CHAPTER 183

Resonant Reflection and Refraction-Diffraction of Surface Waves due to Porous Submerged Breakwaters

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ABSTRACT: A wave equation, taking account of the effects of porous medium, is transformed into coupled parabolic equations. It is assumed in the theory that the mean water depth and the thickness of porous layer are slowly varying and the bottom undulation is rapidly varying compared to the wavelength of surface waves. Though we can utilize the Bragg reflection to reduce transmitted waves behind breakwaters in one-dimensional case, wave heights behind breakwaters do not always decrease due to wave refraction-diffraction in horizontal two-dimensional case. By adding a dissipation term to the coupled parabolic equations, we can calculate the wave breaking deformation.

INTRODUCTION

Porous submerged coastal structures are superior from the view points of seascape, water quality conservation, and fishery resources. If artificial reefs for fish habitat, consisted of blocks, are arranged to form a suitable bar field, we can expect a function of wave control as well as the function of creating fishery resources. Such porous bar fields make wave reflection and transmission smaller than those by impermeable bar fields.

Davies and Heathershaw (1984) studied the refleetion from sinusoidal undulation over a horizontal bottom and derived a solution of reflection coefficient. Mei (1985) and Naciri and Mei (1988) developed theories of wave evolution at and close to the resonant condition by shore-parallel sinusoidal bars and two-dimensional doubly sinusoidal undulations over a slowly varying topography. Kirby (1986) derived a general wave equation which extends the mild slope equation of Berkhoff (1972). These existing theories don't take account of the effects of seabed permeability. Izumiya (1990) obtained an extended mild slope equation for waves propagating over a permeable submerged breakwater. However, since the assumption that the slope of the structure is very gentle is employed, the theory cannot be applied to the case of seabed with rapidly varying undulations.

In this study, a wave equation over porous rippled beds (Mase and Takeba, 1994), taking into account the effects of porous medium, is transformed into coupled para-

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bolic equations of forward- and backward-scattering waves. Numerical calculations are carried out to examine wave transformations or the Bragg scattering by a group of porous submerged breakwaters over constant and sloping beaches in horizontal two-dimension.

WAVE EQUATION OF ELLIPTIC TYPE

Mase and Takeba (1994) and Mase et al. (1995) derived a wave equation over porous rippled beds:

$$\nabla_h \cdot \left(\alpha \nabla_h \hat{\phi}\right) + \alpha k^2 \hat{\phi} - \frac{\cosh^2 kh_s}{D^2}(1-\gamma)\nabla_h \cdot \left(\delta \nabla_h \hat{\phi}\right) = 0 \tag{1}$$

where

$$\alpha = \frac{1}{4kD^2}\left\{\cosh^2 kh_s \sinh 2kh \left(1 + \frac{2kh}{\sinh 2kh}\right) + \gamma \sinh 2kh_s (\cosh 2kh - 1)\right.$$
$$\left. + \gamma^2 \sinh^2 kh_s \sinh 2kh \left(1 - \frac{2kh}{\sinh 2kh}\right) + \gamma \sinh 2kh_s \sinh 2kh \left(1 + \frac{2kh_s}{\sinh 2kh_s}\right)\right\} \tag{2}$$

$$D = \cosh kh_s \cosh kh \left(1 + \gamma \tanh kh_s \tanh kh\right) \tag{3}$$

$$\gamma = n/(\tau + if) \tag{4}$$

$$\omega^2 = gk \frac{\tanh kh + \gamma \tanh kh_s}{1 + \gamma \tanh kh \tanh kh_s} \tag{5}$$

where $\hat{\phi}$ is the complex amplitude of velocity potential, δ is the rapidly varying undulation, h and h_s are slowly varying water depth and thickness of porous layer, ∇_h is the horizontal gradient operator, f is the linearized friction factor, n is the porosity, τ is the inertia coefficient, and ω is the angular frequency. The effects of the porous medium are taken into account through the complex wavenumber k given by Eq.(5) and the complex coefficients α and γ.

