### CHAPTER 200

#### WAVE IMPULSE PREDICTION FOR CAISSON DESIGN

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## **Abstract**

Problems associated with the measurement of wave impact pressures are described and the poor comparison between the results of previous studies is outlined. It is argued that the force impulse form and magnitude are more relevant parameters for caisson design than maximum impact pressures. Data from small and large scale laboratory measurements are used to develop a simplified design approach for caisson breakwaters based on the limited magnitude of the force impulse. Comparison between large scale data and predictions based on small scale results show reasonable agreement.

## Introduction

The form of breakwater selected for a particular site depends upon many physical, economic and political factors. Rubble mound constructions are popular in shallow water because they have proved reliable and easy to repair but they are expensive in deep water. Due to the growth of ocean going vessels and the associated need to provide mooring facilities in deeper water there is increasing interest in vertically sided breakwaters. Historically these have had a masonry construction, although in recent decades cellular caissons have provided a preferred alternative. The most common failure mode of caisson breakwaters has been sliding due to the impact of breaking waves (Oumeraci, 1994, Franco, 1994). Such impacts are highly complex events which can not yet be described theoretically. Small scale laboratory wave impacts are therefore a valuable source of information for both designers and those seeking to improve design methods. Due to the sensitivity of breaker shape to wave and physical boundary conditions and the transient and localised nature of impact pressures, results are frequently test specific. If this problem was successfully addressed then confidence in predictions of prototype loading conditions based on such small scale laboratory tests would be greatly enhanced.

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## Scatter of impulse pressures

Even the pressures generated by nominally identical consecutive breakers of a monochromatic wave train are only predictable in a statistical sense. Hayashi and Hattori, (1958) stated that "The initial shock pressure varies so enormously from test to test that it almost seems to have nothing to do with our theoretical considerations". There are many possible reasons for this, some due to random fluctuations in wave form, others due to test conditions and procedures.

Denny (1951) investigated high frequency random fluctuations in the wave form and took measurements of peak pressure from waves of various heights with two different levels of disturbance. This showed that the distribution of peak pressures normalised to wave height was identical for waves of different heights and similar disturbance, but for waves with less disruption, the distribution included much higher relative pressures. This effect may to due to differences in entrapped aeration, variations in the velocity field or some other process. The degree of disturbance and therefore the degree of pressure attenuation will depend on test facilities such as the wave generator and absorption system, and test procedures such as whether a solitary or regular wave train is tested, and how long the water is left to calm between tests.

Variation in pressure records between experiments will be compounded by effects specific to the wave channel and others resulting from the choice of equipment and procedure. The breaker shape depends, for the two dimensional case, mainly on the incident wave conditions, period, height and depth, and the physical boundary conditions mainly foreshore topography and permeability. Any difference in these between tests could be expected to alter impact pressures. The characteristics of the pressure transducers and their position are also significant because peak pressures are generally highly transient and localised. The number of transducers, their size and positions are significant to the spatial resolution, whilst frequency response and sample rate influence temporal resolution.

## Impulse repeatability

The wide variation in peak pressure was first addressed by Bagnold who noted that the pressure rise impulse (see Figure 1) was far more repeatable. This observation was used to develop a concept of an 'effective length' of a piston like body of fluid extending horizontally into the wave from the front face. Because it was believed that the generation of the impulse was caused by momentum focusing through the pocket, it was the pocket dynamics and essentially its thickness which determined the impulse duration. Bagnold was unable to prove his model by measuring the air pocket. Also it is now known that an air pocket is not a necessary condition for the occurrence of an impulse load. The work has since been developed by other researchers, including Kirkgoz who measured a similar effective length for flat fronted waves as Bagnold found for air pocket waves, although with wide variation, and Weggel, et al (1970) who tried to show, but were unable to establish, that effective length varied with wave steepness.

