CHAPTER 251

A CLOSED-FORM SOLUTION FOR TURBULENT WAVE BOUNDARY LAYERS

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ABSTRACT: A general, closed-form solution is presented to the linearized equation describing the velocity in a turbulent boundary layer. The solution is valid for any type of time-varying free-stream velocity, although the focus of the paper is on oscillatory flows generated by surface waves propagating over the sea bottom. A primary objective is to employ the solution for waves with an asymmetric velocity field, where nonlinear effects are small enough to be neglected in the linearized turbulent boundary layer equation. The general solution is developed to more convenient forms for a few special cases, including a sinusoidally varying free-stream velocity and a velocity given by stream function theory. The solution is tested against two different data sets from oscillatory tunnels; one set involves a sinusoidal and the other set a cnoidal free-stream velocity.

INTRODUCTION

The oscillatory boundary layer that develops when surface waves propagate over the sea bottom has many important implications for flow-dependent phenomena in the coastal zone. Examples of such phenomena are wave energy dissipation due to bottom friction and the initiation and transport of sediment (Grant and Madsen 1986). The velocity changes with elevation at a high rate in the wave boundary layer, which typically causes large shear stresses, high dissipation rates, and strong turbulence intensities. An oscillatory free-stream velocity generates a boundary layer with a characteristic thickness that is much smaller than the corresponding boundary layer under a uni-directional current. Thus, the near-bottom velocity gradients and the associated shear stresses are much larger under oscillatory waves than under a uni-directional current, which suggests that the waves dominate the flow close to the

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bottom in many cases. However, because the oscillatory wave motion often does not give rise to any net transport of for example sediment, but primarily has a stirring effect, a superimposed current may have a marked influence on the net transport rate, although the current itself may be to weak to mobilize and transport the sediment on its own.

In nature the boundary layer under waves will almost always be turbulent (Nielsen 1992). Most simple approaches to calculate the velocity in the wave boundary layer employ the eddy viscosity concept to model the turbulence, where the viscosity is taken to be a function of the elevation above the bottom (Kajiura 1968, Grant and Madsen 1979, Brevik 1981, Myrhaug 1982). These models mainly differ in the formulation of how the eddy viscosity ν varies with the elevation z; the most simple models are based on a linear variation in ν with z (Grant and Madsen 1979), whereas more complex models assume several different layers, each having a separate equation to relate ν and z (Kajiura 1968). In reality, ν should also depend on time and Trowbridge and Madsen (1984a) developed a model where a time-varying eddy viscosity was employed. However, in most models the prediction of the velocity in the wave boundary layer is not overly sensitive to the formulation of ν, and a simple model such as the one suggested by Grant and Madsen (1979) often yields satisfactory results. This is especially true if the velocity calculations are performed with the aim of computing sediment transport rates; existing equations for calculating the sediment transport rate include large uncertainties that do not warrant excessively detailed flow computations (Madsen and Wikramanayake 1991).

The main objective of the present study is to develop a simple, analytical model of the flow in an oscillatory boundary layer under rough turbulent conditions that may be employed for situations where the free-stream velocity is not purely sinusoidal. Such velocity fields are generated by nonlinear shoaling waves, which could produce a net transport of sediment due to the asymmetry in the wave velocity. In deriving the analytical model, it will be assumed that the effects of the nonlinear terms in the momentum equations are negligible, implying that the linearized boundary layer equation may be used (Nielsen 1992). A simple eddy viscosity formulation in accordance with Grant and Madsen (1979) is employed to model the turbulent stresses. The proposed model is tested with data from Jonsson (1980) for a sinusoidal free-stream velocity and with data from Nadaoka et al. (1994) for an asymmetrical free-stream velocity of cnoidal type.

The present study focusses on boundary layer development under oscillatory waves, but the general solution presented for the velocity profile in the boundary layer is valid for any time-varying free-stream velocity. A first-order approach is taken (Grant and Madsen 1979) where the governing equation is linearized in order to obtain a closed-form solution. Thus, for cases where the free-stream velocity is generated by nonlinear waves second-order effects, such as the mass transport in the boundary layer (Trowbridge and Madsen 1984b), are not included in the solution.

