CHAPTER 285

Rip Current Generation on a Plane Beach

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ABSTRACT: In order to establish a wave-current interaction model for rip currents, a set of first-order linearized governing equations describing the 2-D nearshore circulation is derived from the equations of nearshore currents by the perturbation method. Wave refraction induced by the generated currents is fully considered. The solutions to the field equations of rip currents both in the surf zone and shoaling zone are obtained, and expressed in the form of Gaussian hypergeometric functions and modified Bessel functions, respectively. The matching condition of the solutions at the breaking point determines the spacing of the rip currents. Comparison of the computed rip current spacing, circulation pattern, and rip current discharge, with laboratory and field data shows a satisfactory agreement especially in the so-called instability region of the surf similarity parameter.

INTRODUCTION

The change in momentum flux of incoming waves can be described by the offshore and longshore gradients of the radiation stresses which act as the driving forces in the nearshore circulation system. Therefore to formulate the driving forces in the momentum equations for the generation of nearshore currents is of importance (Dingemans, Radder and De Vriend, 1987). Two main causes of the driving forces are the so-called wave-current interaction, and structural interaction (Dalrymple and Lozano, 1978).

The first investigation of nearshore currents as 2-D horizontal circulation generated by the interaction between the incoming waves and the resulting rip currents was by Le Blond and Tang (1974), whose theory applies to a plane beach and normally incident waves. Similar to their work, Iwata (1976) developed a theory which assumed the nonuniformity of bottom friction between surf and shoaling zones. However, these two theories still failed in obtaining the nondimensional alongshore spacing of rip currents as an eigenvalue of the governing equations. Iwata further attempted to find

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currents as an eigenvalue of the governing equations. Iwata further attempted to find an asymptotic solution for both the surf and shoaling zones. Finally, he obtained the real eigenvalue from the characteristic equation which is derived by matching conditions at the breaking point, and found that its value is a function of a parameter determined by the bottom friction coefficient, the surf zone width and the breaker height.

Mizuguchi (1976) also attempted to obtain the eigenvalues from the characteristic equation without using any approximation such as asymptotic solution as in Iwata's work, by considering both the uniformity and nonuniformity of the bottom friction. However, no eigenvalue was obtained. He concluded that this failure occurred due to the exclusion of lateral mixing in the governing equation, and because the contribution of the bottom friction was not sufficient to represent the dissipative effect in the nearshore current. Consequently, he reformulated the bottom friction term to be a function of the distance from the shoreline, similar to the lateral mixing. Dalrymple and Lozano (1978) argued that no reason exists to justify this formulation, so that the real eigenvalue obtained as a function of a parameter proportional to the so-called surf zone similarity parameter and inversely proposional to the bottom friction was invalid.

Dalrymple and Lozano (1978) presented two wave-current interaction models. One was similar to the theory of Le Blond and Tang (1974), in which changes in local wave length due to currents were considered. However, their assumption of an extremely small refraction angle, which implied that no longshore variation in wave orthogonals was allowed, resulted in no rip-current formation. In the second model, the wave-current interaction effect was considered through wave refraction due to current, and the formation of longshore periodic nearshore circulation cells was calculated numerically. The obtained eigenvalue was a function of a parameter expressed by the ratio of beach slope to bottom-friction coefficient. The relationship between the eigenvalue and this parameter showed that the rip-current spacing increases as the parameter increases and vice-versa. Comparison with the rip current spacing of Balsillie (1975) obtained by field measurement showed a good agreement in the region of small values of the parameter, which implied that the theoretical eigenvalue of Dalrymple and Lozano was partly suitable for the prediction of rip-current spacings generated by the incidence of infragravity waves, as it was stated by Balsillie that almost all of the data are catagorized into this type of waves. However, when compared with the field measurements collected by Sasaki (1977), which were made in wider regions such as the regions of instability and edge waves, the theoretical spacing showed a lower value than that of field measurements.

The present study investigates the steady-state nearshore circulation on a plane beach based on the interaction between normally-incident waves and the resulting rip currents. A mathematical formulation of the governing field equations for rip currents on a plane beach is made by means of the wave-current interaction model, including the formulation of the driving forces. The governing equations of 2-D nearshore currents on the plane beach are first established by employing the conservation laws for mass, momentum and wave action. The so-called mild slope equation (MSE), which is able to calculate wave transformation due to interaction with the nearshore currents, such as wave refraction, is applied to formulate the driving forces in the momentum equations. Nevertheless, to simplify the analysis without sacrificing the generality of the theory, the lateral mixing terms are omitted. Using a perturbation method for a small parameter of beach slope, the field equations of rip currents are formulated. At the first order of approximation, the field equations of rip currents are obtained in both the surf and shoaling zones, where the wave refraction due to currents is fully considered. With some additional assumptions, solutions to the field equations are obtained both in the surf and shoaling zones in terms of the Gaussian hypergeometric function and the modified Bessel function, respectively. The matching condition for these solutions at the breaking point makes it possible to determine the eigenvalues of the characteristic equation of the derived field equations.

The theoretical results of the rip current characteristics such as the rip current spacing and flow patterns, are compared with both the previous theoretical results, laboratory and field data. The comparison of the theoretical rip current spacings and those of field data showed a satisfactory agreement. Current patterns in a nearshore circulation cell are also calculated numerically through the determination of integration constants in the stream function based on energy budgeting at the breaking point, which resulted in a reasonable value of rip discharge, both at laboratory and field scales.

