# **CHAPTER 264**

# THE INFLUENCE OF ROLLERS ON LONGSHORE CURRENTS

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### <u>Abstract</u>

A computational model is developed for depth-averaged cross-shore and longshore currents which includes the effects of the surface roller generated by wave breaking. The creation and evolution of the roller itself is modeled explicitly (Dally and Brown, 1995), and convective acceleration terms are included in both the crossshore and longshore momentum equations. Lateral mixing is parameterized in terms of the local cross-shore current and a turbulent eddy viscosity, as proposed by Svendsen and Putrevu (1994); however, a new model for eddy viscosity is proposed which contains contributions from both the roller-induced and bed-induced turbulence The laboratory measurements of quasi-uniform longshore currents reported by Visser (1991) are used to calibrate and verify the model. For driving the model, it is shown that using stream function wave theory produces significantly better results than linear wave theory. Also, comparisons of longshore current distributions with and without the roller terms included show that the roller plays an essential role in faithfully modeling the longshore current. The calibrated model also produces accurate results for the set-up/set-down using stream function theory, for the limited data available from Visser (1991).

# Introduction

Recent investigations of the roller have demonstrated its importance in mean cross-shore currents (undertow); e.g., see Svendsen (1984b), Okayasu et al. (1988), Deigaard et al. (1991), and Dally and Brown (1995). We now focus attention on

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the potentially important influence of the aerated roller on longshore currents, both in driving longshore flows as well as in cross-shore mixing.

For shore normal waves, Dally and Brown (1995) showed that the roller momentum flux ( $R_{xx}$ ) was comparable to the Radiation Stress ( $S_{xx}$ ) associated with the organized wave motion, and that their gradients tend to balance in the outer surf zone. This balance is responsible for the landward shift in the initiation of setup, as well as the landward shift in the location of the peak of the undertow, that have been observed in laboratory data (Bowen, et al., 1968; Nadaoka and Kondoh, 1982). In the present study it will be shown that with obliquely incident waves, the gradients of the cross-product momentum terms ( $R_{xy}$  and  $S_{xy}$ ) also tend to balance in the outer surf zone. This results in a landward shift in the peak of the longshore current, which is also evident in laboratory data (Visser, 1991).

Although lateral mixing of the longshore current has been an important modeling issue since the 1970's, it was not until recently that a physically realistic mechanism for observed cross-shore mixing, i.e. residual convective acceleration induced by the vertical structure of the cross-shore and longshore currents, was proposed by Svendsen and Putrevu (1994). In the present study the influence of the roller, which enhances vertical mixing and consequently *reduces* lateral mixing, will also be modeled and explored.

### General Governing Equations for Longshore and Cross-shore Currents

Assuming steady currents, longshore uniformity, and no flow through the shoreline, a simple equation for the mean, depth-integrated, period-averaged, cross-shore current below the mean water level (U) is developed by integrating the conservation of mass equation. This relation is given by

$$U = \frac{-(Q_{wx} + Q_{rx})}{(h + \overline{\eta})}$$
(1)

The Stokes Drift term  $(Q_{wx})$  is provided by the wave theory selected to drive the model, the roller volume flux  $(Q_{rx})$  is provided by an energy-based roller model, and the still water depth (h) is known. The elevation of the mean water level  $(\overline{\eta})$  is as yet unknown.

Again, assuming steady state conditions, applying longshore uniformity, and neglecting wind stress, the period-averaged, depth-averaged momentum equations for the cross-shore and longshore directions are

$$-\frac{\partial}{\partial x}\left[U^{2}(h+\overline{\eta})\right]+\frac{1}{\rho}\left(\frac{\partial}{\partial x}S_{xx}+\frac{\partial}{\partial x}R_{xx}\right)+g(h+\overline{\eta})\frac{\partial\overline{\eta}}{\partial x}=\frac{1}{\rho}\left(\overline{\tau_{zx}}\Big|_{-h}\right)$$
(2)

and

$$\frac{\partial}{\partial x} \left( \mathbf{D}_{c} \mathbf{h} \frac{\partial \mathbf{V}}{\partial \rho \mathbf{x}} \right) - \frac{1}{\rho} \left( \frac{\partial}{\partial x} \mathbf{S}_{xy} + \frac{\partial}{\partial x} \mathbf{R}_{xy} \right) = \frac{1}{\rho} \left( \overline{\tau_{zy}} \Big|_{-\mathbf{h}} \right)$$
(3)

where S is the Radiation Stress, R is the roller momentum flux,  $D_c$  characterizes a horizontal mixing mechanism,  $\tau$  is the time-averaged bed stress, and V is the depthaveraged longshore current. The first term in Eq. 3 represents a lateral mixing mechanism due to the vertical structure of the cross-shore and longshore currents (Svendsen and Putrevu, 1994) with  $D_c$  approximated by,

$$D_{c} = \frac{0.5 \text{ h}^{2} \text{U}^{2}}{\nu_{t}}$$
(4)

where  $v_t$  is a turbulent eddy viscosity. The eddy viscosity model developed herein contains vertical mixing introduced by both the aerated surface roller and the bottom, as described below.

