the occurrence of resonance at the bay.

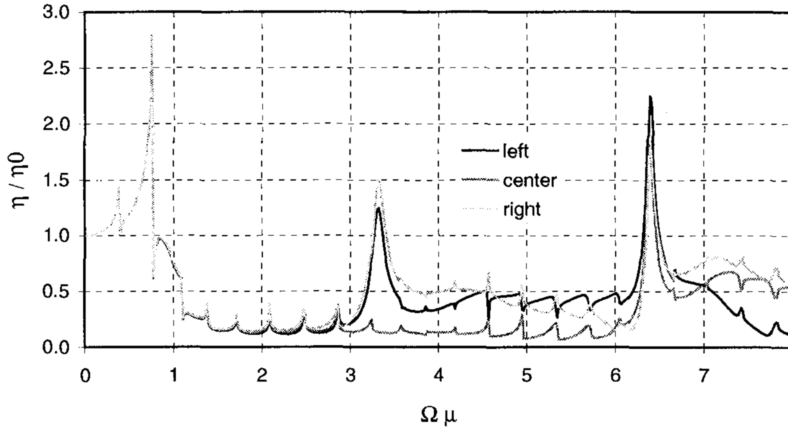
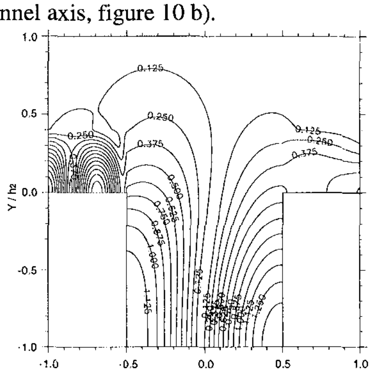


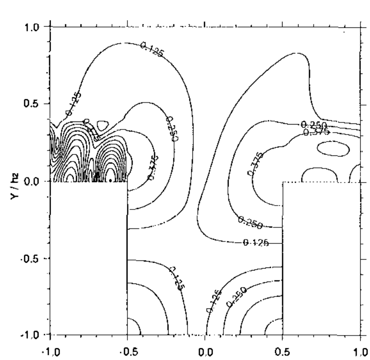
Figure 9. Channel with ledge, $\alpha = 0.4$, $\gamma = 0.1$ and $\alpha / \beta = 0.05$, that is interrupted by square bay, $\mu = \nu = 1.0$. Amplitude of free surface elevation inside bay for zero-mode incident wave.

The oscillation modes excited at those frequencies have nodal lines perpendicular to the channel boundary and is shown in figure 10 a). These peaks do not coincide with an integer number of half wavelengths across the bay width (the natural frequencies of the bay) but are slightly above it.

Next to each of those sharp peaks, at a higher frequency, a broad peak can always be noticed in figure 9. It corresponds to the excitation of an oscillation mode that has nodal lines parallel to the channel axis, in addition to the nodal lines perpendicular to the channel axis, figure 10 b).



a)



b)

Figure 10. Amplitude of free surface elevation when zero-mode incident wave in channel with ledge, $\alpha = 0.4$, $\gamma = 0.1$ and $\alpha / \beta = 0.05$, meets gap in the ledge created by square bay, $\mu = \nu = 1.0$. a) $\Omega\,\mu = 3.320$ and b) $\Omega\,\mu = 4.206$

All these peaks imply an oscillation that involves the bay only. However, since the domain considered here is not semi-infinite but a wide channel, some of the energy radiated from the bay entrance is reflected at the far channel wall and may contribute to the build-up of the oscillation at the region in front of the bay. This could be the cause for the oscillation involving the whole channel around the bay area, shown in figure 11, associated to the sharp peak at $\Omega\,\mu$ = 0.759. Further investigation of this case is continuing.

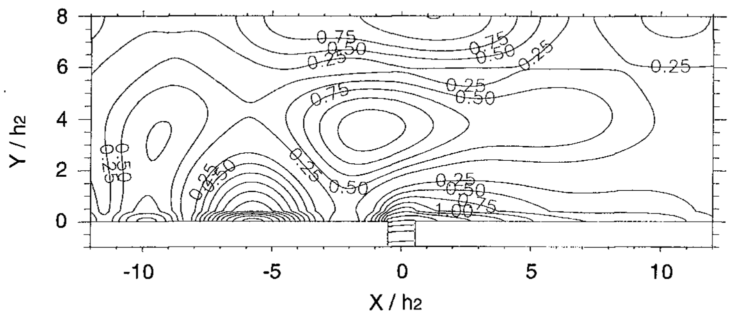


Figure 11. Amplitude of free surface elevation when zero-mode incident wave, with $\Omega\,\mu$ = 0.759, in channel with ledge, α= 0.4, γ= 0.1 and α / β= 0.05, meets gap in the ledge created by square bay, $\mu = \nu$ = 1.0.

Conclusions

We use a channel with a ledge along one of its walls to investigate the effects of a sudden change in the bottom geometry that enables the presence of trapped waves on the propagation of those waves.

The results obtained in this channel allow us to infer on the general trend of the response to similar problems in a semi-infinite ocean. However, in some parameter ranges they show a strong dependence on the channel width. The variability of the results with the dimensionless frequency Ω, show how careful one must be when interpreting results from scale model tests where long waves are to be produced. Since long waves are difficult to attenuate at the boundaries of the model, the effect of the wave tank walls is similar to the far channel wall in our model.

Acknowledgement

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