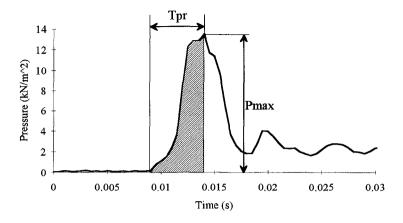


Figure 1. Definition of pressure rise impulse (shaded area)

## Pmax and Tpr

The consistency of wave pressure impulse has often been expressed as a relationship between the maximum impact pressure Pmax, and the time for the pressure to rise to it, Tr of the form;

$$Pmax = K.Tpr^H$$

where K is a constant. Published examples include;

 $Pmax = 232Tpr^{-1}$ , Weggel *et al*, (1970).

The upper envelope of 12 laboratory 2D measurements with a beach slope 1/20 and a discontinuous wave wall. Regular waves were used to form one breaker type, the heights and periods are not specified although the steepness is given as 70 mm/sec. Six pressure transducers were used which were located 19 mm apart. No information is provided on their characteristics or sample rate or the measures taken to ensure a calm water surface.

## $Pmax = 250Tpr^{-0.9}$ , Kirkgoz (1990).

The best fit line to 70 laboratory 2D measurements with a beach slope of 1/10 and a continuous wave wall. Regular waves were used in still water depth of 0.61 m with an average height of 0.259 m and a 2 second period, these conditions produced plunging breakers. 10 pressure transducers with a 19 mm active surface diameter at 30 mm centres were sampled at 40 kHz. 1 hour was left between each test to allow the water surface to calm.

# **Pmax = 261Tpr** $^{-0.65}$ , Witte.

The upper limit function to approximately 100 laboratory 2D events with a beach slope of 1/6. Regular waves of height 0.25 m and period 2 seconds were generated in a still water depth of 0.61 m (depth at wall 0.16 m). 25 waves were generated per train to prevent re-reflection. Steep, vertical and air pocket breaker types were examined. 14 pressure transducers with a minimum gap of 0.015 m and surface diameter of 3.7 mm were sampled at 62.5 kHz.

**Pmax = 400Tpr**<sup>-0.75</sup>, Hattori *et al*, (1994).

The upper limit function to a large number of laboratory events including a range of breaker shapes. The beach slope was 1:20 and the water depth at the wall was 50 mm. Regular wave trains were generated with an energy absorbing paddle with heights which varied from 40 to 120 mm and periods of 1.5, 1.7 and 2.0 seconds. Six pressure transducers were sampled at 5 kHz and located at 10 mm centres around still water level.

**Pmax = 485Tpr**<sup>-1</sup>, Weggel *et al*, (1970).

The test conditions were as described above, except the wave steepness was 17.9 mm/sec.

 $Pmax = 3100Tpr^{-1}$ , Blackmore and Hewson (1984).

The upper limit function to approximately 80 full scale marine wave impacts on a sea wall. The wave heights ranged from 0.8 to 1.3 m with periods of from 2.67 to 8.5 seconds. The waves had mostly broken at impact. Seven 25 mm diameter pressure transducers were positioned at around 1 m intervals.

These functions appear because, as Bagnold noted, the pressure impulse is limited, if Pmax is large, then Tpr tends to be small and *vice versa*. There are too many variables between each test to determine the effect of each on K and H although there seems to be some relationship between K and the size of the wave. For example the constant of 3100 calculated from the prototype sea wave impacts of Blackmore and Hewson is significantly higher than those derived from the laboratory data. The fact that this relationship between Pmax and Tpr has been different in each case is probably due, at least in part, to the difficulties of recording consistent impact pressures between tests.

This pressure impulse appears to be a more consistent feature of impact loading that the pressure maxima, it may therefore be a more reliable basis for design if the structure response to the impulse is properly understood.

## Structural Dynamics.

Structural response to loading is fundamentally important to design. Goda used a dynamic model to develop his method for breakwater design (Goda, 1974 and 1994) which has since become the standard in many countries. Oumeraci and Kortenhaus (1994) created a similar model which was calibrated with near prototype measurements recorded in the Grosser Wellen Kanal (GWK), see below. This was used to show how the structure responded to different force time histories. It was demonstrated that, for impacts without significant air pocket generated force oscillations, the dynamic response could be accurately predicted by simplifying the force time history to a triangular impulse described by the maximum force, Fmax, the impulse duration, Td and the rise time Tr. The results were presented in the form of dynamic amplification factors, ratios of force maxima to the equivalent static load which would have caused the same displacement (Fstat). The dynamic amplification factors depend upon the ratios of Td and Tr to the natural period (Tn) of the structure

in question. In this way structural dynamic response can be accounted for and the problem for the designer is reduced to ensuring integrity under a known static load.

