CHAPTER 170

Overtopping of waves at a wall: a theoretical approach

M. Jervis [1] and D.H. Peregrine [2].

Abstract

The flow of water due to the overtopping of a vertical wall by waves is modelled. The waves are computed with an accurate irrotational flow solver. The case where a jet of water is projected up the face of the wall is considered. A simple estimation is made from the computation of the amount of water that can pass over the crest of a wall of finite height. The results for overtopping volume per wave show a roughly exponential decaying dependence on the height of the wall above the still water level. Results are given for waves of differing height and also for various sizes of berm in front of the wall. The effects of surface tension are included to investigate the possibility of errors in scaling experimental results to prototype scale. These are larger than expected.

Introduction

In many locations sea walls and breakwaters are built to prevent water from the open sea spreading inland, or disturbing harbours and their installations. There are many reasons for estimating how much water may overtop a wall of given height in given wave conditions. With sea walls, erosion of the back side may be the greatest danger, whereas damage to people and port installations may be of concern behind a breakwater.

Previous study of overtopping at vertical walls has been almost entirely based on small scale experiments (e.g. Franco 1994, Juhl & Sloth 1994). This is a theoretical study of wave overtopping. We focus on the case where waves slosh against a vertical wall sending a jet of water up to a height which may be as much as three times the height of the incident wave. Numerically accurate irrotational flow computations model the upward jet, then a simple model described in the next section is used to estimate how much of the water in the jet would overtop a wall of given height.

The results we present here are all for steep solitary waves meeting a wall. The solitary wave is the largest wave that can propagate on water of a given depth. As well as considering a vertical wall, bounding water of constant depth,

[1] Research student, School of Mathematics, University of Bristol.

[2] Professor of Applied Mathematics, School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK. (*D.H.Peregrine@bristol.ac.uk*)

cases of walls with berms are also presented. There is interest in several aspects of the hydrodynamics of such flows, but we concentrate on the overtopping. The effect of a berm depends on both its horizontal extent and height. Small low berms have little effect. Larger berms change the wave behaviour.

There are two types of behaviour characteristic of extreme standing waves, or waves reflecting (Thais & Peregrine, in preparation). One is the thin sheet like jet that we see in figure 1, but a berm can also give the other almost "table-like" elevation as in figure 6, or wave breaking at or before the wall. The computations show strong variation of overtopping depending on the berm size, but are not capable of dealing with the cases of wave breaking.

A further variation is introduced to give an indication of scale effects. Often, experiments are performed on rather shallow water, e.g. 0.1m or less especially when a three-dimensional wave field is being modelled. In such cases surface tension is likely to have a significant influence on the tip of wave jets. By including surface tension in the program to calculate the incident wave and jet this scale effect is quantified and found to be more important than originally expected.

Overtopping model

The program used is that of Cooker, Peregrine, Vidal & Dold(1990) which is based on the accurate irrotational flow solver of Dold & Peregrine(1986). The computation assumes inviscid, irrotational and incompressible flow, solving Laplace's equation with a boundary-integral method, and using the fully non-linear boundary conditions to time step the computation. High-order numerical approximations are used.

The reflection of waves by a vertical structure such as a breakwater means that the waves in front of the structure are well modelled by standing or near-standing waves if no breaking occurs. Breakwaters are sometimes designed so as to guarantee the formation of standing waves in front of the structure. In this way the chances of the occasional very high impact pressures which might lead to structural failure are reduced. The appearance of the standing waves becomes more like a reflecting solitary wave as water depth is decreased. The largest waves that can approach any structure are shallow water waves whose crests are very similar to solitary waves. Thus we use solitary waves as incident waves for the examples here.

Impact on a vertical wall is modelled by considering the symmetrical collision of two steep solitary waves. That is, the initial condition in the computation corresponds to two accurate solitary waves heading towards each other, but sufficiently far apart that there is no initial interaction between them. An initial solitary wave of any height can be accurately modelled using Tanaka's (1986) method. The program can include simple deformations to a flat bed, such as beaches and half ellipses. A berm of quarter-elliptical shape in front of the wall is modelled by placing a semi-ellipse on the bottom directly below the line of symmetry of the surface, where the two solitary waves collide. Results for such cases are also reported. For sufficiently steep waves a vertical jet forms at the wall (see figure 1).