Even though the mass transport may be small it could still be of significance for calculating the sediment transport.

THEORETICAL CONSIDERATIONS

General Solution

Employing the simple eddy viscosity model by Grant and Madsen (1979), the linearized turbulent boundary layer (TBL) equation may be written (Nielsen 1992),

$$\frac{\partial}{\partial t}(u_w - u_b) = \frac{\partial}{\partial z}\left\{\kappa u_{*m} z \frac{\partial}{\partial z}(u_w - u_b)\right\} \tag{1}$$

where $u_w(z,t)$ is the velocity in the TBL, $u_b(t)$ the free-stream (wave) velocity, t time, z a vertical coordinate, κ von Karman's constant (=0.40), and u_{*m} a constant, representative bottom shear velocity. With the boundary conditions $u_w=0$ for $z=z_o$, where z_o is the characteristic height of the bottom roughness, and $u_w=u_b$ for $z \to \infty$, Equation 1 has the following general solution,

$$u_w = \int_o^t \frac{\partial}{\partial t}(u_b(t-\xi))\, I_u(\xi,z)\, d\xi + u_{bo} I_u(t,z) \tag{2}$$

where u_{bo} denotes u_b at $t=0$ and,

$$I_u(t,z) = \frac{2}{\pi}\int_o^\infty e^{-\frac{1}{4}y^2\kappa u_{*m} t} \frac{J_o(y\sqrt{z_o})Y_o(y\sqrt{z}) - J_o(y\sqrt{z})Y_o(y\sqrt{z_o})}{J_o^2(y\sqrt{z_o}) + Y_o^2(y\sqrt{z_o})} \frac{dy}{y} \tag{3}$$

in which J_o and Y_o are zero-order Bessel functions of the first and second kind, respectively, and y is a dummy integration variable. Equation 3 corresponds to the solution for a time-independent free-stream velocity employed at $t=0$ (that is, $u_b(t)=u_{bo}$=constant); thus, the solution for any $u_b(t)$ is obtained through the superposition of the response from an infinite number of temporal changes in u_b, as expressed by the convolution integral in Equation 2. Figure 1 illustrates I_u in non-dimensional form, and the curves may be interpreted as velocity profiles at different times indicating the TBL growth through the water column.

The shear stress at any elevation z may be derived from,

$$\frac{\tau}{\rho} = \kappa u_{*m} z \frac{\partial u_w}{\partial z} = \kappa u_{*m} z \left\{\int_o^t \frac{\partial}{\partial t}(u_b(t-\xi))\, I_s(\xi,z)\, d\xi + u_{bo} I_s(t,z)\right\} \tag{4}$$

where,

$$I_s(t,z) = \frac{1}{\pi\sqrt{z}} \int_o^\infty e^{-\frac{1}{4}y^2\kappa u_{*m}t} \frac{-J_o(y\sqrt{z_o})Y_1(y\sqrt{z}) + J_1(y\sqrt{z})Y_o(y\sqrt{z_o})}{J_o^2(y\sqrt{z_o}) + Y_o^2(y\sqrt{z_o})} dy \qquad (5)$$

in which J_1 and Y_1 are first-order Bessel functions of the first and second kind, respectively. The integral $I_s \to \infty$ as $t \to 0$, but the approach towards infinity is slow and the singularity is easy to handle within the convolution integral in Equation 4. However, if $u_{bo} \neq 0$ an infinite shear stress will be obtained at $t=0$, because $u_w=0$ at $z=z_o$ simultaneously as $u_w=u_{bo}$ an infinitesimal distance above. The integrals in Equations 3 and 5 are time-consuming to evaluate, so in order to speed up the calculation of u_w and τ in Equations 2 and 4, respectively, it is convenient to derive look-up tables for I_u and I_s.