Eq.(1) contains the existing models such as the mild slope equation by Berkhoff (1972) and the general wave equation by Kirby (1986), see Mase and Takeba (1994).

COUPLED PARABOLIC EQUATIONS

We need the boundary condition along a closed curve surrounded an analytical domain to solve Eq.(1) of elliptic type. In problems of predicting wave transformations over sloping topography, we cannot set the shoreward boundary condition a priori. A parabolic approximation method developed by Radder (1979) is useful for such problems. Here, following Kirby (1986), we transform Eq.(1) into coupled parabolic equations.

Multiplying Eq.(1) by the gravity acceleration g makes the dimension of αg same as that of CC_g (phase velocity × group velocity). Hereafter, αg and $g\cosh^2 kh_s / D^2$ are described as α and β, respectively. Manipulating Eq.(1) yields

$$\{\alpha-\beta(1-\gamma)\delta\}\phi_{xx}+\{\alpha_x-\beta(1-\gamma)\delta_x\}\phi_x+\alpha k^2\phi+(\alpha\phi_y)_y-\beta(1-\gamma)(\delta\phi_y)_y=0 \quad (6)$$

and Eq.(6) is rewritten as

$$\phi_{xx}+\nu^{-1}\nu_x\phi_x+\nu^{-1}\alpha k^2\phi+\nu^{-1}(\alpha\phi_y)_y-\nu^{-1}\beta(1-\gamma)(\delta\phi_y)_y=0 \quad (7)$$

where

$$\nu=\alpha-\beta(1-\gamma)\delta \quad (8)$$

$$\nu_x=\alpha_x-\beta(1-\gamma)\delta_x+O(k\delta)^2 \quad (9)$$

$$\nu^{-1}=\alpha^{-1}\left\{1+\frac{\beta}{\alpha}(1-\gamma)\delta+O(k\delta)^2\right\} \quad (10)$$

Let's introduce the following pseudo-operator:

$$\mu^2\phi=k^2\left[\left\{1+\frac{\beta}{\alpha}(1-\gamma)\delta\right\}\phi+\frac{1}{k^2\alpha}(\alpha\phi_y)_y-\frac{\beta(1-\gamma)}{k^2\alpha}(\delta\phi_y)_y\right]+O(k\delta)^2 \quad (11)$$

Treating μ^2 to be a numerical value and taking the square root of it gives

$$\mu\phi=k\left[\left\{1+\frac{\beta}{2\alpha}(1-\gamma)\delta\right\}\phi+\frac{1}{2k^2\alpha}(\alpha\phi_y)_y-\frac{\beta(1-\gamma)}{2k^2\alpha}(\delta\phi_y)_y\right]+O(k\delta)^2 \quad (12)$$

Now we express the potential ϕ as a sum of the forward-scattered potential, ϕ^+, and the backward-scattered potential, ϕ^-, as

$$\phi=\phi^++\phi^- \quad (13)$$

and express their derivatives as

$$\phi_x^+=i\mu\phi^++F(\phi^+,\phi^-) \quad (14)$$

$$\phi_x^-=-i\mu\phi^--F(\phi^+,\phi^-) \quad (15)$$

where $F(\phi^+, \phi^-)$ is a coupling term, and μ is a kind of wavenumber. Using Eq.(11), Eq.(7) is expressed as follows:

$$\phi_{xx}+\frac{\nu_x}{\nu}\phi_x+\mu^2\phi=0 \quad (16)$$

$F(\phi^+, \phi^-)$ is found to be

$$F\left(\phi^+, \phi^-\right) = -\frac{(\mu\nu)_x}{2\mu\nu}\left(\phi^+ - \phi^-\right) \tag{17}$$

by substituting Eqs.(13)~(14) into Eq.(16). The pseudo-operator $\mu\nu$ can be given by

$$\mu\nu = k\left\{\alpha - \frac{\beta}{2}(1-\gamma)\delta\right\} + O(k\delta)^2 \tag{18}$$