We are not able to explicitly model a wall of finite height using the present code. Substantial changes would be needed to do this. However, we make use of a feature of common to all jets arising from unsteady waves. That is once the jet forms the pressure field within the jet is very weak. This means that almost all the motion of water in the jet is close to purely inertial. Thus we assume that

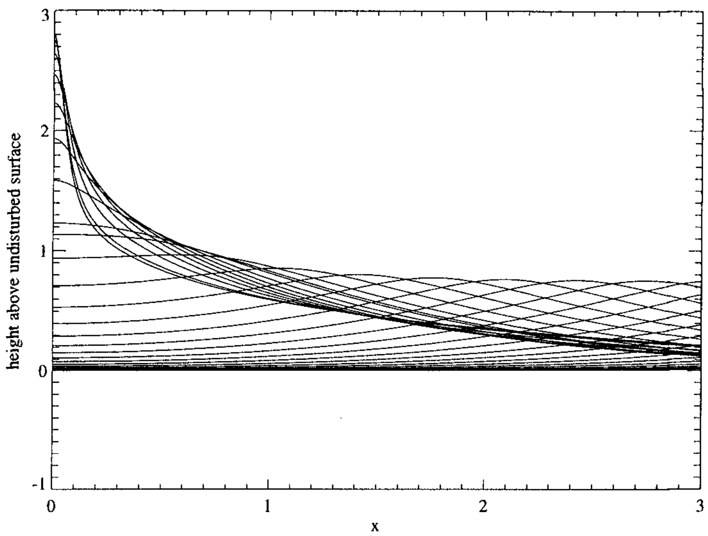


Figure 1: Solitary wave collision with vertical wall. Wave height, $H = 0.75d$

once the jet has formed, the motion of any fluid particle in the jet is governed by its initial momentum and gravity. Portions of fluid are modelled by considering their motion once above the crest of the wall as if they are a free particles moving under gravity, that is

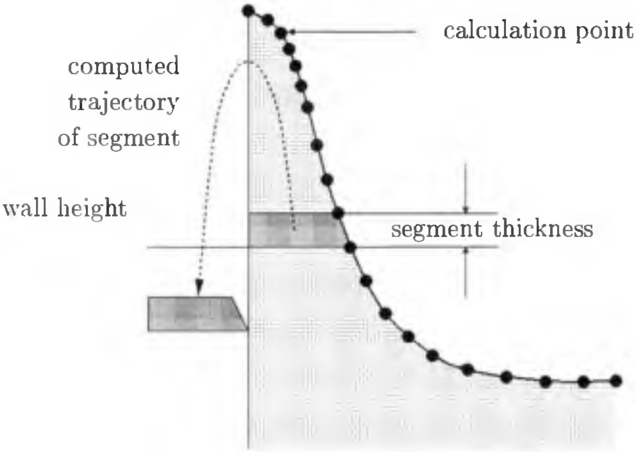
$$y(t) = y_0 + v_0 (t - T_0) - gt^2/2 \quad (1)$$

$$x(t) = x_0 + u_0 (t - T_0) \quad (2)$$

where the fluid particle has position (x_0, y_0) and velocity (u_0, v_0) at a time T_0 when it passes the crest of the wall.

As a jet ascends a wall there is some slight pressure against it. This is due to motion towards the wall as each portion of the jet stretches and thins. As the water passes the top of the wall this slight pressure drops to atmospheric, but the motion towards the line of wall continues. This means any portion of fluid passing the top of the wall has some momentum in a direction towards the wall which can carry it over the wall face and cause overtopping. Thus our model is as follows:

- Take horizontal slices of water as they pass the level chosen to represent the top of the wall.
- Work out their horizontal momentum.
- Treat each slice as a particle moving under gravity as in equations (1) and (2).
- Consider each slice as it returns to the level of the top of the wall. Use the position of its centre of mass to decide how much, if any, of the water has moved horizontally past the face of the wall and hence counts as overtopping.
- Add the contribution from all the slices that flow past the top of the wall to give an estimate of the total volume of water overtopping due to that wave.



This calculation could be further refined by allowing for the contraction of each slice under its initial velocity field. However, such contraction is influenced by the slight pressures exerted by adjacent slices, so that to try and include it would mean going beyond the simple modelling we wish to present here.

Slices were identified by stepping in time. The length of each time step and the vertical velocity together determining the thickness of each slice. Too short a time-step means good resolution but long computation times, too long a time step gives poor resolution. A time step size was chosen such that the thickness was always less than $10^{-4}d$, where d is the undisturbed water depth.

Limitations of simple model

In these calculations it is assumed that solitary waves are an accurate representation of the likely steepest incoming waves (see previous section). For the collision of solitary waves only the case of no surface tension and constant depth has been previously studied. When the waves are steepened by interaction with the berm a range of motions can develop, including flip-through motions as described by Cooker & Peregrine (1990). Flip-through motion is very sensitive to wave shape and can produce very violent jets. The present model is only applied to situations in which the free surface calculated by the potential flow solver has reached or passed the time of maximum run-up before breaking down. Thus we cannot model overtopping from breaking waves such as in figure 2, or the strong 'flip-through' impacts described by Cooker & Peregrine since in such cases we cannot compute up to the time of maximum elevation. On the other hand these give very thin jets. If the wave has already broken any resulting jet is small and mostly fails to overtop the structure. Thus the cases to which our model is restricted are those which happen to be responsible for a significant amount of overtopping.