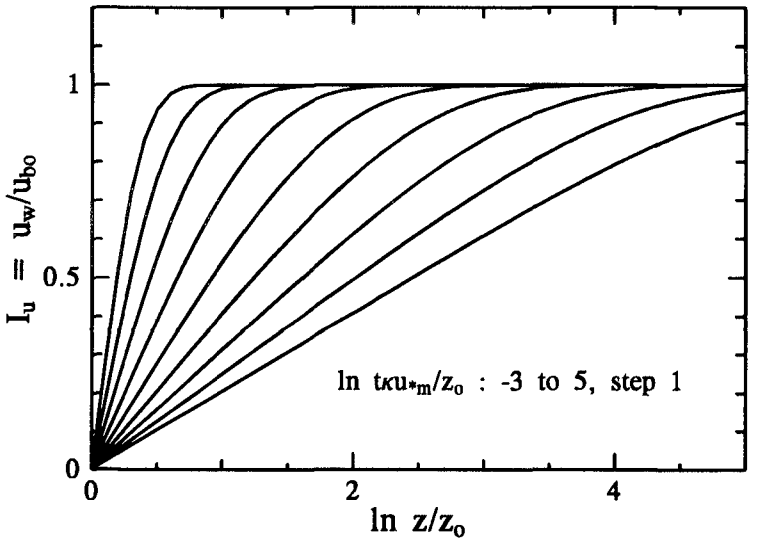

Figure 1. The integral I_u as a function of non-dimensional distance and time.

Equation 2 is a general solution of Equation 1 for any type of free-stream velocity, although from a physical point of view the solution only makes sense for a wave boundary layer where the assumptions behind Equation 1 are applicable. There is only one free parameter in the solution, namely the roughness length scale z_o, which in the case of rough turbulent flow over a flat bed is typically set to $k_n/30$, where k_n is the equivalent Nikuradse sand grain roughness (Grant and Madsen 1979). After it has been defined, the representative shear velocity u_{*m} may be obtained implicitly from the solution. Grant and Madsen (1979) studied the TBL under a sinusoidal free-stream velocity and used the maximum bottom shear stress τ_{bmax} during a wave period to define $u_{*m}=(\tau_{bmax}/\rho)^{1/2}$, where ρ is the water density. For

more complex variations in the free-stream velocity other choices to define u_{*m} may be more appropriate, such as the mean absolute value of the bottom shear stress during a cycle (τ_{bav}).

Sinusoidal Free-Stream Velocity

Equation 2 was derived using Laplace transform technique and for elementary $u_b(t)$ more convenient forms than Equation 2 may be obtained. If the free-stream velocity is sinusoidal and described by $u_b=u_o cos\omega t$, where u_o is the velocity amplitude at the bottom and ω is the angular frequency, the following solution satisfies Equation 1 and the boundary conditions,

$$u_w = u_o \cos\omega t - u_o \frac{N_o(2\sqrt{f\frac{z}{z_o}})}{N_o(2\sqrt{f})} \cos\left(\omega t + \Phi_o(2\sqrt{f\frac{z}{z_o}}) - \Phi_o(2\sqrt{f}\right) - \frac{2u_o}{\pi}\int_o^\infty e^{-\frac{1}{4}\frac{\omega t}{f}y^2} \frac{y^3}{y^4+(4f)^2} \frac{J_o(y\sqrt{\frac{z}{z_o}})Y_o(y) - Y_o(y\sqrt{\frac{z}{z_o}})J_o(y)}{J_o^2(y)+Y_o^2(y)} dy \quad (6)$$

where N_o and Φ_o is the modulus and phase, respectively, of the zero-order Kelvin function $ker_o x + i\, kei_o x$, and $f=\omega z_o/\kappa u_{*m}$. The second term in Equation 6 is a transient that is dampened out quickly for small values on f; in most cases this term is negligible already after a wave period. The shear stress at steady-state conditions is given by (see first part of Equation 4),