THE BASIC EQUATIONS OF NEARSHORE CURRENTS

The MSE derived by Kirby (1984) which includes an additional wave energy dissipation term, $i\omega_r W\Phi$, is employed to formulate the driving forces. The terms in the MSE of Kirby are as follows: W is the ratio of the wave energy dissipation rate D to the total wave energy E, C is the wave celerity, C_g is the group velocity, where both values are assumed to be nearly equal due to the shallow water approximation, and D/Dt is the Lagrangian derivative. By introducing velocity potential of linear wave theory, $\Phi = \phi \exp(-i\omega t)$, and $\phi = (ga/i\omega_r)\exp(iS)$, where S is the phase function, into the MSE of Kirby in the steady state condition, the MSE is written as:

$$\left\{\frac{\partial}{\partial x_j}\left(C C_g \frac{\partial}{\partial x_j}\right) + 2i\omega U_j \frac{\partial}{\partial x_j}\right\}\phi = -\left\{i\left(\omega\frac{\partial U_j}{\partial x_j} + \omega_r W\right) + \left(\omega^2 + k^2 C C_g - \omega_r^2\right)\right\}\phi \tag{1}$$

Following lengthy algebraic procedures using the expressions for the radiation stresses by Dingemans, Radder and De Vriend (1987), and for the depth-integrated mass transport by Crapper (1984) (see Tsuchiya and Suriamihardja, 1989), the driving force F_i can be obtained as:

$$F_i = \frac{\rho\omega_r}{2g}\frac{\partial S}{\partial x_i} W\phi\phi^* - \frac{\rho}{4g} d \frac{\partial}{\partial x_i}\left(\phi\phi^* \frac{\omega_r^2}{d}\right) + \frac{\rho\omega_r}{2g}\phi\phi^* \frac{\partial S}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) - \frac{\rho}{4g} C C_g \frac{\partial}{\partial x_i}\left(\frac{\phi\phi^*}{A}\frac{\partial^2 A}{\partial x_j^2}\right) \tag{2}$$

where ϕ^* is the complex conjugate of ϕ. The first term in the right side of (2) is the rotational term, and is the contribution from the dissipation of wave energy. The second is the irrotational term, and the third term describes the interaction between wave-induced mass transport and currents. The fourth term represents the additional effect resulting from the diffraction of waves. Dingemans, Radder and De Vriend (1987) demonstrated that only these rotational terms are able to generate non-zero depth-averaged current velocities. For simplicity, the horizontal components of the nearshore current are assumed to be independent of depth. Therefore the conservation laws to be employed in this study are presented in depth-integrated form (For example, Phillips, 1966; Dolata and Rosenthal, 1984; and Crapper, 1984). Neglecting the wave diffraction effect and assuming the shallow water condition, the steady state mass and momentum equations can finally be reduced to

$$\frac{\partial}{\partial x_j}\left(\rho U_j d\right)=0 \tag{3}$$

$$U_j\frac{\partial U_i}{\partial x_j}+R_i=-g\delta_{ij}\frac{\partial\bar{\eta}}{\partial x_j}+\frac{D}{\rho\omega_r d}\frac{\partial S}{\partial x_i} -\frac{\partial}{\partial x_i}\left(\frac{E}{2\rho d}\right)+\frac{E}{\rho\omega_r d}\frac{\partial S}{\partial x_i}\left(\frac{\partial U_j}{\partial x_i}-\frac{\partial U_i}{\partial x_j}\right) \tag{4}$$

where $$R_i=\frac{1}{\rho d}\left(\frac{\partial}{\partial x_j}\left(\rho d\tau_{ij}\right)+f_{ij}U_j\right)\qquad E=\frac{1}{2}\rho g\omega_r^2\phi\phi^*$$

in which U_i are the vertically-averaged horizontal current velocity components, τ_{ij} are the lateral mixing terms, η is the mean water level, ρ is the density of water, d is the water depth, g is the acceleration of gravity, t is time, x_j are the horizontal coordinates x and y for $j=1$ and 2, and f_{ij} are the bottom friction coefficients following Iwata's (1976) formulation in the case of normal incidence and are written as

$$\left.\begin{aligned}\begin{pmatrix}f_{xx} & 0\\ 0 & f_{yy}\end{pmatrix}\begin{pmatrix}U\\ V\end{pmatrix}&=\frac{2}{\pi}K_*\sqrt{gd}\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}U\\ V\end{pmatrix}, \quad x\le x_B\\ \begin{pmatrix}f_{xx} & 0\\ 0 & f_{yy}\end{pmatrix}\begin{pmatrix}U\\ V\end{pmatrix}&=\frac{2}{\pi}K_*\sqrt{gd_B}\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}U\\ V\end{pmatrix}, \quad x\ge x_B\end{aligned}\right\} \tag{5}$$

where $K_* = 1.41\,(\gamma / k.\,k_e\,)^{-2/3}$ in which k_e is the bottom roughness, k is the wave number and γ is the ratio of wave amplitude to local water depth, and suffix B stands for the position of wave breaking and indices x and y represent the seaward x and alongshore y directions. The conservation of wave action, which can be derived directly from (1), and wave number conservation which is equivalent to the irrotationality condition for wave number, are written as:

$$\frac{\partial}{\partial x_j}\left\{\frac{E}{\omega_r}\left(U_j+C_j\right)\right\}+\frac{D}{\omega_r}=0;\qquad \frac{\partial k_j}{\partial x_i}=\frac{\partial k_i}{\partial x_j} \tag{6}$$

