STOCHASTIC MODELLING OF NONLINEAR WAVES IN SHALLOW WATER

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Abstract

In this paper we shall study and model the nonlinear transformation of frequency wave spectra using two different types of stochastic models. The nonlinear processes considered include triad wave interaction and dissipation due to depth-induced wave breaking. The two stochastic models are the two-equation model proposed by Kofoed-Hansen and Rasmussen (1998) and the one-equation Lumped Triad Approximation (LTA) originally proposed by Eldeberky and Battjes (1995). Model results are compared with laboratory experiments and results obtained by the underlying deterministic time-domain Boussinesq model. The two stochastic models are found in good agreement with measurements of wave height (H_{m0}) and wave period (T_{01}). In case of wave transformation on a horizontal bottom, the LTA model fails as the rapid oscillations are neglected. The two-equation model predicts the energy transfer to sub-harmonics and non-resonant interaction excellently. In the inner surf zone and where the nonlinearity is strong, only the underlying deterministic model predicts the spectra and higher order wave statistics accurately.

Introduction

In recent years, considerable effort has been put on modelling of shallow water phenomena such as quadratic nonlinear wave interaction and depth-induced wave breaking using stochastic models. One of the major objective is to extend third generation wind-wave models, like the well-known WAM model developed for oceanic waters and shelf seas, to coastal waters where triad interaction and wave breaking are the dominating phenomena, see eg Cavaleri and Holthuijsen (1998). The starting point is typically

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deterministic evolution equations for the complex amplitudes of a weakly nonlinear wave field using either Boussinesq type equations (eg Freilich and Guza, 1984; Madsen and Sørensen, 1993) or the Laplace equation with leading order nonlinearity in the surface boundary conditions (eg Agnon and Sheremet, 1997; Madsen and Eldeberky, 1998).

In this study we have adopted the Boussinesq type equations with enhanced linear dispersion characteristics derived by Madsen and Sørensen (1993). Eldeberky and Battjes (1996) extended their equations by including energy dissipation due to depth-induced wave breaking. Based on these deterministic formulations, coupled stochastic evolution equations for the power spectrum and bispectrum have been derived recently by Eldeberky (1996) and Kofoed-Hansen and Rasmussen (1998) assuming that the trispectrum may be formulated as products of the power spectrum. The stochastic evolution equations suggested by Herbers and Burton (1997) are derived on basis of an extension of Freilich and Guza's (1984) deterministic model with lowest order dispersion and nonlinearity. Kofoed-Hansen and Rasmussen (1998) solved numerically the coupled evolution equations in order to calculate the wave spectrum and higher order nonlinear measures such as skewness and asymmetry (integral measures of the wave shape). For application in coastal energy-based wave models, Eldeberky and Battjes (1995) introduced a series of simplifications in order to avoid solving the evolution equation for the bispectrum. This resulted in a simple source function accounting for the effect of triad wave interaction, which can be used in conventional phase-averaged transport equations for the wave energy spectrum.

Stochastic Nonlinear Models

Time-domain Boussinesq type equations

The stochastic models considered in this paper are based on Boussinesq type equations derived by Madsen et al (1991) and Madsen and Sørensen (1992). These equations incorporate enhanced linear dispersion characteristics and shoaling properties, which are important for an accurate representation of the nonlinear energy transfer. This paper considers unidirectional waves propagating normally to the bottom contours (one horizontal dimension). The depth-integrated equations of continuity and momentum can then be formulated as

$$\frac{\partial \eta}{\partial t}+\frac{\partial P}{\partial x}=0 \qquad (1)$$

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial x}\left(\frac{P^2}{d}\right)+gd\frac{\partial \eta}{\partial x}-Bg\,h^3\frac{\partial^3\eta}{\partial x^3}-\left(B+\frac{1}{3}\right)h^2\frac{\partial^3 P}{\partial x^2\partial t}$$
$$-\frac{\partial h}{\partial x}\left(2Bgh^2\frac{\partial^2\eta}{\partial x^2}+\frac{1}{3}h\frac{\partial^2 P}{\partial x\partial t}\right)+R_{xx}=0 \qquad (2)$$

where η is the surface elevation, P the depth-integrated horizontal velocity, h the still water depth, g the acceleration due to gravity, d the instantaneous total water depth, and B the dispersion coefficient. Variable x is the horizontal space coordinate and t denotes time. The inclusion of wave breaking is based on the surface roller concept, see Madsen et al (1997). The effect of the roller on the wave motion is taken into account by the excess momentum term (term R_{xx} in Eq. 2) originating from a non-uniform velocity profile due to the presence of the roller. The equations have lowest order nonlinearity as the classical Boussinesq equations. For a dispersion coefficient of B=1/15, accurate dispersion characteristics are obtained for kh less than about 3.2.