$$\frac{\tau}{\rho} = -\kappa u_o u_{*m} \sqrt{f\frac{z}{z_o}} \frac{N_1(2\sqrt{f\frac{z}{z_o}})}{N_o(2\sqrt{f})} \cos\left(\omega t - \frac{\pi}{4} + \Phi_1(2\sqrt{f\frac{z}{z_o}}) - \Phi_o(2\sqrt{f})\right) \quad (7)$$

where N_1 and Φ_1 is the modulus and phase, respectively, of the first-order Kelvin function. Grant and Madsen (1979) previously derived the steady-state part of the solution for a sinusoidal free-stream velocity (see also Kajiura 1968).

To permit rapid evaluation of the maximum bottom shear stress it is convenient to define a wave friction factor f_w from $\tau_{bmax}=0.5\rho f_w u_o^2$ (Jonsson 1966, 1980). In accordance with Grant and Madsen (1979), who defined u_{*m} based on τ_{bmax}, f_w may be obtained through the expression for τ_{bmax} given by Equation 7. An alternative approach would be to use τ_{bav} in u_{*m} instead. This would yield a smaller τ_{bmax}, because the turbulent mixing is less if τ_{bav} is used to define u_{*m}, also implying that f_w is smaller than if the conventional definition of u_{*m} is employed. Figure 2 displays f_w as a function of the bottom excursion amplitude A_b $(=u_o/\omega)$ over the grain size k_n for a u_{*m} defined based on τ_{bmax} and τ_{bav}.

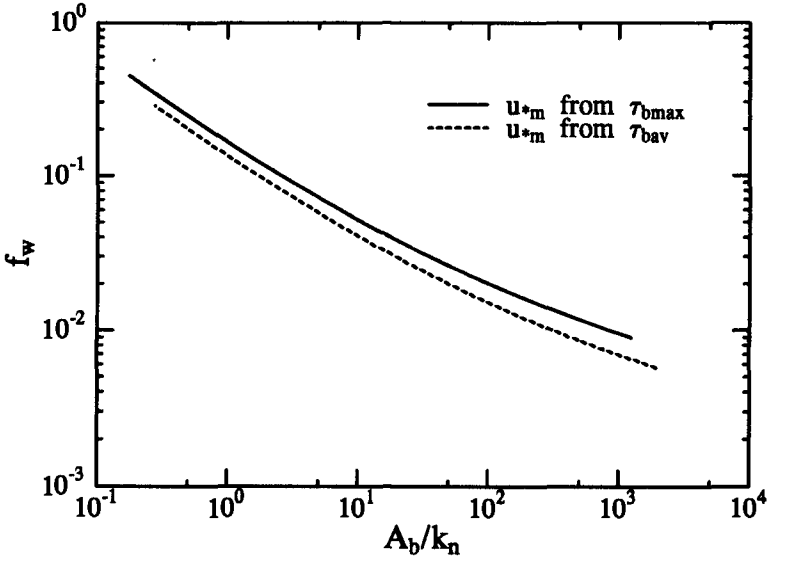

Figure 2. Friction factors for a sinusoidal wave velocity, where the friction velocity is defined based on τ_{bmax} respective τ_{bav}.

Free-Stream Velocity from Stream Function Theory

Stream function theory (Dean 1965) is convenient for describing nonlinear wave properties, because the theory is valid from deep water up to wave breaking. The bottom orbital velocity at a point under a wave described by stream function theory may be expressed as,

$$u_b = -\frac{2\pi}{L}\sum_{n=1}^{N} nX(n)\cos(n\omega t) \tag{8}$$

where L is the wavelength, $X(n)$ stream function coefficients, and N the order of the theory employed. The velocity given by Equation 8 is a sum of sinusoidal components and the steady-state solution to Equation 1 with this u_b is,

$$u_w = \sum_{n=1}^{N} u_n \left\{ \frac{N_o(2\sqrt{nf\frac{z}{z_o}})}{N_o(2\sqrt{nf})} \cos\left(n\omega t + \Phi_o(2\sqrt{nf\frac{z}{z_o}}) - \Phi_o(2\sqrt{nf}) \right) - \cos(n\omega t) \right\} \tag{9}$$