Our model is inaccurate for wall heights below the height of jet formation, or for waves where no significant jet is formed, i.e. where the force due to internal pressure on a fluid particle is not negligible compared to gravity. Thus for example it does not apply to surging over low walls. In practice this means that

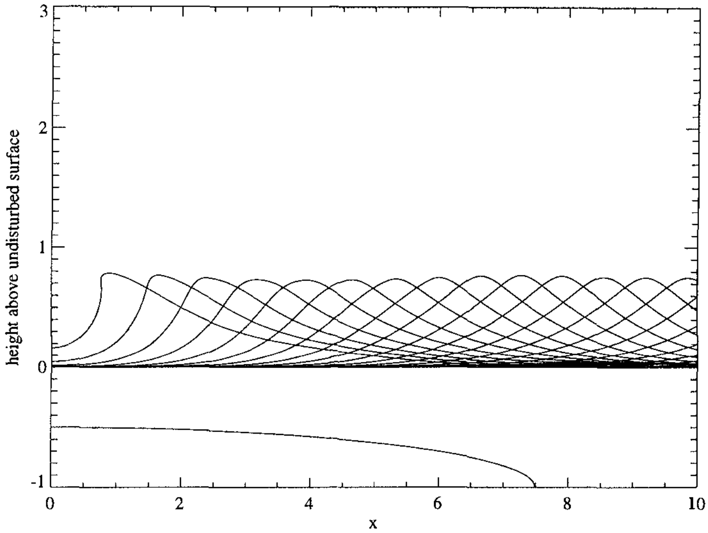


Figure 2: Solitary wave steepening over wide berm.

only solitary waves of steepness greater than $0.5d$ were used, and the results are restricted to the higher end of the dimensionless freeboard range (approximately to above R/H=1.8 in the calculations presented here).

Results with gravity waves.

Since these calculations assume two-dimensional waves the overtopping volumes calculated are volume per unit length of the wall, Q. The undisturbed water depth, d, is used as the unit of length in computations, so for example the wave height, H, is a dimensionless ratio of wave height to that water depth. Following the presentation of results in previous papers on overtopping rates, the quantities presented in the following section are in terms of the overtopping volume Q/H^2, and dimensionless wall crest free board R/H, where R is the height of the top of the wall above the undisturbed water level. Note, for these solitary wave computations the undisturbed level corresponds to trough level in a periodic or irregular wave train.

Figure 3 shows the results of dimensionless overtopping against dimensionless crest free board for the case of a horizontal bed. The dimensionless overtopping volume is plotted on a logarithmic scale. For solitary waves of amplitude 0.5 or less, no significant jet is formed at the wall. Thus our model is less accurate for the wave of lower amplitude for which it is only likely to give a rough indication of overtopping volumes. The variation with wave height reflects the greater height of run-up at wall for the highest waves. This is described in Cooker *et al.* (1997). Dimensionless overtopping volume increases with wave height.

For the case of a berm in front of the wall figures 4 and 5 show the results of dimensionless overtopping volume against dimensionless crest free board for a solitary wave with amplitude $0.7d$ for berm widths d and $2d$. These include a range of results for berms of different heights. Rather suprisingly, for berms of

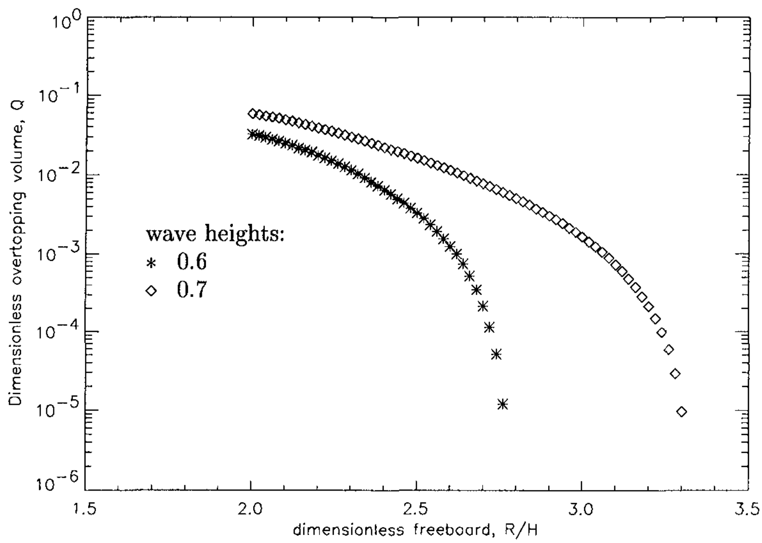


Figure 3: Solitary wave overtopping on water of constant depth; different wave heights, H

width d, $2d$, the overtopping decreases with increasing berm height. For these short berm widths, the higher berm interferes with the water motion near the wall. The influence on the shape of the resultant jet shape is small in the case of length d, but strong for the berm of length $2d$ (see figure 6 for example), making it increasingly wide and short.