Deterministic evolution equations

Now the surface elevation is represented as a Fourier series

$$\eta(x,t)=\sum_{p=-\infty}^{\infty} A_p(x)e^{i(\omega_p t-\psi_p(x))} \tag{3}$$

where A_p is a complex amplitude, ω_p is the angular frequency, $\psi_p(x)$ is a phase function and i denotes the imaginary unit. Following Madsen and Sørensen (1993), the spatial evolution of the complex amplitude, A_p, is to leading order given by the differential equation

$$\frac{dA_p}{dx}=(L_p\frac{dh}{dx}+\gamma_p)A_p+i\sum_{m=-\infty}^{\infty} J_{m,p-m}A_m A_{p-m}e^{-i\delta\psi(x)} \tag{4}$$

which describes the spatial evolution of a weakly nonlinear wave field on a mildly sloping bottom. In the derivation it has been assumed that the amplitudes are slowly varying in space. The first term on the right-hand side of Eq. (4) represents the linear shoaling and the dissipation due to wave breaking (γ_p), whereas the second term describes the nonlinear triad wave interaction (bound waves as well modulated free waves). Here $\delta\psi(x)=\int(k_{m+p}-k_m-k_p)dx$ is the phase-mismatch and may be considered as a measure of the departure from exact resonance. Variable k_p denotes the wave number. The detailed expression for L_p and $J_{m,m-p}$ is given in Kofoed-Hansen and Rasmussen (1998). Eq. (4) is identical to Eq. (7.1) in Madsen and Sørensen (1993) when γ_p= 0, and is the starting point for the derivation of the stochastic models suggested by Eldeberky and Battjes (1995), Eldeberky (1996) and Kofoed-Hansen and Rasmussen (1998).

Two-equation stochastic model

Stochastic evolution equations for the various order of spectra are derived by manipulating the deterministic evolution equations for the complex amplitude followed by ensemble-averaging. At lowest order, this procedure leads to an evolution equation for the power or wave energy spectrum including terms involving the next order spectrum, ie the bispectrum. An evolution equation for the bispectrum is derived at the following order, where terms including the trispectrum appear. In order to close the system of equations we write the trispectrum as products of the power spectrum, the well-known Gaussian closure approximation. Hence, the evolution equation for the power spectrum and bispectrum constitutes the two-equation stochastic model describing the spatial evolution of a weakly nonlinear unidirectional wave field propagating on a mildly sloping bottom, see Kofoed-Hansen and Rasmussen (1998) for a thorough derivation. The coupled evolution equations are written as

$$\frac{dF_p}{dx} = 2\left(L_p \frac{dh}{dx} - \gamma_p \right) F_p - 2 \sum_{m=-\infty}^{\infty} J_{m,p-m} \Im\left(B_{m,p-m}\right) \tag{5}$$

$$\begin{aligned} \frac{d\,B_{m,p-m}}{dx} &= \left(\left(L_m + L_{p-m} + L_p\right)\frac{dh}{dx} - (\gamma_m + \gamma_{p-m} + \gamma_p) - i\delta k(x) \right) B_{m,p-m} \\ &\quad + 2i\,J_{m,p-m}\left(\frac{k_m}{k_p} F_p F_{p-m} + \frac{k_{p-m}}{k_p} F_p F_m - \frac{k_p}{k_p} F_m F_{p-m} \right) \end{aligned} \tag{6}$$

where F_p and $B_{m,p-m}$ denote the discrete power and bispectrum, respectively. The bispectrum describes the degree of coupling and phase relationship in triads of nonlinearly interacting wave components. It describes statistically the shape of shoaling waves, ie skewness ~ $\Re(B_{m,p-m})$ and asymmetry ~ $\Im(B_{m,p-m})$, see Elgar and Guza (1985). $\Re$ and $\Im$ denote the real and imaginary part, respectively. The discrete spectrum is converted into a continuous spectrum by dividing the power spectrum by the frequency resolution and the bispectrum by the frequency resolution squared, respectively.