where $u_n = 2\pi nX(n)/L$. The corresponding shear stress is given by,

$$\frac{\tau}{\rho} = \kappa u_{*m}\sqrt{f\frac{z}{z_o}}\sum_{n=1}^{N} u_n\sqrt{n}\,\frac{N_1(2\sqrt{nf\frac{z}{z_o}})}{N_o(2\sqrt{nf})}\cos\left(n\omega t-\frac{\pi}{4}+\Phi_1(2\sqrt{nf\frac{z}{z_o}})-\Phi_o(2\sqrt{nf})\right) \quad (10)$$

A wave described by stream function theory is uniquely defined by the two ratios h/L_o and H/L_o (Dean 1990), where h is water depth, H wave height, and the subscript o denotes deepwater conditions. Waves with identical values on h/L_o and H/L_o yield the same dimensionless velocity $u_b/(H/T)$; thus, the quantity H/T appears as a normalizing "velocity". A friction factor derived for a stream function wave will depend not only on the normalized roughness k_n/H, but also on h/L_o and H/L_o. The friction velocity may be computed by using τ_{bav}, which is obtained from time integration of the absolute shear stress over a wave period T.

RESULTS

Data From Jonsson (1980)

Measurements by Jonsson (1980) of u_w in a water tunnel for a sinusoidally varying u_b with the amplitude u_o was employed as a first step to validate Equation 1 for describing the velocity in the TBL. It was verified that Equation 2 produced identical results to Equation 6, and the steady-state portion of the solution was used for the comparison with the data. Jonsson (1980) presented data for two cases: 1) $u_o=2.11$ *m/s*, $T=8.39$ *s*, $k_n=2.3$ *cm*, and 2) $u_o=1.53$ *m/s*, $T=7.20$ *s*, $k_n=6.3$ *cm*. Comparison between the analytical solution and the measurements was performed for the phases t/T = *1/2, 5/8, 3/4, 7/8,* and *1*. The Reynolds number *Re* for Cases 1 and 2 were 6.0 10^6 and 2.7 10^6, respectively, based on A_b and u_o. The roughness values given by Jonsson were employed and there were no free calibration parameters. Two different definitions of u_{*m} were used in the comparison, namely $u_{*m}=(\tau_{bmax}/\rho)^{1/2}$ and $u_{*m}=(\tau_{bav}/\rho)^{1/2}$.

Figures 3 and 4 display the comparison between the analytical solution and the measurements for Cases 1 and 2, respectively. In general, the difference between the two formulations for u_{*m} is small, although using τ_{bav} seems to consistently produce somewhat better agreement with the data. Some of the overshoot effect in the data is not entirely captured by the analytical solution, especially for Case 2. The computed wave friction factors f_w for Cases 1 and 2 were 0.019 and 0.033, respectively, when τ_{bmax} was employed, and 0.014 and 0.025 when τ_{bav} was used. Using different formulations for u_{*m} will not change u_w as much as it will affect the calculation of the shear stress.