THE FIELD EQUATION OF RIP CURRENT

Perturbation Scheme

Before ordering the equations using the perturbation method, it is convenient to nondimensionalize them. The water depth d_B at the breaking point is selected as the representative length to facilitate the nondimensionalization. The process is defined as:

$$\left.\begin{array}{ll} (x, y, \overline{\eta}, d) = d_B(x^*, y^*, \overline{\eta}^*, d^*) & (U, V, C) = \sqrt{g d_B}(U^*, V^*, C^*) \\ (\omega, \omega_r) = \sqrt{g/d_B}(\omega^*, \omega_r^*) & (k, k_x, k_y) = d_B^{-1}(k^*, k_x^*, k_y^*) \end{array}\right\} \tag{7}$$

where an asterisk represents the dimensionless quantities.

. The beach slope s is selected as a parameter of perturbation in ordering the equations, and the series expansion, in which asterisks have been dropped for convenience, are given as:

$$d = s(d_0 + s\zeta_1 + s^2\zeta_2 + \ldots) \quad ; \quad a = s(a_0 + sa_1 + s^2 a_2 + \ldots) \tag{8a}$$

$$U = s^{\frac{1}{2}}\left(0 + sU_1 + s^2 U_2 + ..\right) \quad ; \quad V = s^{\frac{1}{2}}\left(0 + sV_1 + s^2 V_2 + ..\right) \tag{8b}$$

$$\mathbf{k} = \left(k_{0x} + sk_{1x} + s^2 k_{2x} + ..\right)\mathbf{i} + \left(0 + sk_{1y} + s^2 k_{2y} + ..\right)\mathbf{j} \tag{8c}$$

$$\omega_r = s^{\frac{1}{2}}\{k_{0x}C_0 + s(k_{1x}C_0 + k_{0x}C_1) + ..\}; \; S = (S_0 + sS_1 + s^2 S_2 + ..) \tag{8d}$$

$$C = s^{\frac{1}{2}}(d_0 + s\zeta_1 + s^2\zeta_2 + \ldots)^{\frac{1}{2}}\sqrt{(\tanh kd_B)/(kd_B)} \tag{8e}$$

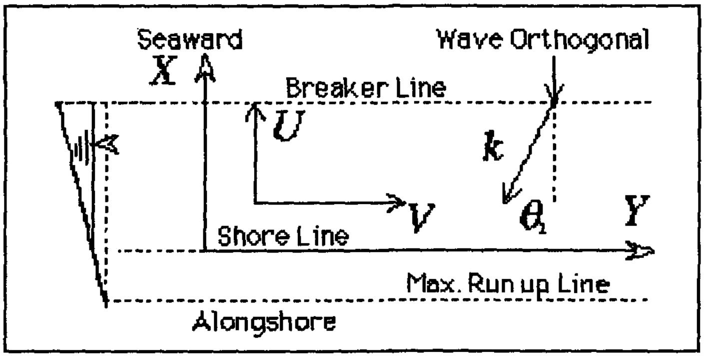

Figure-1 The coordinate system and geometry for nearshore current vectors U, V, and wave number k.

The Conservation of Wave Number

The normally-incident waves refract due to their interaction with rip currents. This may mean that the wave number direction intersects the beach obliquely. Therefore, based on Figure 1 the wave number can be given as:

$$\mathbf{k} = -(|\mathbf{k}|\cos\theta_1)\ \mathbf{i} - (|\mathbf{k}|\sin\theta_1)\ \mathbf{j} \tag{9}$$

where $|\mathbf{k}|$ is the magnitude of vector $\mathbf{k}$. Equation (9) can be expanded with respect to the perturbation parameter s, and as was mentioned above that the group velocity is to be nearly equal to the wave celerity in the surf zone, it was assumed to be in the same direction as the wave number vector. Consequently, it can also be expanded by the perturbation parameter. The conservation equation for the wave number is finally formulated for each order of the perturbation parameter (see Tsuchiya and Suriamihardja, 1989) as:

$$\left.\begin{aligned} &O(s^0) \equiv \frac{\partial k_{0x}}{\partial \tilde{y}} = 0 \\ &O(s^1) \equiv \frac{\partial k_{1x}}{\partial \tilde{y}} = \frac{\partial k_{1y}}{\partial \tilde{x}} \Rightarrow \frac{\partial}{\partial \tilde{y}}(C_1 - U_1) = -\frac{\partial}{\partial \tilde{x}}(V_1) + \frac{1}{\tilde{x}} V_1 \end{aligned}\right\} \tag{10}$$

where $\tilde{x} = m(x + x_s) = d_0$; $\tilde{y} = my$; $m = (1 + \mathrm{d}\zeta_0 / \mathrm{d}\tilde{x})$

m is the slope of mean water level, and x_s is the maximum run-up position.