One-equation model

Based on the evolution Equations (5) and (6), Eldeberky and Battjes (1995) derived a parameterised model for application in conventional phase-averaged models, the Lumped Triad Approximation, LTA. They introduced a series of simplifying assumptions in order to avoid the computation of the evolution equation for the bispectrum. First, they integrated this equation, then they neglected the rapid oscillations involving the wave number mismatch. Finally, they restricted the formulation to self-self interaction and parameterised the biphase in terms of the local Ursell number as originally suggested by Doering and Bowen (1995). The evolution equation for a continuous wave spectrum can be written as

$$\frac{dF_p}{dx} = 2\left(L_p \frac{dh}{dx} - \gamma_p \right) F_p + S_p - S_{2p} \tag{7}$$

where the quadratic term, S_p, is expressed by

$$S_p = \alpha\, c_p J^2_{p/2,p/2} \sin\left|\beta_{p/2,p/2}\right| \left[F^2_{p/2} - 2F_p F_{p/2}\right] \tag{8}$$

The relation between the biphase, $\beta_{p,p}$, and the Ursell number is given by

$$\beta_{p,p} = -\frac{\pi}{2} + \frac{\pi}{2} tanh\left(\frac{0.2}{Ur}\right), \quad Ur = \frac{g}{8\sqrt{2}\pi^2} \frac{H_{m0}T_{01}^2}{h^2} \tag{9-10}$$

where $H_{m0} = 4\sqrt{m_0}$ and $T_{01} = m_0/m_1$. The moments are calculated as $m_n = \int_0^\infty f^n F df$ and the tuning parameter α appearing in Eq. (8) is of order 1. The variable, c_p, denotes the phase speed.

Dissipation due to wave breaking

The dissipation rate, γ_p, incorporated in the stochastic models is either taken as frequency independent as suggested by Eldeberky and Battjes (1996) or as a frequency squared dependent dissipation term as suggested by Chen et al (1997). In general the dissipation rate, γ_p, should be treated as a complex quantity. Here we restrict ourselves to consider the dissipation as purely real.

The coupled set of stochastic evolution equations for the power spectrum and bispectrum are solved numerically using standard numerical integration techniques and with linear upwind boundary conditions, ie $B_{m,p-m} \equiv 0$.

Numerical Results and Comparison with Experimental Data

Results of numerical simulations using the two stochastic models are compared with experimental data and results obtained by the underlying phase-resolving time-domain deterministic model.

Submerged bar

The measurements of Beji and Battjes (1993) are used to evaluate the two stochastic models for propagation of non-breaking waves over a trapezoidal submerged bar. Figure 1 illustrates their experimental setup. We consider the case with a very narrow-banded target spectrum. At WG1 this spectrum has a peak frequency of $f_p = 0.4$ Hz and a significant wave height of $H_{m0} = 0.023$ m, see Figure 2. A spatial resolution of 0.1 m is used and the frequency resolution is 0.03906 Hz. The tuning parameter α used in LTA is set to 1.

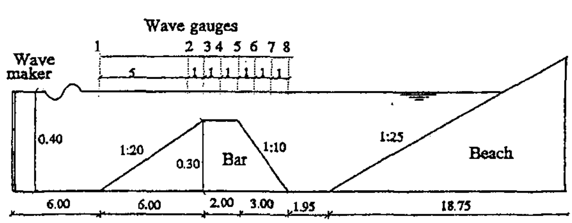


Figure 1. Layout of experiment of Beji and Battjes (1993). All lengths are given in meter.

Figure 2 presents a comparison of simulated and measured frequency spectra at locations WG1 and WG5. It has previously been shown (eg Kofoed-Hansen and Rasmussen, 1998 and Eldeberky, 1996) that the power spectrum is very accurately modelled on the uphill slope of the bar using the two stochastic models. On the top of the bar (see Figure 2) the two-equation model shows too much energy transfer to particularly the second harmonic components. The reason for the discrepancies is due to violation of the basic assumption of quasi-Gaussianity. At the crest section of the bar, the medium is almost non-dispersive for the primary waves ($k_p h$ ~ 0.26). The LTA model underestimates the energy transfer towards higher harmonics ($f > 2f_p$) as well as to lower frequencies ($f < \frac{1}{2} f_p$) as a consequence of the introduced simplifications.

The spatial evolution of quantities such as the significant wave height, H_{m0}, and the characteristic mean wave period, T_{01}, determined by the two stochastic models is compared measured data and the results obtained with the time-domain Boussinesq model in Figure 3.

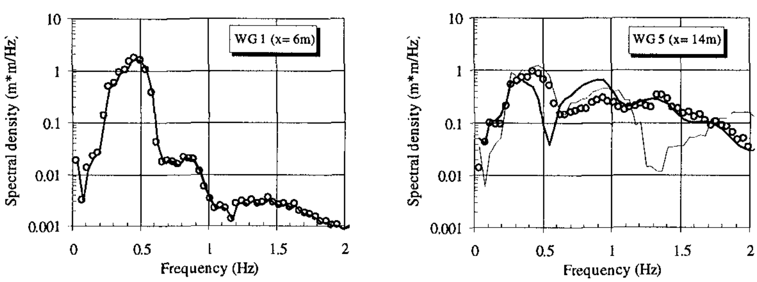


Figure 2. Comparison of frequency spectra from numerical simulations and measurements. (—) two-equation stochastic model, (–) LTA model and (ooo) experimental data by Beji and Battjes (1993).