Differentiating Eq.(18) with respect to x yields

$$(\mu\nu)_x = (k\alpha)_x - \frac{1}{2}\left\{k\beta(1-\gamma)\delta\right\}_x \tag{19}$$

Using Eq.(12) and Eqs.(17) ~ (19), Eqs.(14) and (15) can be expressed as follows:

$$\begin{aligned}\phi_x^+ = ik\Bigg[&\left\{1 + \frac{\beta}{2\alpha}(1-\gamma)\delta\right\}\phi^+ + \frac{1}{2k^2\alpha}\left(\alpha\phi_y^+\right)_y \\ &- \frac{\beta}{2k^2\alpha}(1-\gamma)\left(\delta\phi_y^+\right)_y\Bigg] - \left\{\frac{(k\alpha)_x}{2k\alpha} - \frac{\beta}{4\alpha}(1-\gamma)\delta_x\right\}\left(\phi^+ - \phi^-\right)\end{aligned} \tag{20}$$

$$\begin{aligned}\phi_x^- = -ik\Bigg[&\left\{1 + \frac{\beta}{2\alpha}(1-\gamma)\delta\right\}\phi^- + \frac{1}{2k^2\alpha}\left(\alpha\phi_y^-\right)_y \\ &- \frac{\beta}{2k^2\alpha}(1-\gamma)\left(\delta\phi_y^-\right)_y\Bigg] + \left\{\frac{(k\alpha)_x}{2k\alpha} - \frac{\beta}{4\alpha}(1-\gamma)\delta_x\right\}\left(\phi^+ - \phi^-\right)\end{aligned} \tag{21}$$

The relation between the potential amplitude and the wave amplitude is

$$\phi^+ = -\frac{ig}{\omega}Ae^{ik_0x} \tag{22}$$

$$\phi^- = -\frac{ig}{\omega}Be^{-ik_0x} \tag{23}$$

where k_0 is a reference wavenumber. Substituting Eqs.(22) and (23) into Eqs.(20) and (21) yields

$$\begin{aligned}&A_x + \left[ik_0 - ik\left\{1 + \frac{\beta(1-\gamma)\delta}{2\alpha}\right\} + \frac{(k\alpha)_x}{2k\alpha} - \frac{\beta(1-\gamma)\delta_x}{4\alpha}\right]A \\ &- \frac{i}{2k\alpha}\left(\alpha A_y\right)_y + \frac{i\beta(1-\gamma)}{2k\alpha}\left(\delta A_y\right)_y = \left\{\frac{(k\alpha)_x}{2k\alpha} - \frac{\beta(1-\gamma)\delta_x}{4\alpha}\right\}Be^{-2ik_0x}\end{aligned} \tag{24}$$

$$B_x + \left[-ik_0 + ik\left\{1 + \frac{\beta(1-\gamma)\delta}{2\alpha}\right\} + \frac{(k\alpha)_x}{2k\alpha} - \frac{\beta(1-\gamma)\delta_x}{4\alpha} \right] B$$
$$+ \frac{i}{2k\alpha}(\alpha B_y)_y - \frac{i\beta(1-\gamma)}{2k\alpha}(\delta B_y)_y = \left\{ \frac{(k\alpha)_x}{2k\alpha} - \frac{\beta(1-\gamma)\delta_x}{4\alpha} \right\} A e^{2ik_0 x} \qquad (25)$$

Eqs.(24) and (25) are the coupled parabolic equations for forward- and backward-scattered waves. Detail deviation is seen in Mase et al. (1995).