There are a series of problems to be overcome before adopting this approach. The form and magnitude of the force impulse has to determined from the incident wave conditions and the physical boundary conditions. At present this can only be achieved through physical model tests. The natural variation in the magnitude and form of the impulses measured during such tests must therefore be accounted for and a reliable and appropriate method of scaling must be determined.

#### Scale effects

A comparison of similar wave impacts at significantly different scales have been conducted by Sakakiyama (1994) for the case of a submerged breakwater in front of a vertical wall, and by Fuhrboter (1986) for sloping revetments. To the authors knowledge no such data has been published for the case of a vertical wall experiencing direct wave breaking. The case for the existence of scale effects appears to be that field measurements provide relatively smaller peak impact pressures than have been measured in the laboratory and scaled with the Froude law. Aeration is often suggested as a probable cause although it seems probable that the above mentioned difficulties associated with measuring consistent impact pressures are at least partly responsible.

#### Aeration

Entrained aeration has a strong effect on the compressibility characteristics of water. It affects the rate of momentum exchange between the wave and wall (Blackmore and Hewson, 1984) and reduces the speed of sound in the fluid, altering the propagation of pressure waves (Topliss, 1994, Griffiths, 1994). It depends strongly on the chemistry and biology of the water. Bubbles of air in fresh water tend to be formed larger than in sea water due to increased surface tension, they also coalesce more readily and burst more easily at the surface. No direct measurements have yet been made of breaker aeration so its effects can only be assumed, however it has been shown that artificial aeration can reduce impact pressures (Crawford et al, 1994, Walkden et al, 1995).

If entrained aeration does affect impact pressures this does not necessarily mean that the impulse magnitude is affected. The force impulse represents the momentum of a proportion of the wave. Although aeration reduces water density, the wave mass remains essentially the same due to bulking. Also the wave kinematic pattern is probably not strongly effected, there is therefore reason to believe that the wave impulse might be reasonably independent of entrained aeration and so possibly free of associated scale effects.

## Physical experiments

Measurements of wave impact forces were conducted at both small and near prototype scale in order to develop a way of accounting for wave impulse loading, for impacts without significant air pocket oscillations. The following subjects were investigated;

The magnitude and form of the force impulse,

Linearisation of the impulse,

The significance of K and H,

Scale effects.

Transformation from dynamic loads to equivalent static loads.

## University of Plymouth (UoP) small scale wave impact tests

Experiments were conducted in a 20 m long, 1.2 m deep and 0.9 m wide flume fitted with an energy absorbing wedge type wave paddle (Bullock and Murton, 1989). Waves shoaled over a 1:4.5 sloping impermeable foreshore and broke onto a vertical wall made of 25 mm acrylic sheet. Five sealed gauge Kulite type 219 pressure transducers with an active surface diameter of 18 mm were used to record the pressures at the wall at 2 kHz. Three of these had a pressure range of from 0 to 100 kN/m², the others were designed for pressures of up to 400 kN/m². The transducer centres were 40, 55, 70, 90 and 110 mm above the toe of the wall, although during experiments the transducer at 90 mm was damaged. Data was recorded on a P.C. by means of a Microlink logger. Video records were made of each test.