Zeroth-order equations and their solutions

The leading terms of the governing equations produced by the perturbation expansion are given for the surf zone and shoaling zone, respectively. In the surf zone, the zeroth order equations of momentum and wave action conservation are given respectively as:

$$d_0\left(\frac{\mathrm{d}\zeta_0}{\mathrm{d}x}\right) = \frac{D_0}{\omega_{0r}}\frac{\mathrm{d}S_o}{\mathrm{d}x} - d_0\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{E_0}{2d_0}\right); \qquad -\frac{\mathrm{d}}{\mathrm{d}x}\left(E_0 C_0\right) + D_0 = 0 \tag{11}$$

In order to solve the above equations, the wave amplitude a_0 in the surf zone is assumed to be proportional to the local depth to leading order: $a_0 = \gamma d_0$ where γ is an empirical constant. This formulation is employed in surf zone models. The wave energy dissipation term D is hypothesized as:

$$D = \left(5\gamma^2/4\right)\left\{md_0^{\frac{3}{2}} + s\left(3md_0^{\frac{1}{2}}\zeta_1/2 + d_0^{\frac{3}{2}}\left(\partial\zeta_1/\partial x\right) + \ldots\ldots\right)\right\} \tag{12}$$

where D_0 in (11) is equal to $(5\gamma^2 m/4)\, d_0^{3/2}$

The wave amplitude in the shoaling zone is obtained from the leading order of the equation of wave action conservation as $a_0 = B_0^{1/2} d_0^{-1/4}$, where B_0 is an integration constant. The wave set-up is obtained from (11) (see Tsuchiya and Suriamihardja, 1989) as:

$$\zeta_0(x) = -\left\{\left(\tfrac{3}{2}\gamma^2\right)x + x_s\right\} m \tag{13}$$

where $m = (1 + \frac{3}{2}\gamma^2)^{-1}$; $x_s = \gamma^2\left\{\frac{3}{2} - \frac{1}{4}\left(1 + \frac{3}{2}\gamma^2\right)/\left(1 - \frac{3}{2}\gamma_B^2\right)\right\} x_B$, and $\gamma_B = B_0\, x_B^{-5/4}$

and x_B is the distance from the shoreline to the breaking point.

The first-order equations of rip currents in the surf zone

The mass conservation equation is given from the first equation of (3), considering (8) for its expansion and using variables defined from mass conservation, of which the stream function can be defined as follows:

$$\frac{\partial}{\partial \tilde{x}}(\tilde{x}\, U_1) + \frac{\partial}{\partial \tilde{y}}(\tilde{x}\, V_1) = 0 \Rightarrow (\tilde{x} U_1) = \frac{\partial}{\partial \tilde{y}}(\Psi) \text{ and } (\tilde{x} V_1) = -\frac{\partial}{\partial \tilde{x}}(\Psi) \tag{14}$$

The approximation may be made to the wave dispersion relation, to evaluate C_1. It is necessary, of course, to obtain a simple relation between C_1 and ζ_1 in order to reduce higher-order differentation terms. The simple relation can be derived from the established set of four equations. In order to approximate the relation along with the four equations, however, the x-direction of the first order of momentum equation (4) can be used by considering the most effective terms, thus:

$$C_1 = q\tilde{x}^{-\frac{1}{2}}\zeta_1; \qquad q = \frac{1}{2}\sqrt{(\tanh kd_B)/(kd_B)} \tag{15}$$

if the other non-leading terms are neglected, q may equal to 1/2 which is equivalent to that of Darlymple and Lozano's approximation. However, based on this approximation, the value of q might be somewhat less than 1/2.

When the horizontal mixing terms remain in (4), the field equation of rip currents may become a fourth-order partial differential equation. Without loss of generality in the formation of rip currents, as previously treated in theoretical approaches to the formation of rip currents, the horizontal mixing terms are neglected, but the bottom friction terms are included. By use of the four equations using (15) to replace C_1 the first-order equation of momentum can finally be obtained (see Tsuchiya and Suriamihardja, 1989 for the detailed reduction) as:

$$\begin{pmatrix} M^{\kappa}_{xx} & 0 \\ 0 & M^{\kappa}_{yy} \end{pmatrix}\Psi = \begin{pmatrix} \beta & 0 \\ 0 & \alpha \end{pmatrix}\frac{\partial^2}{\partial \tilde{x}\partial \tilde{y}}\left(\tilde{x}^{\frac{3}{2}}\zeta_1\right) - \frac{5\gamma^2}{4}\begin{pmatrix} (1+2q) & 0 \\ 0 & q\kappa \end{pmatrix}\frac{\partial}{\partial \tilde{y}}\left(\tilde{x}^{\frac{1}{2}}\zeta_1\right) \tag{16}$$

where
$$\begin{cases} M^{\kappa}_{xx} = \left\{\frac{1}{4}\gamma^2\frac{\partial}{\partial \tilde{x}}\left(\tilde{x}\frac{\partial^2}{\partial \tilde{y}^2}\right) - \frac{15}{8}\gamma^2(1-\kappa)\tilde{x}^{-1}\frac{\partial}{\partial \tilde{x}} - \frac{7}{8}\gamma^2\left(1+\frac{20\kappa}{7}\right)\frac{\partial^2}{\partial \tilde{y}^2}\right\} \\ M^{\kappa}_{yy} = \left\{\frac{1}{4}\gamma^2\frac{\partial}{\partial \tilde{x}}\left(\tilde{x}\frac{\partial^2}{\partial \tilde{y}^2}\right) - \frac{5}{2}\gamma^2(1-\kappa)\tilde{x}^{-1}\frac{\partial}{\partial \tilde{x}} + \frac{3}{8}\gamma^2\left(1-\frac{10\kappa}{3}\right)\frac{\partial^2}{\partial \tilde{y}^2}\right\} \end{cases}$$

and $\alpha = \left\{1 + \left(\frac{3}{8} + \frac{1}{4}q\right)\gamma^2\right\}; \quad \beta = \left\{1 + \left(\frac{13}{8} + \frac{1}{4}q\right)\gamma^2\right\}; \quad \kappa = \frac{4K}{5\gamma^2 ms}$