BRAGG SCATTERING DUE TO POROUS SUBMERGED BREAKWATERS

Procedure of numerical calculations

Eqs.(24) and (25) were finite-differentiated by the Crank-Nicolsen method. The procedure of numerical calculations is as follows:

1) Setting $B = 0$ in the right hand side of Eq.(24), we calculate Eq.(24) for A in the forward direction;
2) Using the calculated A for the right hand side of Eq.(25), we calculate Eq.(25) for B in the backward direction;
3) Using the calculated B in the previous step, we solve Eq.(24) for A;
4) Using the calculated A in the prcvious step, we solve Eq.(25) for B.

The procedure of 3) and 4) is repeated until getting convergence of the calculated results of A and B. A preliminary calculation revealed that four repetition was enough to reach the convergence.

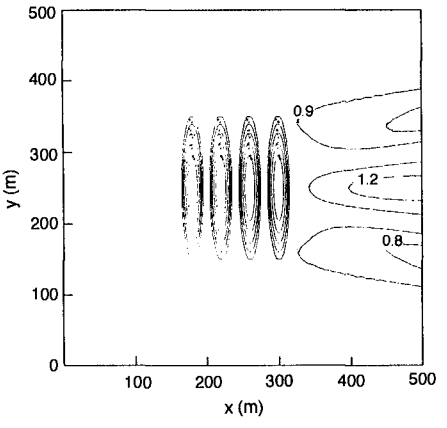
Numerical conditions

The following conditions were adopted in the numerical calculations:

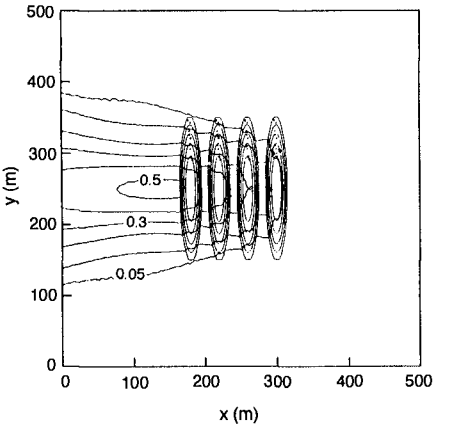
1) The analytical domain is 500 m × 500 m;
2) The model beaches are constant water depth of 8 m, and 1/25 uniform slope;
3) Impermeable and permeable submerged breakwaters of elliptic shape (see, Mase et al., 1995), are installed at the interval of 40 m over constant and sloping beaches;
4) The height of the breakwaters is changed by 1.5 m and 2.5 m;
5) Characteristics of porous medium are selected as $n = 0.4$, $\tau = 1.0$, and $f = 1.0$;
6) Waves propagate in the direction of x axis. The incident wave amplitude is 1 m, and the wave period is changed by 8 s, 10 s, and 12 s.

Calculated results and discussions

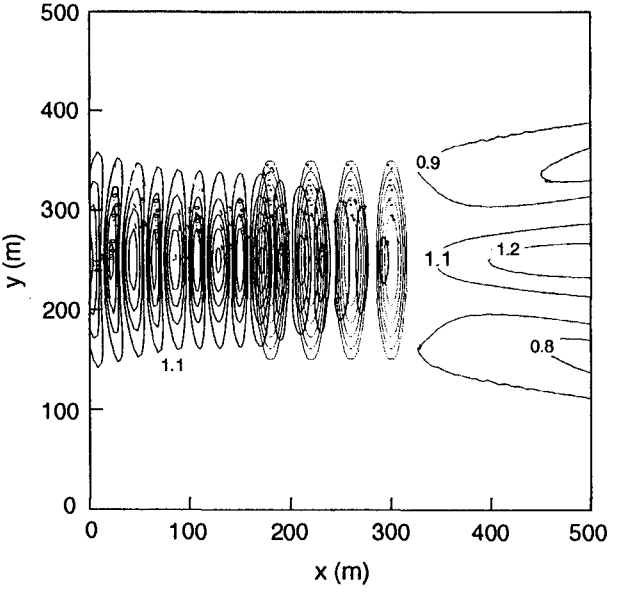
When the wave period is 10 s, the resonant Bragg reflection condition is satisfied in the constant water depth. The calculated results to be shown hereafter are those in the case of wave period of 10 s. Figure 1(a) shows the contour of the forward-scattered wave amplitude, Fig.1(b) the backward-scattered wave amplitude, and Fig.1(c) the total wave amplitude for the case of impermeable submerged breakwaters of which height is 2.5 m. It is seen from Fig.1(a) that the amplitude becomes large behind the elliptic breakwaters similar to the case of a large shoal. Fig.1(b) indicates that the breakwaters generate the reflected waves. In Fig.1(c), the two-dimensional standing wave pattern can be seen.