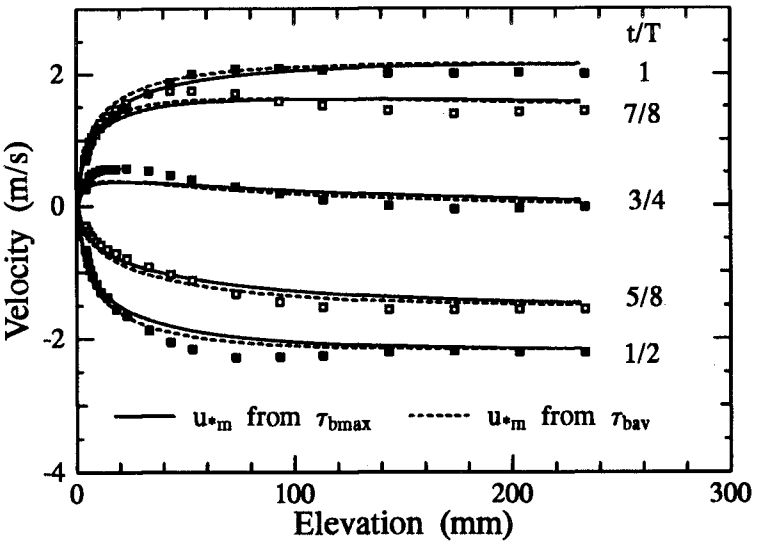

Figure 3. Calculated and measured velocity in the turbulent boundary layer for Case 1 from the data by Jonsson (1980).

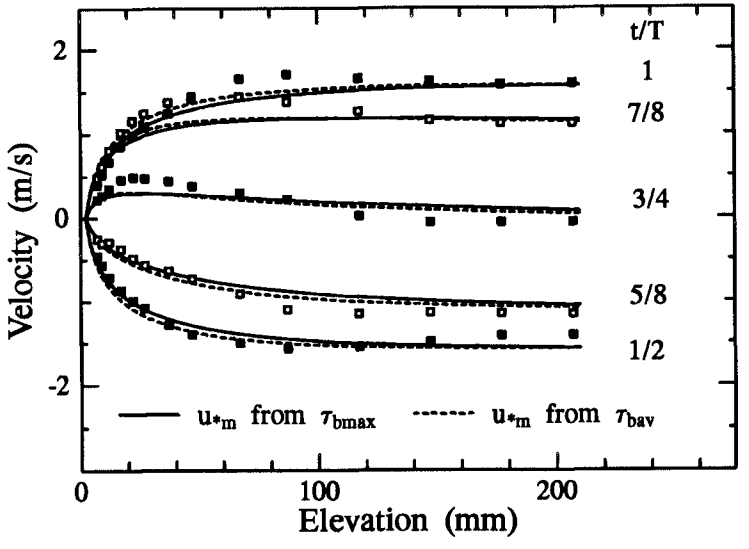

Figure 4. Calculated and measured velocity in the turbulent boundary layer for Case 2 from the data by Jonsson (1980).

Data From Nadaoka et al. (1994)

Nadaoka et al. (1994) measured u_w in an oscillatory tunnel using air for a free-stream velocity that was asymmetric. The measurements used here to evaluate the TBL model involved a velocity u_b that was of cnoidal type with a positive peak velocity of 2.50 m/s, a negative peak velocity of 1.05 m/s, and a period of 5 s. A cnoidal wave producing a non-dimensional time variation in u_b corresponding to the experimental conditions implies an Ursell number of $U_r=57.8$, although during the experiment u_b was generated to agree with the velocity induced by a hyperbolic wave. In general, such a strongly nonlinear wave provides a severe test for the linearized TBL equation; neglecting the nonlinear terms in the governing equation assumes that the particle velocity is small compared to the wave phase speed (Madsen and Wikramanayake 1991), which may not be the case for strongly nonlinear waves. However, for data obtained in oscillatory tunnels the spatial gradients should be small enough to permit that the nonlinear terms are neglected.

Instead of using a cnoidal or hyperbolic wave to describe u_b in the solution given by Equation 2, u_b was approximated using a wave described by 20-order stream function theory ($H=5.26$ m, $T=10.3$ sec, and $h=8.22$ m; the resulting velocity as a function of non-dimensional time is shown in Figure 5 together with the generated hyperbolic velocity variation during the experiment). Stream function, cnoidal, and hyperbolic theory produced essentially identical variation in u_b with time, but the former theory allows direct calculation of u_w from Equation 9 for steady-state conditions without having to compute for the transient phase, which is necessary if the general solution in Equation 2 is employed. The bed consisted of spray-painted aluminum and was judged to be hydraulically smooth during the experiments. Thus, the length scale z_o is independent of the boundary roughness and may be calculated from $z_o=(3.3\nu_a/u_{*m})/30$ (Madsen and Wikramanayake 1991), where ν_a is the kinematic viscosity for air. The air temperature was about 10 deg during the experiment and the corresponding Reynolds number was $Re=2.8\,10^5$. The origin of the vertical axis in the measurements was assumed to approximately coincide with z_o.