By eliminating ζ_1 from (16), the first-order field equation of rip currents can finally be obtained as:

$$\left(\hat{P}\frac{\partial^2}{\partial \tilde{x}^2} - \hat{Q}\frac{\partial^2}{\partial \tilde{x}^2}\frac{\partial^2}{\partial \tilde{y}^2} - \hat{R}\frac{1}{\tilde{x}}\frac{\partial}{\partial \tilde{x}} - \hat{S}\tilde{x}\frac{\partial}{\partial \tilde{x}}\frac{\partial^2}{\partial \tilde{y}^2} - \hat{T}\frac{\partial^2}{\partial \tilde{y}^2}\right)\Psi = 0 \tag{17}$$

where
$$\hat{P}=\left\{1+2q\left(1-\frac{\beta\kappa}{2\alpha}\right)\right\}\left\{q(1-\kappa)+\left(q-\frac{1}{2}\right)\right\}+\frac{3q}{2}\left(\frac{4\beta}{3\alpha}-1\right)(1-\kappa)$$
$$\hat{Q}=\frac{q}{5}\left(\frac{\beta}{\alpha}-1\right)$$
$$\hat{R}=\left\{1+2q\left(1-\frac{\beta\kappa}{2\alpha}\right)\right\}\left\{2\left(q-\frac{1}{2}\right)+\frac{3q}{2}(1-\kappa)\right\}+\frac{9q}{4}\left(\frac{4\beta}{3\alpha}-1\right)(1-\kappa)$$
$$\hat{S}=\frac{3q}{5}\left(\frac{2}{3}+\frac{\beta}{\alpha}\right)+2\kappa q\left(1-\frac{\beta}{2\alpha}\right)$$
$$\hat{T}=\left\{1+2q\left(1-\frac{\beta\kappa}{2\alpha}\right)\right\}\left\{q(1-2\kappa)-\frac{1}{2}\right\}+\frac{q}{4}\left(1+\frac{\beta}{\alpha}\right)+q\kappa\left(1-\frac{\beta}{2\alpha}\right)$$

The first-order equation of rip currents in the shoaling zone

In the shoaling zone the governing equations differ from the surf zone equations principally in the wave energy dissipation term and in the formulation of the bottom-friction stresses. In this zone, the driving forces are presented in irrotational form, which may be incapable of generating currents of the first order. The remaining terms in the cross-differentiated equations originate from the friction terms. Consequently, the momentum equations in the shoaling zone play an important role in the decay of currents produced in the surf zone. As was previously indicated in (14), a stream function in the shoaling zone can be equally defined. The momentum equations in the x and y directions and cross-differentiated momentum equations are given respectively by

$$\begin{pmatrix}\frac{2\kappa}{s^{3/2}\tilde{x}} & 0\\ 0 & \frac{2\kappa}{s^{3/2}\tilde{x}}\end{pmatrix}\begin{pmatrix}U_1\\ V_1\end{pmatrix}=-\left(1-\frac{3\gamma_B^2}{8}\right)\begin{pmatrix}\frac{\partial}{\partial\tilde{x}}\\ \frac{\partial}{\partial\tilde{y}}\end{pmatrix}\left(\zeta_1-\frac{B_0}{4}\tilde{x}^{-\frac{5}{2}}\zeta_1+\frac{B_0^{\frac{1}{2}}}{2}\tilde{x}^{-\frac{5}{4}}a_1\right) \tag{18}$$

$$\left(2\frac{\partial^2}{\partial\tilde{y}^2}+\frac{\partial^2}{\partial\tilde{x}^2}-\frac{2}{\tilde{x}}\frac{\partial}{\partial\tilde{x}}\right)\Psi=0 \tag{19}$$

THE SOLUTIONS OF THE FIELD EQUATIONS OF RIP CURRENTS

The solution in the surf zone

The solutions to (17) for the surf zone and (19) for the shoaling zone can be obtained by means of the method of separation of variables. The stream function Ψ to be solved for can be expressed by $\Psi(\tilde{x},\tilde{y})=\Xi(\tilde{x})Y(\tilde{y})$. Substituting this expression into (17), the equation can be transformed into two equations as

$$\left.\begin{aligned}&\left[\left\{1+\frac{\hat{Q}}{\hat{P}}(\lambda\tilde{x})^2\right\}\frac{\mathrm{d}^2}{\mathrm{d}\tilde{x}^2}+\left\{\frac{\hat{S}}{\hat{P}}(\lambda\tilde{x})^2-\frac{\hat{R}}{\hat{P}}\right\}\frac{1}{\tilde{x}}\frac{\mathrm{d}}{\mathrm{d}\tilde{x}}-\frac{\hat{T}}{\hat{P}}\lambda^2\right]\Xi(\tilde{x})=0\\ &\left(\frac{\mathrm{d}^2}{\mathrm{d}\tilde{y}^2}+\lambda^2\right)Y(\tilde{y})=0\end{aligned}\right\} \tag{20}$$

where λ is the separation constant which is the eigenvalue corresponding to the number of rip currents. Introducing the new variable