(a) Incident Wave Amplitude



(b) Reflected Wave Amplitude



(c) Total Wave Amplitude

Fig.1 Wave Transformation due to Impermeable Submerged Breakwaters over Constant Water Depth

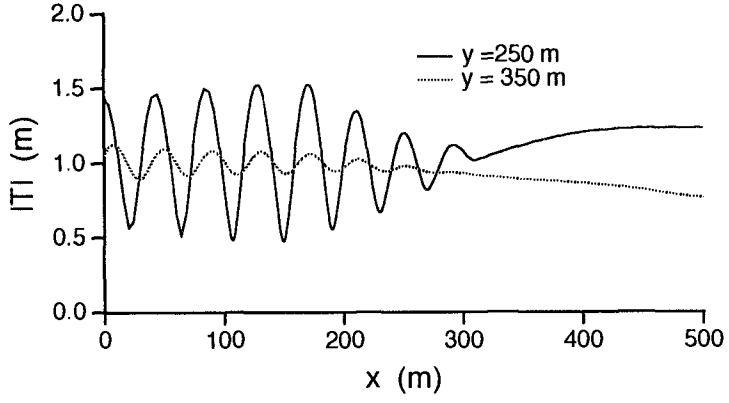
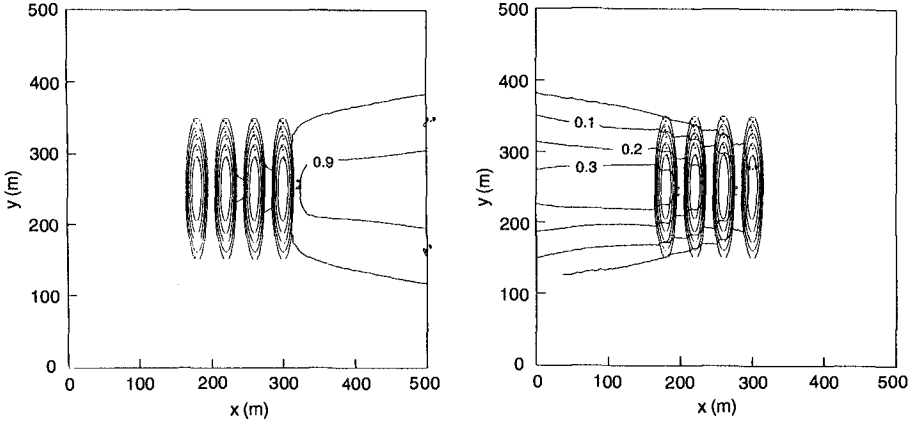