Since smooth turbulent flow prevailed during the experiment, z_o could be obtained from ν and u_{*m} and no calibration was needed to estimate the bed roughness. The representative shear velocity was based on τ_{bav}, which was determined through time integration over T. A value of $u_{*m}=0.065$ m/s was thus calculated implying $z_o=0.024$ mm. Figure 6 displays measured and calculated velocity profiles for selected phase values of t/T. The maximum positive peak in u_b occurred at about $0.18t/T$, the maximum negative peak at $0.68t/T$, and zero velocity at $0.36t/T$. The model captures the overall features of the velocity variation in the boundary layer, but the overshoot effect is not well predicted by the model, especially during the phase of flow reversal in the boundary layer in connection with large gradients in the wave velocity. The simple eddy viscosity model employed in the linearized TBL equation is most likely the reason for the disagreement between

the model and the measurements (Sleath 1987), although lack of detailed information on z_o and the use of stream function theory to describe u_b may also contribute to the discrepancy.

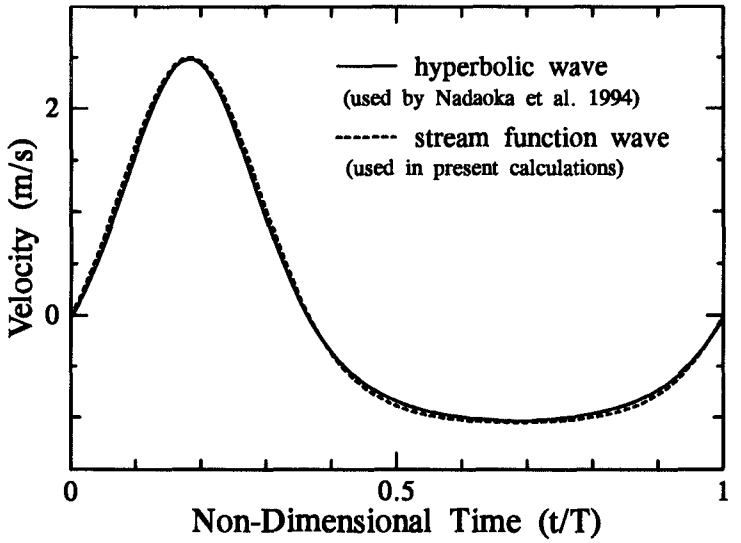

Figure 5. Hyperbolic wave velocity generated by Nadaoka et al. (1994) in their experiment on turbulent boundary layers and the velocity from a stream function wave that approximates the hyperbolic wave.

Figure 7 shows the computed normalized shear stress as a function of time at selected elevations based on the experimental conditions from Nadaoka et al. (1994). The time variation in the shear stress differs significantly from u_b and τ is completely asymmetric. This feature of the shear stress under nonlinear waves is important to include in for example detailed sediment transport calculations that employ the instantaneous shear stress to compute the transport rate.

CONCLUDING REMARKS

The analytical solution presented in Equation 2 is valid for any time-varying free-stream velocity, although from a physical point of view the solution is only interesting when the conditions underlying the linearized TBL equation are fulfilled. For example, the solution will describe the temporal growth of a TBL under a uni-directional current (compare Figure 1); however, the boundary layer will grow infinitely large and at some time the assumption that the layer thickness is small compared to the water depth will be violated and the solution lacks physical meaning. Also, for strongly nonlinear waves where the convective terms in the momentum equations are significant, the linearized TBL equation will yield results that are not