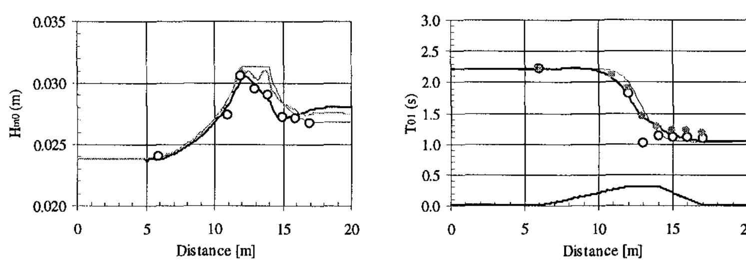


Figure 3. Spatial evolution of characteristic integral measures. (— and •) Deterministic model, (—) two-equation stochastic model, (–) LTA model and (ooo) experimental data by Beji and Battjes (1993). The bathymetry is sketched on the right panel.

From Figure 3 it appears that all three types of model result in similar growth of the wave height up-slope. Here the generated bound waves are phase-locked to the primary free waves and thus having almost the same group velocity. On the horizontal bar crest, two-way exchange of energy between free and bound waves takes place, which results in spatial inhomogeneity. This phenomena is only included in the deterministic as well as in the two-equation stochastic model. As LTA model neglects the rapid oscillations involving the wave number mismatch, the significant wave height will here remain constant. The reduction of the mean wave period over the bar is well predicted by all three types of model. Kofoed-Hansen and Rasmussen (1998) have shown that higher order statistical quantities (ie skewness and asymmetry) can be predicted reasonably well using the two-equation model for this non-breaking bar test. The results will not be shown here.

Barred beach

In this case, we consider the spatial evolution of an incident Pierson-Moskowitz type spectrum over a barred sandy beach (test no LIP 11D, Case C, Arcilla et al, 1994) as illustrated in Figure 4. This figure also indicates the location of the wave gauges. The power spectrum of the measured surface elevation at WG1 yields a significant wave height of H_{m0}= 0.58 m and a peak frequency of f_p= 0.125 Hz. This spectrum is applied as boundary condition for the stochastic models at the boundary 20 m seawards of WG1. The most energetic waves are characterised by having very weak dispersion as $k_p h$= 0.53. The spatial resolution is again set to 0.1 m and the frequency resolution is 0.0097 Hz. The tuning parameter used in LTA is set to α= 0.5, which results in best agreement with the measured data.

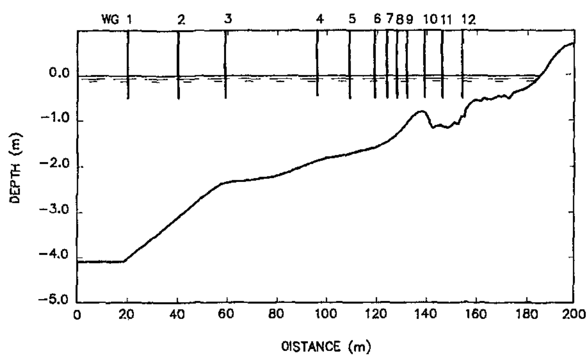


Figure 4. Layout of the large-scale laboratory flume experiment of Arcilla et al (1994).

In Figure 5 the predicted frequency spectrum is compared to the measured spectrum at the four locations WG1, WG3, WG6 and WG11. In this case, the frequency-independent wave breaking dissipation model (suggested by Eldeberky and Battjes, 1996) is used. At WG3 is seen that the two stochastic models predict accurately the measured spectrum until a frequency of approximately $3f_p$. Here mainly one way of energy transfer occurs (as was the case for the bar test). At WG6 and WG11 there is an increasing tendency of too strong transfer rates in both models, which is probably due to violation of the basic assumption of a quasi-Gaussian sea state. Although some discrepancies appear between the model results and measurements, the overall performance is reasonable. Particularly the energy transfer towards low-frequency wave components is excellently predicted by the two-equation model.

The spatial evolution of the significant wave height, the mean wave period, skewness and asymmetry determined by the stochastic models are compared to the measured data and the results obtained with the time-domain Boussinesq model in Figure 6. It is seen that the three models predict almost the same significant wave height and mean wave period in reasonably good agreement with measurements. The LTA model shows a slightly better agreement with the measured mean wave period than the deterministic and two-equation model at a distance of 100-130 m. The mean wave period decreases as the higher order spectral moments increase during the nonlinear shoaling. In a linear model the wave period will be almost constant. Figure 6 also shows that the skewness and asymmetry are in good agreement with the measurements for distances less than, say, 100 m. For larger distances both measures deviate significantly from the measurements most likely as a consequence of too strong nonlinearity.