Fig.2 Spatial Change of Total Wave Amplitude along Two Lines

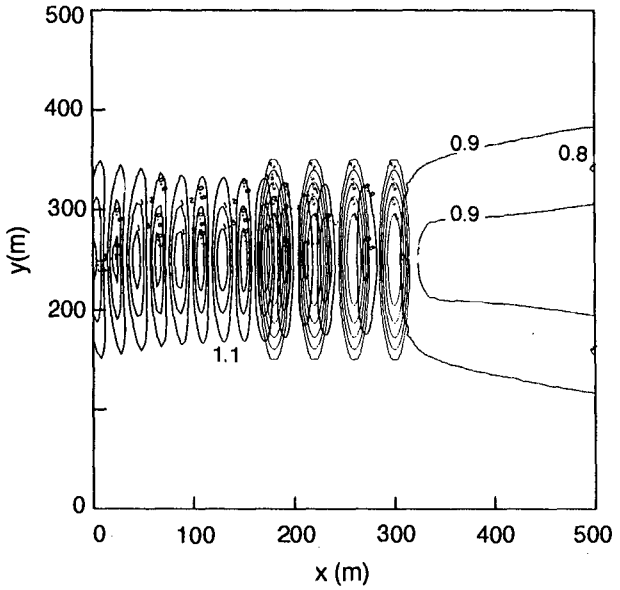


(a) Incident Wave Amplitude

(b) Reflected Wave Amplitude

Fig.3 Wave Transformation due to Permeable Submerged Breakwaters over Constant Water Depth (Continued)

The spatial distribution of the wave amplitude along $y = 250$ m and $y = 350$ m is shown in Fig.2. In an one-dimensional case, we can see that the transmitted waves downstream the ripples are reduced by utilizing the Bragg resonant scattering (Mase and Takeba, 1994); however, in a two-dimensional case, the wave height behind submerged breakwaters does not become small.



(c) Total Wave Amplitude

Fig.3 Wave Transformation due to Permeable Submerged Breakwaters over Constant Water Depth

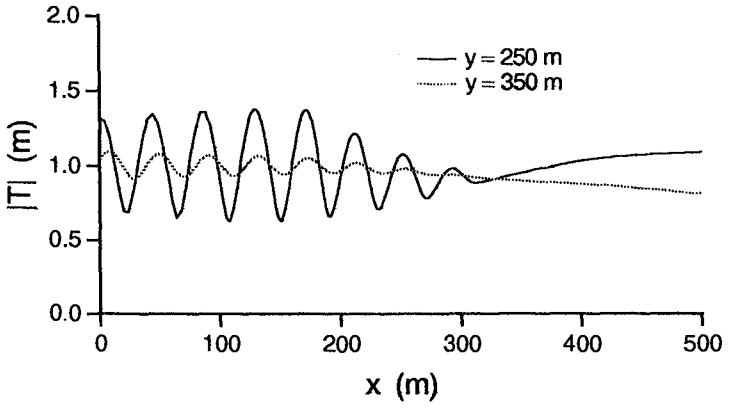


Fig.4 Spatial Change of Total Amplitude along Two Lines

Figure 3 shows the result of the case of permeable breakwaters of 2.5 m in height, where the figure (a) is the contour of the forward-scattered wave amplitude, the figure (b) the back-scattered wave amplitude, and the figure (c) the total wave amplitude. Figure 4 is the spatial change of total wave amplitude along $y = 250$ m (solid line) and $y = 350$ m (dotted line). Comparing these figures with those of Figs.1 and 2, we can see that the standing wave pattern and the increase in wave height behind the breakwaters are weakened due to energy dissipation in the porous medium.

Within the surf zone, wave energy is dissipated. To include the energy dissipation due to wave breaking in the coupled parabolic equations, an energy dissipation term is required. Dally et al. (1985) proposed an energy dissipation model which assumed that there is a stable wave height after breaking equal to some fraction of the water depth and that the rate of energy dissipation in the surf zone is proportional to the difference between the actual wave energy flux and the stable wave energy flux, $(EC_g)_s$. The model is as follows:

$$\frac{d(EC_g)}{dx} = -W = -\frac{K}{h}\left\{EC_g - (EC_g)_s\right\} \tag{26}$$

where E is the wave energy. The stable wave height is given by $H_s = \gamma h$. In this study,