A quadratic bed stress model is adopted:

$$\overline{\tau_{zx}}\Big|_{-h} = \rho \frac{c_f}{8} \overline{u_b (u_b^2 + v_b^2)^{\frac{1}{2}}}$$
(5)

$$\overline{\tau_{zy}}\Big|_{-h} = \rho \frac{c_f}{8} \overline{v_b (u_b^2 + v_b^2)^{\frac{1}{2}}}$$
(6)

where  $u_b$  and  $v_b$  are the total instantaneous velocities at the bed. Following Smith et al. (1993)  $c_f$  is the Darcy-Weisbach friction factor, related to Manning's  $n_m$  and the total water depth by

$$c_{f} = \frac{8 g n_{m}^{2}}{(h + \bar{\eta})^{\frac{1}{3}}}$$
(7)

Manning's resistance coefficient has units of  $(s*m^{-1/3})$  and was determined experimentally by Chow (1959) for a wide variety of channels.

To solve the system of equations, the initial conditions of wave height, wave angle, and bottom topography must be specified. The selected wave theory is then used to determine the mass and momentum fluxes while shoaling and refracting the wave to the next grid. The cross-shore mass and momentum equations are next solved for the undertow and the wave induced set-up/set-down. By iterating between the mass equation, the cross-shore momentum equation, and the roller model, the undertow and the set-up/set-down are determined across the entire transect. The present model employs the wave height decay model of Dally, Dean, and Dalrymple (1985), hereafter referred to as the  $D^3$  model.

Once the cross-shore hydrodynamics are computed, the longshore momentum equation is solved with a forward difference scheme. The longshore momentum equation is solved from onshore to offshore because the boundary condition of V=0 must be applied at the shoreline. Equation (3) is solved across the entire transect using an implicit tridiagonal solution method. Because there is a quadratic dependence on V in the longshore mean bed stress, the longshore momentum equation is iterated until V converges to a selected tolerance for all points across the

transect. The solution method outlined in this section is the same regardless of the wave theory used to drive the model.

#### The Roller Model

The roller model recently developed by Dally and Brown (1995) is used to predict the development and decay of the aerated surface roller in the surf zone, and is based on a depth-integrated and period-averaged energy balance equation. For obliquely incident waves, the roller model, under the condition of longshore uniformity, is expressed by

$$\frac{\partial \overline{F_{w}} \cos \theta}{\partial x} + \frac{\partial}{\partial x} \left( \frac{1}{2} M_{R} c^{2} \cos^{2} \theta \right) = -M_{R} g \beta_{d} \cos \theta$$
(8)

where  $\overline{F_w}$  is the depth-integrated, time-averaged, wave-induced energy flux, x is the cross-shore coordinate positive in the shoreward direction,  $M_R$  is the roller mass flux,  $\theta$  is the local wave angle (computed from Snell's law), g is gravity, and  $\beta_d$  is the roller dissipation coefficient.  $\beta_d$  was calibrated by Dally and Brown (1995) using a number of laboratory data sets, and a value of 0.1 yields good agreement when stream function theory is used to compute the driving terms. The volume flux of the roller, used in the conservation of mass equation (Eq. 1), is calculated from

$$Q_{rx} = \frac{M_R}{\rho} \cos\theta \tag{9a}$$

$$Q_{ry} = \frac{M_R}{\rho} \sin\theta \tag{9b}$$

The momentum flux terms for the roller in used in Eqs. 2 and 3 are calculated from

$$\mathbf{R}_{xx} = \rho \mathbf{Q}_{xx} \mathbf{c} \cos \theta \tag{10a}$$

$$\mathbf{R}_{xy} = \rho \mathbf{Q}_{rx} \mathbf{c} \sin \theta = \rho \mathbf{Q}_{ry} \mathbf{c} \cos \theta \tag{10b}$$