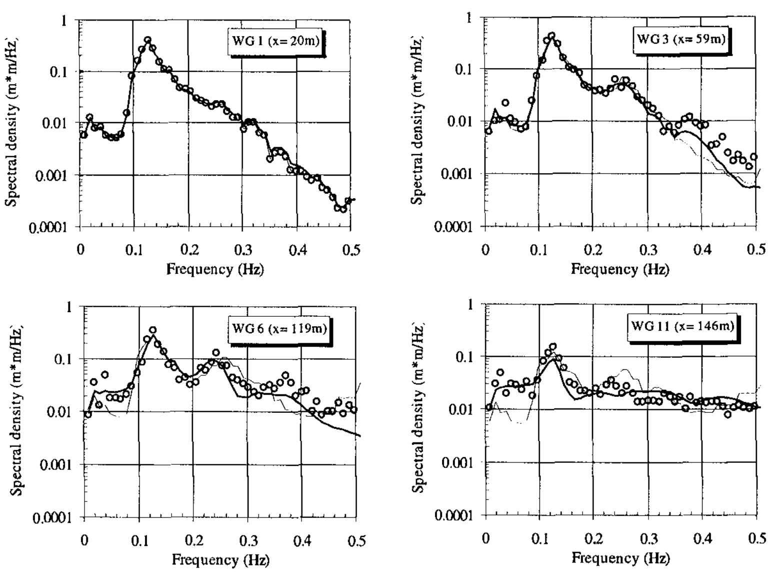


Figure 5. Comparison of frequency spectra from numerical simulations and measurements. (▬) two-equation stochastic model, (—) LTA model and (ooo) experimental data by Arcilla et al (1994).

The assumption of using a wave breaking dissipation rate in proportion to the spectral density (ie independent of the frequency) is not found to be the major reason for the discrepancy between measured and simulated (by the two-equation stochastic model) skewness and asymmetry as suggested by Chen et al (1997). The results of simulations using a frequency-independent and frequency-dependent (f^2) formulation are depicted in Figure 7. As seen from the figure the skewness and asymmetry were not improved by weighting the dissipation towards higher frequencies in this case. It is also seen that by setting $F= 1$ (frequency-independent formulation) in the model by Kaihatu and Kirby (1996), the third-order statistical quantities are almost identical to the results obtained using the model Eldeberky and Battjes (1996). As the basic assumption of Gaussianity is highly violated in the inner surf zone and the statistical closure is highly questionable, only a phase-resolving model (including an advanced wave breaking formulation) is applicable there.

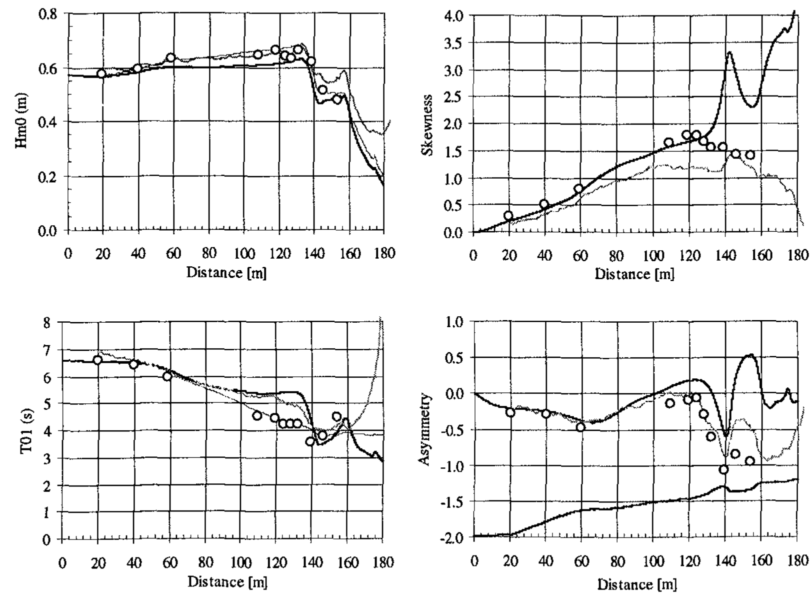
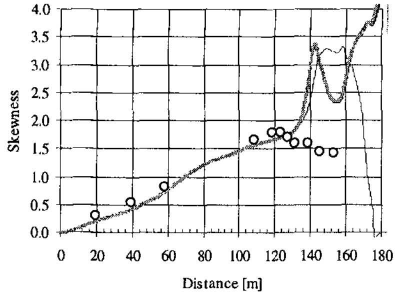


Figure 6. Spatial evolution of characteristic integral measures. (—) Deterministic model, (—) two-equation stochastic model, (–) LTA model and (ooo) experimental data by Arcilla et al (1994). The bathymetry is sketched at the lower right panel.