$$\tilde{\xi} = \left\{1 + \hat{Q}(\lambda \tilde{x})^2 / \hat{P}\right\}^{-1} \tag{21}$$

into the first equation of (20) yields

$$\left[\tilde{\xi}^2(1-\tilde{\xi})\frac{\mathrm{d}^2}{\mathrm{d}\tilde{\xi}^2} - \left\{(\hat{a}+\hat{b}-1)\tilde{\xi} + (2-\hat{c})\tilde{\xi}^2\right\}\frac{\mathrm{d}}{\mathrm{d}\tilde{\xi}} + \hat{a}\hat{b}\right]\Xi(\tilde{\xi}) = 0 \tag{22}$$

where $\quad (\hat{a},\hat{b}) = \left\{(\hat{S}/\hat{Q}-1) \pm \sqrt{(\hat{S}/\hat{Q}-1)^2 + 4\hat{T}/\hat{Q}}\right\}/4\,;\ \hat{c} = (\hat{S}/\hat{Q} + \hat{R}/\hat{P})$

The general solution of this equation can be expressed as a Gaussian hypergeometric function in the form

$$\Xi_{\mathrm{surf}} = A_1\tilde{\xi}^{\hat{a}}\hat{F}(\hat{a},1+\hat{a}-\hat{c};1+\hat{a}-\hat{b};\tilde{\xi}) + A_2\tilde{\xi}^{\hat{b}}\hat{F}(\hat{b},1+\hat{b}-\hat{c};1+\hat{b}-\hat{a};\tilde{\xi}) \tag{23}$$

where A_1 and A_2 are the integration constants to be determined, and the suffix "surf" represents thesurfzone.To solve for regular rip-current spacing as an eigenvalue problem of the equation, a maximum point for (23) must exist in the surf zone. The solutions should have a finite value and a maximum point within $0<\tilde{\xi}<1$. This behaviour can be examined through the following conditions:

$$\Xi(\tilde{\xi})\Big|_{\tilde{\xi}=0} = 0 \quad ; \quad \frac{\partial}{\partial\tilde{\xi}}\Xi(\tilde{\xi})\Big|_{\tilde{\xi}_B\le(\tilde{\xi}=\tilde{\xi}_m)\le 1} = 0 \quad ; \quad \frac{\partial^2}{\partial\tilde{\xi}^2}\Xi(\tilde{\xi})\Big|_{\tilde{\xi}_B\le(\tilde{\xi}=\tilde{\xi}_m)\le 1} < 0 \tag{24}$$

where B indicates the breaking point. It is also confirmed by numerical calculation, that for A_2=0, the conditions will be fulfilled.

The solution in the shoaling zone

The solution in the shoaling zone can be obtained by solving (19) using the method of separation of variables. Introducing the new variables

$$\varphi(\breve{x}) = \breve{x}^{-\frac{3}{2}}\,\Xi(\breve{x}) \quad ; \quad \breve{x} = \lambda\tilde{x}\sqrt{2} \tag{25}$$

Equation (19) is reduced to

$$\left\{\frac{\partial^2}{\partial\breve{x}^2} + \frac{1}{\breve{x}}\frac{\partial}{\partial\breve{x}} - \left(1 + \frac{9}{4\breve{x}^2}\right)\right\}\varphi(\breve{x}) = 0 \quad ; \quad \left(\frac{\partial^2}{\partial\breve{y}^2} + \lambda^2\right)Y(\breve{y}) = 0 \tag{26}$$

The solution to (26) can be expressed using modified Bessel functions of the first and second kinds. By use of (25) the solution can finally be written as

$$\Xi_{\mathrm{shoal}}(\tilde{x}) = B_1\tilde{x}^{\frac{3}{2}} I(\lambda\tilde{x}\sqrt{2}) + B_2\tilde{x}^{\frac{3}{2}} K(\lambda\tilde{x}\sqrt{2}) \tag{27}$$

where B_1 and B_2 are integration constants to be determined, and the suffix "shoal" represents the shoaling zone. To fulfill the condition that $\Xi_{\mathrm{Shoal}} \to 0$ far offshore, therefore, B_1 should be zero.

The boundary conditions and characteristic equation

The eigenvalues can be determined by the matching condition at the breaking point, that is:

$$\frac{d}{d\tilde{x}}\Xi_{surf}(\tilde{x})\Bigg|_{\tilde{x}=\tilde{x}_B} = \frac{d}{d\tilde{x}}\Xi_{shoal}(\tilde{x})\Bigg|_{\tilde{x}=\tilde{x}_B} \quad ; \quad \Xi_{surf}(\tilde{x})\Big|_{\tilde{x}=\tilde{x}_B} = \Xi_{shoal}(\tilde{x})\Big|_{\tilde{x}=\tilde{x}_B} \tag{28}$$

where B indicates the breaking point.

The real and positive eigenvalues λ which satisfy the matching conditions (28) can be determined if the stream functions in both the surf and shoaling zones are decreasing monotonically in the offshore direction, the characteristic equation is given by

$$\frac{\hat{F}(-\hat{b},\hat{c}-\hat{b};1+\hat{a}-\hat{b};\xi_B)}{\hat{F}(1-\hat{b},\hat{c}-\hat{b};1+\hat{a}-\hat{b};\xi_B)} = \left(\frac{\hat{P}}{\hat{a}\hat{Q}}\right)\left\{\frac{(1-\xi_B)/\xi_B}{1+\sqrt{2\hat{P}(1-\tilde{\xi}_B)/\hat{Q}\tilde{\xi}_B}}\right\} \tag{29}$$

By numerical solution of this characteristic equation, the eigenvalues λ can be determined. The eigenvalues λ directly correspond to rip spacing along the shore line.