## GWK large scale wave impact tests

Experiments were conducted in Europe's largest wave flume, the GWK. The physical boundary conditions were not geometrically similar to those of the UoP flume but it was felt that adequately similar breaker geometry would be attainable. The GWK is a large concrete 'U' channel, with its base at ground level, internally it is 324 m long, 5 m wide and 7 m deep. Waves broke onto a sand filled concrete caisson mounted on a rubble foundation over a shallow sloping sand bed. Instrumentation was mounted on and built into a spar which was then attached to the caisson face. Four Kulite type 219 pressure transducers were installed which had a pressure range of 0 to 400 kN/m<sup>2</sup> (see above). They were positioned at 200 mm increments, 0.72 to 1.32 m from the top of the rubble mound. An additional pressure record was made with one of the transducers belonging to the Franzius Institut which was already built into the caisson and located 0.54 m above the rubble mound. Cabling was contained within the spar to protect it from the impacting waves. Pressures were recorded at 2 kHz. Video records of each test from two static and one mobile camera provided information on the breaker forms. Thirty eight tests were conducted in total in water depths ranging from 3.1 m to 3.3 m, including regular and pseudo random wave trains and solitary waves. Wave heights ranged from 0.5 to 1.1 m with periods from 3.5 to 5.5 seconds.

### Results

Due to space limitations only the results from two tests will be shown, one conduced in the UoP flume, and one carried out in the GWK. In the former case 350 waves of height (H) 0.11 m, period 1.25 s and 79 mm water depth at the wall, broke with a near vertical, slightly overhanging wave front.

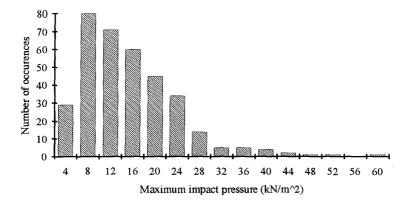


Figure 2. Maximum impact pressures from one transducer (UoP Data)

The data shown in Figure 2 illustrates the typically large spread in Pmax. The mean value is  $13.4~\text{kN/m}^2$ , with a range from  $2.3~\text{kN/m}^2$  to  $58.6~\text{kN/m}^2$ . If this test had been conducted in order to model a design wave impact it might be concluded that the design event may produce a pressure maxima of anything from 2.13~pgH to 54.3~pgH. It is not clear how such results should be interpreted.

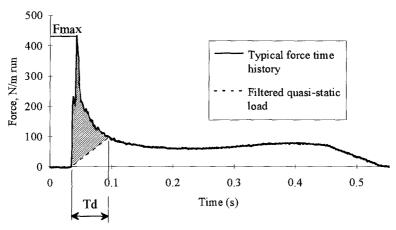


Figure 3. Typical force time history, the shaded area represents the impulse (UoP data)

In order to obtain the impulses, the force time histories were first calculated by spacewise integration of the pressures. The impulse is a dynamic load involving rapid variations in force, therefore a simple low pass filter was used to remove the quasi

static force component, see Figure 3. The distribution of the impulse was found to be compact, Figure 4 shows the results from the UoP test, normalised to the total momentum exchange caused by the wave at the wall. This implies that accounting for the impulse magnitude, with for instance a statistical confidence limit, may be fairly straightforward.

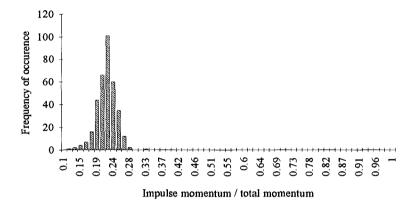


Figure 4. Frequency distribution of force impulse normalised to total momentum (Impulse momentum fraction, UoP data)

Partly because of this consistency, if the data is represented in the Fmax Td domain it clusters around a similar function to those found in the Pmax Tpr domain, as shown in Figure 5.

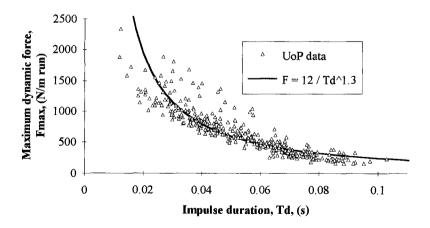


Figure 5. Relationship between maximum dynamic force and impulse duration

The values of Fmax and Td in Figure 5 are not suitable to be used to predict the structure response following the method described by Oumeraci and Kortenhaus. This is because the impulse is linearised to a triangle with base Td and height Fmax. Some of the data shows both a high peak force and a long duration which if used as a basis for inearisation results in an unrealistically high impulse magnitude and an overprediction of structure response. This is because the form of the impulse tends to be concave as the force drops, as can be seen in Figure 3. In order to realistically predict structure response the magnitude of the impulse must be correct.