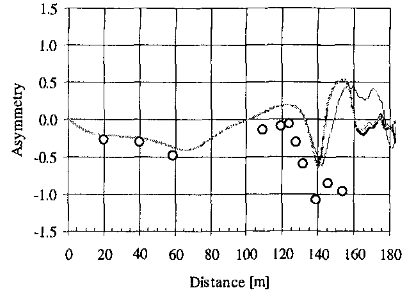


Figure 7. Spatial evolution of skewness and asymmetry simulated by the two-equation model using different wave breaking dissipation models. (—) Kaihatu and Kirby (1996), F=0, (—) Kaihatu and Kirby (1996) model, F=1, (—) Eldeberky and Battjes (1996) model and (ooo) experimental data by Arcilla et al (1994). When F= 1 the dissipation is frequency-independent and for F=0 the dissipation has a frequency squared dependency.

Horizontal bottom

The numerical results for the bar test show that the LTA model predicts no spatial variation of the total wave energy on the horizontal crest contrary to the measured data and the results of the two-equation stochastic model. In the following, we shall examine the long-term evolution of a narrow-banded spectrum at a constant depth a bit more using the two stochastic models. The simulations have been performed using similar parameters as used in Elgar et al (1990), see their Figure 6 p. 11552. The initial spectrum (x=0) consists of a single large primary peak at f_p= 0.0625 Hz as illustrated in Figure 8. The water depth is 2.0 m. As the significant wave height is H_{m0}= 0.169 m, the Ursell number $Ur= (\frac{1}{2}H_{m0}/h)/(k_p h)^2$= 1.33, $k_p h$= 0.18. The frequency resolution is 0.0625 Hz, and the tuning parameter α used in the LTA model is set to 1.

The spatial evolution of the primary spectrum is presented in Figure 8 at x= 0, x= $7L_p$, x= 30 L_p, and x= $70L_p$, where L_p denotes the wave length corresponding to the initial peak frequcncy. The results obtained by the two-equation model shows that harmonics of the primary wave components grow during the initial stages of evolution. As the wave field evolves further, spectral valleys are filled in at the expense of spectral peaks. After about 70 wave lengths, the frequency spectrum is essentially featureless, and almost all traces of the initial spectrum and its harmonics are gone. In this case the beat length for the super-harmonie interaction between the primary peak and its second harmonic is 31-$32L_p$, but as the nonlinear transfer is very strong no clear evidence of recurrence is found in this particular case. Test cases with recurrence is presented and discussed in Rasmussen (1998). The simulated results presented in Figure 8 are in quite good agreement with the results presented by Elgar et al (1990) based on integration of Freilich and Guza's (1984) evolution equations.