Determination of the integration constants in the stream functions

The integration constant of the stream function A_{surf} in the surf zone can be determined from the steady-state wave power conservation in the shoaling and surf zones. The wave energy dissipation rate immediately after breaking is different from that in the inner region. This is revealed by the different wave-elevation decay rates. Therefore, the ratio of wave amplitude to local water depth should be taken into consideration in wave energy budgeting. This difference in the rate of wave energy dissipation is responsible for the structure of the nearshore current. Consequently, the wave power contribution to the generation of the nearshore current initiates at the breaking point. The change in wave power occurring in the region from just before to just after the breaking point can be expressed by:

$$\left(\frac{1}{2\pi}\right)^{\frac{1}{4}}\left(\frac{H'_\infty}{d_B}\right)\left(\frac{L'_\infty}{d_B}\right)^{\frac{1}{4}}\left(\frac{d_B}{d_{0B}}\right)^{\frac{5}{4}} =$$
$$s^{\frac{5}{4}}\left[H^{*2}_{0B}C^*_{0B} + s\left\{2H^*_{0B}H^*_{1B}C^*_{0B} + H^{*2}_{0B}\left(C^*_{1B}-U^*_{1B}\right)\right\}+\ldots\right]^{\frac{1}{2}} \tag{30}$$

Kimura, Goto, and Seyama's (1988) experimental work suggests the condition $\gamma_B > \gamma$. Based on this fact, the first-order approximation of (30) is reduced to:

$$\frac{1}{s}\left(\frac{\gamma_B}{\gamma}-1\right)\tilde{x}_B \cong \left\{\frac{a_{1B}}{\gamma}+\frac{1}{2}\left(C_{1B}-U_{1B}\right)\tilde{x}_B\right\} \Rightarrow \gamma_B = \frac{1}{2}\left(\frac{1}{2\pi}\right)^{\frac{1}{4}}\left(\frac{H'_\infty}{d_B}\right)\left(\frac{L'_\infty}{d_B}\right)^{\frac{1}{4}} \tag{31}$$

Using (31), the integration constant A_{surf} can immediately be evaluated, and the stream function Ψ can finally be expressed as

$$\left.\begin{aligned}\Psi(\xi,\tilde{y}) &= A_{surf}\,\xi^{\hat{a}}\hat{F}(\hat{a},1+\hat{a}-\hat{c};1+\hat{a}-\hat{b};\xi)\cos(\lambda\tilde{y})\\ \Psi(\tilde{x},\tilde{y}) &= \mathrm{Re}\left\{(-1)^{-\hat{a}}\frac{\Gamma(\hat{c})\Gamma(\hat{b}-\hat{a})}{\Gamma(\hat{c}-\hat{a})\Gamma(\hat{b})}\right\}\Psi(\xi,\tilde{y})\end{aligned}\right\} \tag{32}$$

The depth-integrated rip current velocity at first-order is now ready to be expressed numerically as:

$$Q(\tilde{x},\tilde{y}) = \mathbf{i}\ dU(\tilde{x},\tilde{y}) = \mathbf{i}\ (s\tilde{x})s^{\frac{1}{2}}\{0 + sU_1(\tilde{x},\tilde{y}) + s^2U_2(\tilde{x},\tilde{y}) + \ldots\} \qquad (33)$$

Theoretical Results And Comparison With Field Data

A theory of the steady nearshore horizontal circulation cells induced by normally-incident waves on a plane beach has been proposed in this paper. The theoretical results predict two main characteristics of rip currents, i.e. the alongshore spacing and the depth-integrated velocity distribution in the seaward direction. The obtained rip current spacing is compared with the theoretical curve of Dalrymple and Lozano (1978) and the field data of Sasaki (1977) and other investigators where the surf similarity of the waves are categorized into instability region. To compare the theoretical results with the field data of Sasaki (1977) and other investigators, their values of the parameter q at the breaking point should be evaluated using their wave characteristics data, and their values of κ should be evaluated using bottom roughness k_e, bottom slope s, and wave length data at the breaking point, then the results are plotted over the theoretical curves. In the evaluation of κ, Kajiura's expression for bottom friction coefficient was used, as Dalrymple and Lozano used and assumed the bottom roughness as k_e = 0.4 mm.