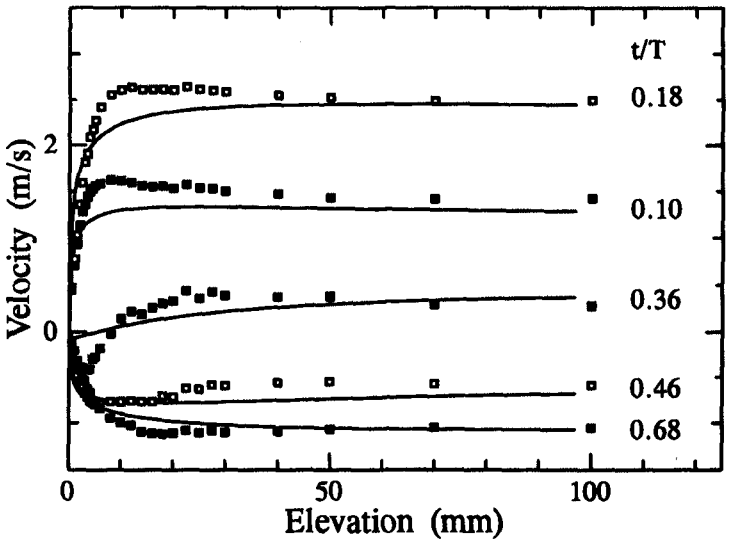

Figure 6. Calculated and measured velocity in the turbulent boundary layer using the asymmetric velocity case from Nadaoka et al. (1994).

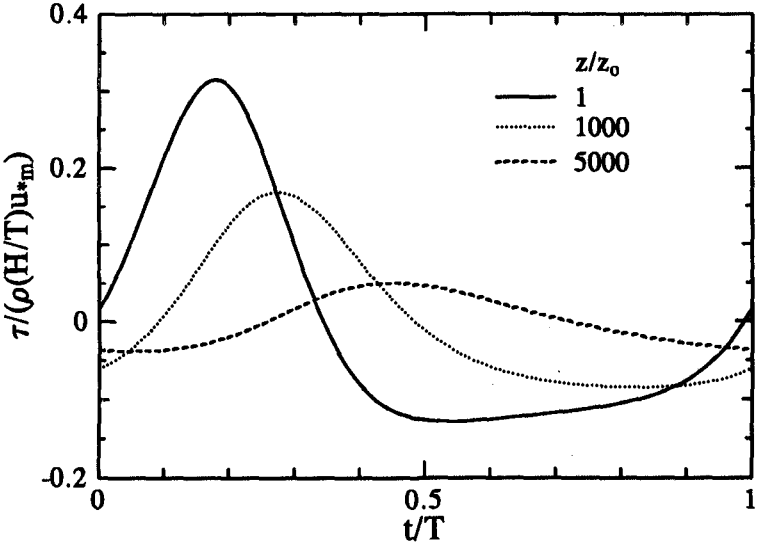

Figure 7. Calculated time variation in the shear stress at selected elevations for the experimental conditions given by Nadaoka et al. (1994).

reliable. For the case tested from the data by Nadaoka et al. (1994) the linearized equation failed to produce accurate predictions, especially during the phase of flow reversal in the boundary layer in connection with large gradients in the wave velocity. This is mainly attributed to the simple eddy viscosity model approach employed which fails to realistically capture the variation in the turbulence during a wave cycle (Sleath 1987). A time-varying eddy viscosity would most likely improve the agreement with the data, but such a formulation would not permit an analytical solution of the governing equations.

However, the simple eddy viscosity model of Grant and Madsen (1979) seemed to produce acceptable results for a sinusoidal wave velocity, even if ν increases without limit with distance from the boundary. This observation pertains mainly to the velocity profile, whereas the shear stress is more sensitive to the eddy viscosity formulation because of the dependence on the velocity gradient. Similarly, the velocity profile is not overly sensitive to the choice of representative friction velocity, in contrast to τ which is more affected by the u_{*_m} used. It was found in the present study that a u_{*_m} based on the mean absolute shear stress during a wave cycle produced somewhat better agreement with data for a sinusoidal wave than using the maximum absolute shear stress.

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