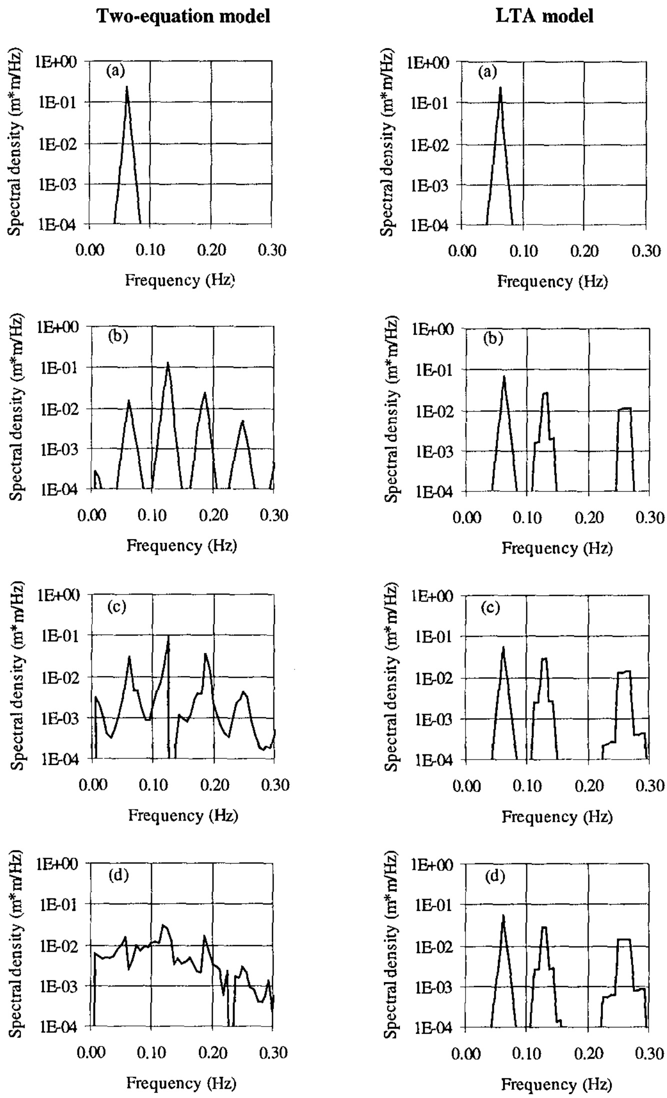


Figure 8. Evolution of a narrow-banded frequency spectrum on a horizontal bottom simulated by the two-equation model (left) and the LTA model (right). The power spectrum is presented at different distances a) x= 0, b) x= $7L_p$, c) x= $30L_p$ and d) x= $70L_p$, where L_p= 70.5 m is the wave length corresponding to the initial (x=0) peak frequency of f_p= 0.0625 Hz. The water depth is h= 2.0 m.

The spectral evolution simulated with the LTA model (see Figure 8), shows initially energy transfer to the second and fourth harmonics. After a few wave lengths (< 10), the wave spectrum remains almost unaltered. From Eq. (8), it is seen that an 'equilibrium' spectrum is obtained, where $F_p = \frac{1}{2}F_{p/2}$, ie the spectral density at the second harmonic is half of the spectral density at the primary peak and so on. Thus, the LTA model is not applicable for simulation of recurrence and white-noise type spectra on constant or nearly constant water depth. This is a result of neglecting the phase-mismatch between free and bound wave components.

Discussion

Having examined the performance of the two stochastic models, we may define the area of model application. As discussed in Kofoed-Hansen and Rasmussen (1998) the two-equation model can be used to predict the evolution of the frequency spectrum and bispectrum and associated characteristic integral measures in shallow water when the Ursell number (based on the peak frequency of the primary waves) is less than 1-2. Beyond that limit, the basic assumption of a quasi-Gaussian sea state is highly violated and the underlying phase-resolving model (including a proper description of the wave breaking process) is more accurate. In cases with a high degree of frequency dispersion, say, $k_p h > 2$, a more accurate description of the dispersion is required as shown in Madsen and Eldeberky (1998).

The LTA model, which is presently used in thc public domain third-generation wind-wave model SWAN, see eg Cavaleri and Holthuijsen (1998), shows excellent agreement with the measurement of the significant wave height, H_{m0}, and mean wave period, T_{01}, for the two cases considered in this paper. The tuning parameter α is set to 1.0 for the bar test and 0.5 for the barred beach test. In case of a constant water depth the model predicts an unphysical long-term spectral evolution of the frequency spectrum, which is different from the expected featureless white-noise type spectrum. As a consequence of the introduced simplifications, the LTA model is mainly appropriate for relatively short evolution distances on sloping bathymetries, where the generation of bound super-harmonics is substantial.

Further, as the LTA model only includes self-self interactions the approach can in general only be applied for uni-modal (and unidirectional) frequency spectra. In cases of sea states involving swells and wind-waves, the LTA model is not expected to model accurately the nonlinear energy exchange between the two frequency regimes.

Although both types of stochastic models are of phase-averaged type, the required spatial resolution is different. Using the two-equation model, the wave number mismatch has to be resolved. Therefore the resolution is typically of the same order as for the underlying deterministic model. The spatial resolution used for solving the conventional phase-averaged energy transport equation including the LTA model (as source function for triad wave interaction) is usually at least 100 times larger.

Summary and Conclusions

This paper compared results of numerical calculations obtained from phase-averaged one- and two-equation stochastic models as well as the underlying deterministic (phase-resolving) model with laboratory experiments in case of a submerged bar (Beji and Battjes, 1993) and on a barred beach (Arcilla et al, 1994). Also the long-term evolution of a narrow-banded frequency spectrum on a horizontal bottom has been examined. The sensitivity of wave breaking formulations on third-order wave statistics derived from the two-equation model has been studied as well.

The simplified one-equation stochastic model (LTA) represents an average effect of triad wave interaction, transferring energy from lower to higher frequencies through self-self interaction. The model is in excellent agreement with measurements of H_{m0} and T_{01} in the two cases considered. The two-equation model also takes into account the energy transfer to sub-harmonics as well as the non-resonant wave interaction and provides third-order statistics too. In general, the agreement between the simulated results and measurements is found to be acceptable, even beyond the domain where Gaussianity may be justified. However, in the inner part of the surf zone, the stochastic model underestimate significantly the skewness and asymmetry. Results of simulations using frequency-dependent formulations did not improved the accuracy. As the basic assumption of Gaussianity is highly violated in the inner surf zone and the statistical closure is highly questionable, only a phase-resolving model is applicable there.