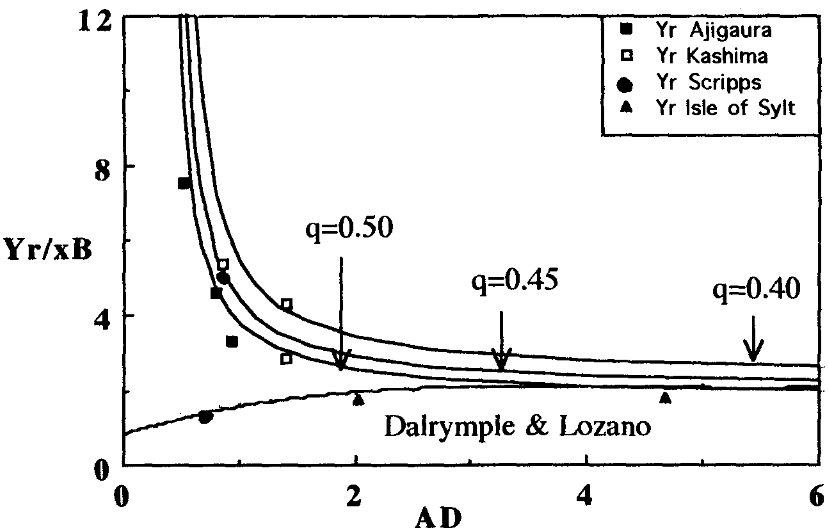

Figure 2. Theoretical curves of dimensionless rip spacing (Y_r/x_B) in terms of Dalrymple number A_D for q-values of 0.40, 0.45, and 0.50 including Dalrymple & Lozano's curve (1978) and rearranged field data of rip currents generated by waves having surf similarity parameter of the instability region.

In comparing the results of this study with those of Dalrymple and Lozano, the parameter κ, which has been previously defined, is transformed into Dalrymple and Lozano's number $A_D = 0.4/\kappa$. Figure 2 shows the curve of dimensionless rip spacing as a function of Dalrymple parameter A_D. The present curves (q=0.5, 0.4, and 0.3) rapidly increase as A_D becomes less than 1, while Dalrymple's curve decreases as A_D goes to 0. The present theoretical results and those of Dalrymple and Lozano differ particularly at small values of A_D. In the course of deriving the characteristics equation in the present eigenvalue problem, the dimensionless rip spacing depends on the values of κ and q. It was defined that q corresponds to wave characteristics and beach slope s, and κ depends on the bottom roughness and wave length at the breaking point. Therefore, we conclude that the dimensionless rip spacing is determined by incoming wave characteristics, i.e.wave height H and wave length L, and morphological characteristics, i.e. beach slope s and bottom surface roughness k_e.

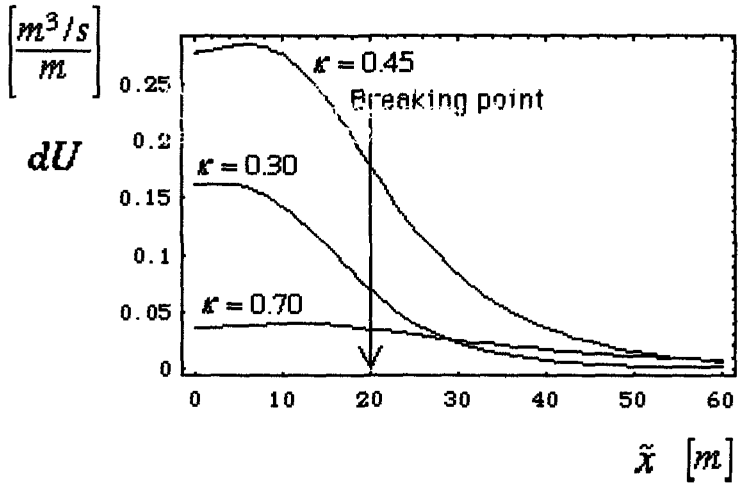

Figure 3. Seaward distribution of rip discharge, where beach slope s = 0.05 and dimensionless rip spacings of 3.12 (κ = 0.30), 4.21 (κ = 0.45), and 7.70 (κ = 0.70) are used.

Figures 3 illustrate the distribution of depth-integrated rip current velocity or seaward rip discharge as a function of seaward distance. The illustrations have dimensionless rip spacings of 7.70, 4.21, and 3.12, a beach slope of 0.05, and q of 0.50.The breaking point is located at 20 meters from run-up line. By using γ_B of 0.50, the numerical value of A_{surf} gives values of rip discharges with reasonable values at both laboratory and field scales.

CONCLUSIONS

In this study a new mathematical model for steady-state horizontal nearshore circulation has been developed, in which the wave-current interaction is taken into consideration. The basic equations consist of the depth-integrated equations for conservation of mass, momentum, wave action, and wave number. The wave-current interaction includes not only wave refraction due to currents but also the work done by radiation stresses against the mean current.

The basic equations are decomposed into the leading order and the instability order using the beach slope as a perturbation parameter. The leading order gave the solution for wave set-up in the surf zone and wave set-down in the shoaling zone. The instability order gave linearized field equations of nearshore currents. It is found that the nearshore circulations were generated by the rotational driving forces in the momentum equation. The solutions were characterized by boundary conditions at a shore line, a breaking line, and far offshore. In the surf zone, the solution was represented by the Gaussian hypergeometric function. In the shoaling zone, the solution was represented by the modified Bessel function. The eigenvalues which correspond to the dimensionless rip spacing are obtained from the characteristics equation extracted from the matching condition at the breaking point.

The present theoretical results for the dimensionless rip spacing were consistent with the scatter of the field data of Sasaki in terms of surf similarity parameter of the region of instsbility. The curves suddenly increase in the range $A_D < 1.0$, and tend to be a constant value for larger values of A_D. The field data on dimensionless rip spacings caused by infragravity waves are difficult to predict by this theory. Outside of this range, the theory gives reasonable predictions for rip spacing caused by wave-current interactions.

Rip discharge is illustrated with exact numerical values, using an integration constant which is obtained from energy budgeting at the breaking point. This budgeting was applied at the suggestion of Kimura et.al, that the wave energy dissipation rate immediately after breaking is larger than the dissipation rate in the inner region. The resulting value is within the range of data at both laboratory and field scales.

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