#### The Eddy Viscosity Model

It is suggested that two sources of turbulence, i.e. roller-induced turbulence and bottom-induced turbulence, should be accounted for in the eddy viscosity model. The measurements of Battjes and Sakai (1981) and Nadaoka and Kondoh (1982) suggest that as a wave dissipates across the surf zone, the turbulence it creates and leaves behind is similar to a turbulent wake. Consequently, an analogy is made that the turbulent wake produced by the roller of a breaking wave is similar to the turbulent wake produced by a cylinder submerged in a moving fluid. From the theory of turbulent wakes, the eddy viscosity in a turbulent wake  $(v_T)$  can be described by (Tennekes and Lumley, 1977)

$$v_{\rm T} = \frac{\kappa}{R_{\rm T}} U_{\rm o} \Theta \tag{11}$$

where  $\kappa$  is Von Karman's constant (equal to 0.4),  $R_T$  is the turbulent Reynolds number (equal to 12.5 based on experimental observations),  $U_o$  is the free stream velocity, and  $\Theta$  is the momentum thickness of the wake. For a coordinate system moving with the breaking wave, it would seem that the wave celerity (c) is analogous to  $U_o$ . Tennekes and Lumley (1977) show that the momentum thickness for a circular cylinder is approximately half the frontal height of the cylinder, for Reynolds numbers between  $10^3$  and  $3 \times 10^5$ . Thus, if the roller is equated to a cylinder, a logical estimate of the momentum thickness, or '*radius*', of the roller ( $\Theta_R$ ) is given by

$$\Theta_{\rm R} = \frac{1}{\sqrt{\pi}} \sqrt{M_{\rm R} \frac{\rm T}{\rho}}$$
(12)

Completing the cylinder analogy, the roller contribution to the eddy viscosity is given by

$$v_{\rm R} = \frac{0.4}{12.5} c \Theta_{\rm R} \tag{13}$$

where  $v_R$  is uniform over depth, which also follows from turbulent wake theory.

With the upper layer turbulence represented, the bottom-induced turbulent eddy viscosity is now developed. Nielsen (1985) suggests that the length scale of the near-bed eddies should be that of the oscillatory water particle excursion, whereas the velocity scale should be that of a friction velocity. We also suggest that the friction velocity is well-represented by the maximum water particle velocity multiplied by a friction factor. Consequently, it appears that the turbulent eddy viscosity contribution from the bed might be modeled by

$$\mathbf{v}_{\rm B} = \sqrt{\frac{c_{\rm f}}{2}} |\mathbf{u}_{\rm max}| \boldsymbol{\xi}_{\rm max} \tag{14}$$

$$u_{max} = \frac{H\sigma}{2\sinh kh}$$
(15)

$$\xi_{\rm max} = \frac{\rm H}{2} \frac{1}{\sinh kh} \tag{16}$$

where,  $c_f$  is the friction factor discussed in the previous section,  $u_{max}$  is the maximum orbital velocity given by linear wave theory in equation (15),  $\xi_{max}$  is the maximum water particle displacement given by linear wave theory in equation (16), H is wave height,  $\sigma$  is angular frequency, and k is wave number. With the bottom-induced eddy viscosity component modeled, the combination of mixing induced by the roller of the breaking wave and mixing induced by bottom turbulence is given by

$$v_{\rm T} = \left(v_{\rm R}^2 + v_{\rm B}^2\right)^{\frac{1}{2}}$$
 (17)

A comparison of the new eddy viscosity model with the eddy viscosity model proposed by Svendsen and Putrevu (1994) (Figure 1) shows an order of magnitude agreement. The new eddy viscosity model does provide a better behaved transition across the breakpoint, with the point of maximum turbulence located inside the surf zone, whereas the Svendsen and Putrevu (1994) eddy viscosity model has a maximum at the breakpoint.



Figure 1 Comparison of the new eddy viscosity model to Svendsen and Putrevu's (1994) eddy viscosity model for Visser's (1991) Case 4 conditions.

## **Calibration**

The numerical model presented here is compared to the laboratory data set of Visser (1991) for calibration and verification. This data set is unique because it is the only experiment to minimize the recirculation present in enclosed basins by utilizing a longshore pumping method, to develop a uniform longshore current. Calibration of the longshore current model entailed the selection of a Manning's friction factor from a range of values presented by Chow (1959) for open channel flow conditions. The empirical decay coefficient ( $\kappa$ ) and the stable wave height coefficient ( $\Gamma$ ), in the  $D^3$  wave decay model are also calibrated to minimize the error in wave height prediction. Manning's friction factor and the  $D^3$  coefficients are

selected based on their relative least squares error to the measured data defined by equation (18) where B represents the variable in question:

$$\mathbf{E}_{v} = \left(\frac{\sum \left(\mathbf{B}_{\text{meas}} - \mathbf{B}_{\text{calc}}\right)^{2}}{\sum \left(\mathbf{B}_{\text{meas}}\right)^{2}}\right)^{\frac{1}{2}}$$
(18)

Calibration results for the longshore current model are presented for two wave theories in tables 1 and 2. The numerical model results with stream function theory are presented in contrast to the model results for linear wave theory.