The results of the simulations performed on a horizontal bottom indicate that the LTA model is not applicable for prediction of long-term evolution on a nearly horizontal bottom in shallow water. As the two-equation model retains the phase-mismatch between free and bound waves, the frequency spectrum considered here tends to evolve towards a white-noise type spectrum after several wave lengths.

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References

Agnon, Y. and A. Sheremet (1997), "Stochastic nonlinear shoaling of directional spectra", *J. Fluid Mech.*, 345, 79 -99.

Arcilla, A.S., J.A. Roelvink, B.A. O'Connor, A. Reniers and J.A. Jimenez (1994), "The Delta Flume'93 experiment", *Proc. Coastal Dynamics '94*, ASCE, 488-502.

Beji, S. and J.A. Battjes (1993), “Experimental investigation of wave propagation over a bar”, *Coastal Eng.*, 19, 151-162.

Cavaleri, L. and L.H. Holthuijsen (1998), "Wave modelling in the WISE Group", *Proc. 26th International Coastal Engineering Conference, Copenhagen, Denmark*, ASCE.

Chen, Y., R.T. Guza and S. Elgar (1997), "Modeling spectra of breaking surface waves in shallow water", *J. Geophys. Res.*, 102, C11, 25035-25046.

Doering, J.C. and A.J. Bowen (1995), "Parameterisation of orbital velocity asymmetries of shoaling waves using bispectral analysis", *Coastal Eng.*, 26, 15-23.

Eldeberky, Y. (1996), "Nonlinear transformation of wave spectra in the nearshore zone", *PhD. thesis*, Delft University of Technology, The Netherlands, 203 pp.

Eldeberky, Y. and J.A. Battjes, (1995), "Parameterisation of triad interaction in wave energy models", *Proc. Coastal Dynamics '95*, Gdansk, Poland, ASCE, 140-148.

Eldeberky, Y. and J.A. Battjes (1996), "Spectral modelling of wave breaking. Applications to Boussinesq equations", *J. Geophys. Res.*, 101, C1, 1253-1264.

Elgar, S. and R.T. Guza (1985), "Observations of bispectra of shoaling surface gravity waves", *J. Fluid Mech.*, 161, 425-448.

Elgar, S., M.H. Freilich and R.T. Guza (1990), "Recurrence in truncated Boussinesq models for nonlinear waves in shallow water", *J. Geophys. Res.*, 98, C7, 11547-11556.

Freilich, M.H. and R.T. Guza (1984),"Nonlinear effects on shoaling surface gravity waves", *Phil. Trans R. Soc. London*, A311, 1-41.

Herbers, T.H.C. and M.C. Burton (1997), "Nonlinear shoaling of directionally spread waves on a beach", *J. Geophys. Res., 102, C9, 21101-21114.*

Kaihatu, J.M. and J.T. Kirby (1996), "Effects of mode truncation and dissipation on predictions of higher order statistics", *Proc. 25th International Coastal Engineering Conference*, Orlando, FL, USA, ASCE, 123-135.

Kofoed-Hansen, H. and J.H. Rasmussen, (1998), "Modelling of nonlinear shoaling based on stochastic evolution equations", *Coastal Eng.*, 33, 203-232.

Madsen, P.A. and Y. Eldeberky (1998), "A new formulation of deterministic and stochastic evolution equations for three-wave interactions involving fully dispersive waves", *Proc. 26th International Coastal Engineering Conference*, Copenhagen, Denmark, ASCE.

Madsen, P.A., R. Murray and O.R. Sørensen (1991), "A new form of the Boussinesq equations with improved dispersion characteristics", *Coastal Eng.*, 371-388.

Madsen, P.A. and O.R. Sørensen, (1992), "A new form of the Boussinesq equations with improved dispersion characteristics, Part 2. A slowly-varying bathymetry", *Coastal Eng.*,18, 183-204.

Madsen, P.A. and O.R. Sørensen, (1993), "Bound waves and triad interactions in shallow water", *Ocean Eng.*, 20, 359-388.

Madsen, P.A., H.A. Schäffer and O.R. Sørensen (1997), "Surf zone dynamics simulated by a Boussinesq type model. Part I: Model description and cross-shore motion of regular waves", *Coastal Eng., 32,* 255-288.

Sørensen, O.R., P.A. Madsen and H.A. Schäffer (1998), "Nearshore wave dynamics simulated by Boussinesq type models", *Proc. 26th International Coastal Engineering Conference*, Copenhagen, Denmark, ASCE.

Rasmussen, J.H. (1998), "Deterministic and stochastic modelling of surface gravity waves in finite depth", *PhD. thesis*, Technical University of Denmark.