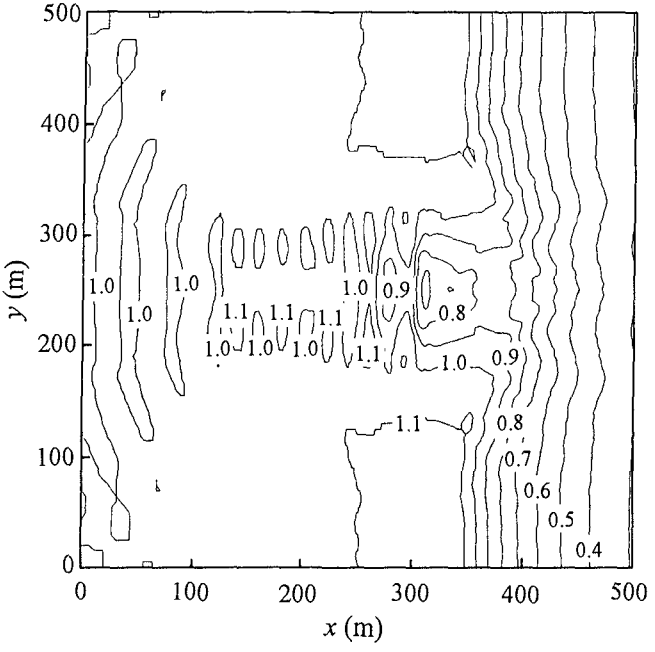


Fig.5 Wave Transformation due to Permeable Submerged Breakwaters over Sloping Beach

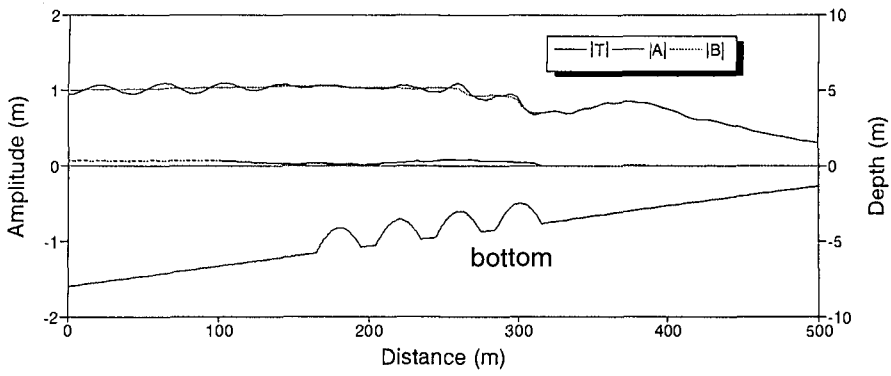


Fig.6 Spatial Change of Total Amplitude along Line of $y = 250$ m

$K = 0.15$ and $\gamma = 0.4$ were adopted. And the wave breaking condition of $H_b = 0.78h$ was employed, where wave height H is defined as $2|A|$.

A coefficient of energy dissipation is defined as

$$W_r = W / E \tag{27}$$

In order to include energy dissipation in the coupled parabolic equations, the term of $W_r A/(2C_g)$ was added in the left hand sidc of Eq.(24).

Figure 5 shows the wave transformation over a uniform sloping beach existing a group of permeable submerged breakwaters. Figure 6 shows the spatial distribution of wave amplitude along the line of $y = 250$ m. Wave breaking occurs around $x = 300$ m, and wave begins to decrease and again increases toward the shore. Second wave breaking occurs around x=380m, and wave amplitude continues to decrease.

CONCLUSIONS

In order to deal with wave transformations due to permeable submerged breakwaters, we developed a wave equation of elliptic type taking account of the effects of porous medium, and the wave equation was transformed into coupled parabolic equations. It was assumed, in the theory, that the mean water depth and the thickness of porous layer were slowly varying and the bottom undulation was rapidly varying compared to the wavelength of surface waves.

Numerical examples of the Bragg scattering were shown in horizontal two-dimensional case. Wave amplitudes became large behind a group of submerged breakwaters due to the wave refraction-diffraction, even when the resonant Bragg reflection condition was satisfied. When the breakwaters were permeable, the standing wave pattern and the increase in wave height behind the breakwaters were weakened due to energy dissipation in the porous medium.

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