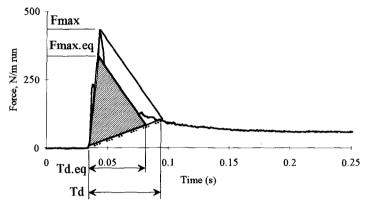


Figure 6. Method of processing the linearised force impulse

It was therefore decided to process the values of Fmax and Td to parameterise both the magnitude and the form of the impulse. Figure 6 demonstrates the method which was used. Td.eq and Fmax.eq are equivalent values which were calculated so that the area of the triangle they described is equal to the impulse magnitude, and the triangle they form is similar to the original linearised impulse. An alternative more ideal approach might account for the relationship between removed impulse frequencies and the natural frequency of the structure. The data in Figure 7 has been processed in this way.

The power of Td.eq in the best line fit in Figure 7 (shown with the dashed line) found through least squares regression, is 0.92. It can be seen that Td.eq<sup>-1</sup> also provides a good fit. If the latter value is used then the constant assumes the value 16.3 and has units of Ns/m run or impulse per metre and is twice the average impulse magnitude. This data can now be used to account for dynamic response in the way Oumeraci and Kortenhaus suggest because both the impulse magnitude and its form are parameterised by Fmax.eq and Td.eq.

## Comparison with large scale test results

Similarity of physical boundary conditions between the large and small scale experiments was not possible. The small scale foreshore was impermeable and had a

regular slope, whereas the large scale foreshore was permeable and included a berm. If the incident wave conditions were geometrically similar then the breaker shapes would not be and no comparison between the results would be meaningful. It was therefore decided to base a scale comparison on the breaker shape and the wave momentum. The video records were used to match similar breaker shapes at small and large scale, then the wave momenta were used to obtain the scale ratio. The average momentum measured for the UoP waves was 39.2 Ns/m run and for the GWK data, 4545 Ns/m run, which provides a length scale of 6.7.

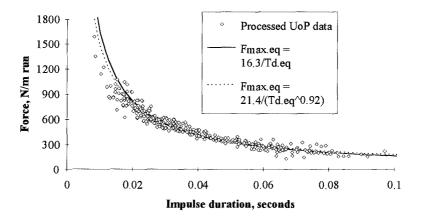


Figure 7. Impulse momentum and shape parameterised by F eq and Teq

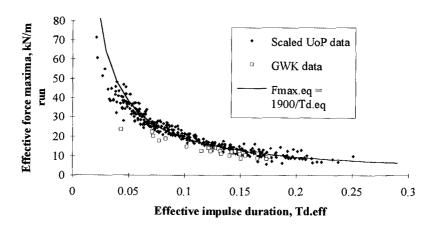


Figure 8. Comparison between scaled UoP data and GWK data

The values of Fmax.eq and Td.eq measured in the UoP flume were scaled according to Froude and compared to values obtained from the GWK test in Figure 8. The match is not perfect but when the lack of boundary condition similarity is considered the results might be considered close.

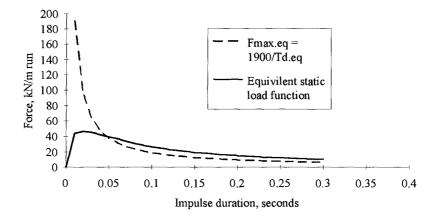


Figure 9. Equivalent static load function calculated from the dynamic amplification factors of Oumeraci and Kortenhaus, 1994. It is assumed that Tn = 0.1 and Tr = 0

In Figure 9 the dynamic force function Fmax.eq = 1900/Td.eq has been multiplied by a dynamic amplification function to obtain the equivalent static load function. The maxima of this (in this case 45 kN/m run) might be considered the largest equivalent static force the impulse can generate and therefore the design load for this breaker.