	D <sup>3</sup> Coefficients		Least squares error in wave height	
Case #	κ	Г	Stream Function Theory	Linear wave theory
4	0.15	0.2	7.89 %	5.80 %
5	0.15	0.4	12.45 %	16.70 %
6	0.2	0.3	4.09 %	4.51 %
7	0.2	0.2	9.06 %	7.03 %

**Table 1** Selected  $D^3$  wave decay model coefficients and relative least squares errorfrom Visser's (1991) measured data.

Least squares error in longshore current velocity							
Case #	n <sub>m</sub> =0.01	n <sub>m</sub> =0.011	n <sub>m</sub> =0.013				
Stream Function Theory							
4	9.42 %	6.85 %	17.13 %				
5	19.10 %	13.05 %	16.92 %				
6	25.35 %	26.65 %	31.92 %				
Linear Theory							
4	19.02 %	20.16 %	25.29 %				
5	10.85 %	8.84 %	16.71 %				
6	33.13 %	34.97 %	39.59 %				

Least squares error in longshore current velocity							
Case #	$n_m = 0.015$	n <sub>m</sub> =0.017	$n_{\rm m} = 0.02$				
Stream Function Theory							
7	59.32 %	43.89 %	25.75 %				
	Li	near Theory					
7	43.27 %	31.35 %	17.01 %				

**Table 2** Least squares percent error for the range of Manning's frictioncoefficients  $(n_m)$  from Chow (1959) for Visser's (1991) cases 4 through 7.

Because Visser did not measure the cross-shore current distribution, it is impossible to asses the validity of the cross-shore velocity predictions. However, based on the work of Dally and Brown (1995), the cross-shore current is expected to be reasonably represented. Sensitivity testing of the longshore current model shows that the magnitude of the longshore current is not overly dependent on Manning's friction factor. For the smooth bottom, cases 4-6, the normal friction factor value provides accurate results; however, for the gravel bottom in case 7 a value larger than Chow's (1959) suggested maximum provides better results.

Visser's experiments 1-3 were performed on a beach with a slope of 1/10. Because stream function theory is limited by the breaker height to water depth ratio of 0.78, cases 1-3 were not investigated because the breaking waves in these experiments exceeded this ratio. Because case 8 was reported without wave height measurements, this case was also discarded.

### **Results and Discussion**

The most obvious way to asses the effect of the roller on the longshore current is to contrast the results to a model without the roller term. Figure 2 shows the calibrated longshore current model results for Case 4, with and without the roller term in the cross-shore and longshore momentum equations. It is noted that the roller contribution to turbulence is still included. The model that neglects the roller term in the momentum equations shows a dramatic decrease in the longshore current velocity inside the surf zone and the location of the maximum current is shifted offshore. A similar effect occurs in the model predictions for all the cases studied.

To asses the importance of the roller contribution to turbulence in the surf zone, results are presented that neglect the roller contribution to the eddy viscosity model. It is noted that the roller contribution to the longshore momentum equation is included in this comparison so that the effect of the roller turbulence on lateral mixing may be examined exclusively. Figure 3 clearly shows that the roller-induced turbulence has a significant effect on the overall longshore current distribution. The absence of the roller turbulence in the eddy viscosity model decreases the maximum velocity in the longshore current and shifts this point landward. Also, outside the breakpoint, a slight increase in the longshore current is produced without the rollerinduced turbulence included.



Figure 2 Visser's (1991) Case 4 model comparisons with and without the roller contribution to the longshore momentum equation for (a) Stream Function theory and (b) Linear Wave theory.



Figure 3 Visser's (1991) Case 4 model comparisons with and without the roller contribution to the eddy viscosity model for (a) Stream Function theory and (b) Linear Wave theory.

It is important to note that the predicted cross-shore current driven by linear wave theory is much greater than the stream function theory cross-shore current. According to Dally and Brown (1995) linear theory in general overpredicts the Stokes drift, whereas stream function theory provides a better prediction of this wave-induced mass transport. Although Visser (1991) did not measure the cross-shore velocity component, it is reasonable to conclude that the hydrodynamic model that uses stream function theory is the better model for calculating the cross-shore and longshore currents.

### Conclusions

The major findings of this investigation are as follows: (1) The nonlinear mixing term described by Svendsen and Putrevu (1994) provides an adequate amount of lateral mixing, with realistic eddy viscosity values, in comparison to the laboratory data for longshore currents collected by Visser (1991). (2) The addition of roller momentum to the longshore equation of motion is necessary for accurate longshore velocity predictions. (3) The contribution of turbulence from the roller is an important aspect of realistically modeling the turbulent eddy viscosity. These findings affirm the premise that the roller plays a significant role in driving the longshore current and in shifting the maximum longshore velocity inside the breakpoint.

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