## Conclusions and discussion

- (I) Maximum impact pressures are highly variable and difficult to compare between tests. This is due to differences in test procedure and conditions as well as natural random scatter.
- $(\Pi)$  The magnitude of the force impulse is more consistent and more relevant to structural stability than a transient and localised pressure maxima.
- (III) The form of the force impulse is highly variable, this may be due to aeration effects, variations in breaker kinematics or some other unknown effect.
- (IV) Clustering occurs in the Fmax Td domain due to points (II) and (III). This can be described with the dynamic force function  $Fmax = K.Td^H$ , where K is a constant.
- (V) If the impulse is linearised to parameterise its magnitude and form by Fmax.eq and Td.eq then K is twice the mean impulse area, and H assumes the value -1.
- (VI) Scale effects can be reduced if experiments are conducted in large flumes however unless sea water is employed, similarity in aeration characteristics and therefore compressibility can not be assumed.

- (VII) It was not possible to properly assess scale effects on the impulse because of a lack of geometric similarity, however a reasonably close comparison between large scale results and predictions based on small scale tests was obtained suggesting that scale effects on the impulse may be minimal.
- (VIII) If Tr is assumed to be zero, and the structure natural frequency is known, then the maxima of the product of the dynamic force function and a dynamic amplification function might be considered the maximum static load for design purposes.

The analysis and interpretation presented is highly simplified and is intended to illustrate the basis of an approach. Areas of necessary further development include;

- (I) Allowing non zero rise times.
- The rise time can be measured fairly simply and used to select a more appropriate, dynamic amplification function. The ratio Tr/Td may be affected by scale.
- (II) Accounting for significant air induced force oscillations.
- The entrapment of a long and large oscillating air pocket is recognised as resulting in a significant and dangerous loading condition. It is probably a relatively rare event because it requires a normal direction of approach and a relatively clean wave form. A similar danger may arise due to group dynamics of bubble clouds.
- (II) The mean dynamic force function was used to predict equivalent static loads. For design purposes the loading should be calculated on the basis of the worst case with a chosen probability of occurrence within the design life. If the model test conditions have been chosen to represent events with a known return period then the smooth distribution of measured impulses should allow a confident estimate of a realistic design impulse magnitude.
- (III) The method of impulse definition.

The use of a low pass filter to define the impulse seems to work well. The cut off frequency should be decided upon with a consideration of the way the structure will respond to that frequency. Similarly the process of obtaining Td.eq and Fmax.eq could be developed with more consideration of the structure dynamics.

(IV) Directionality.

The waves angle of approach has not been dealt with although its inclusion may be relatively straightforward. This method is based on momentum transfer at the plane of the caisson face which might be expected to vary relatively simply with angle of approach.

(V) Accounting for the structure response.

The conversion of dynamic loads to equivalent static loads has been based on the dynamic amplification factors published by Oumeraci and Kortenhaus (1994). These were derived by reducing the description of the caisson motion to a simplified single degree of freedom model. This was intended to "constitute a useful guide for judgement in developing design load specifications". There are many limitations which have been discussed in their paper regarding their model and modelling of structural dynamics in general. In the light of their comments it would be preferable, though more complex, to use a numerical model rather than a dynamic amplification function to account for structure response.

(VI) Assessment of the effect of breaker shape on the impulse momentum fraction. This varies with breaker type so it will reach a measurable maxima at some breaking condition. This might be multiplied by the design wave momentum in order to calculate a worst case impulse momentum. From this the constant K is obtained, and so the dynamic load function is known. The effect of variations in the incident wave and physical boundary conditions on the impulse momentum fraction should be easier to asses than for impact pressure maxima because of its more compact distribution. It is also far less sensitive to the characteristics of measurement equipment and test procedures. For these reasons scale effects may be also be easier to asses.

The main advantage of this approach is that it provides a means of interpreting the results of physical model tests in a way which is meaningful for structural design. This means that nature can be left to resolve the complexities of the breaker shape, a problem which can not yet be fully solved through numerical modelling.

## Future work

Measurements of entrained and entrapped air are in progress at the University of Plymouth in order to examine their effect on the impulse form and magnitude and wave impact pressures. This work is being conducted both in the laboratory and in the field (Crawford, et al 1994, Walkden, et al, 1995 and Bird et al, 1997). It is hoped that the results will help explain the distribution of data along the dynamic load function, provide insights into the relationship between entrapped air and the force impulse and provide data with which to test possible scale effects.

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