It can be seen in figure 6 that the berm forces the water level at the wall up earlier than in the case with no berm. The result is that the strong accelerations normally seen at this time are reduced and the water just sloshes smoothly against the wall. In this case the reduction in the violence of the collision also corresponds to a reduced impact pressure at the wall. However, since the motion of the water is directed up by the berm at the wall this may give larger overtopping volumes at low crest free-boards. This is beyond the scope of our current model. For berms of width $3d$, the effect seen with the $2d$ berm and that of wave steepening appear to cancel each other. The overtopping predicted is very close to that with no berm. For wider berms still, wave steepening is the dominant effect, which increases the violence of the motion and the corresponding overtopping volume.

Figure 7 shows the dimensionless overtopping volume against dimensionless crest free board for berms of fixed height but different width. For dimensionless free-boards up to 2.5 there is little differcnce; it is only in the free-boards near the maximum run-up height that the influence of the berms can be clearly seen. Results show that a berm of width less than $3d$ decreases Q, compared to the no berm case. Over wider berms, the wave steepens giving a stronger jet on impact. A roughly exponential relation between individual dimensionless overtopping volume and dimensionless free-board (i.e. a straight line on the graph) can be see over much of the valid range.

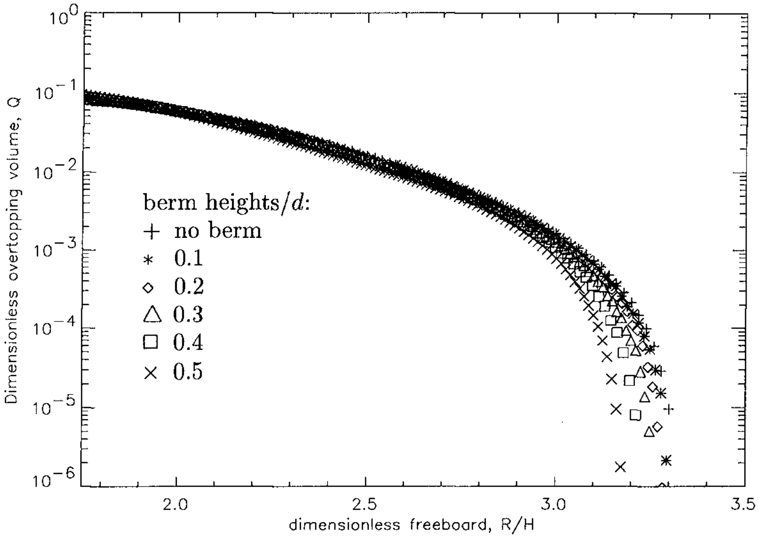


Figure 4: Solitary wave overtopping; berm width $= d$, different berm heights. $H = 0.7d$

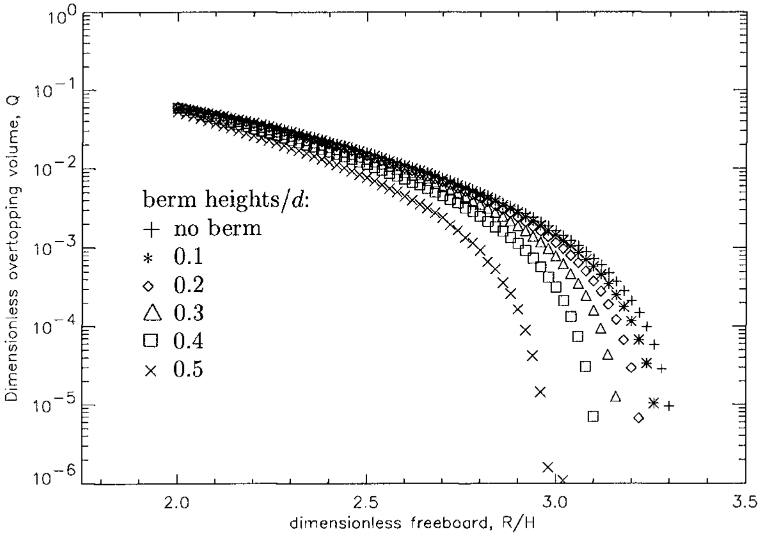


Figure 5: Solitary wave overtopping; berm width $= 2d$, different berm heights. $H = 0.7d$

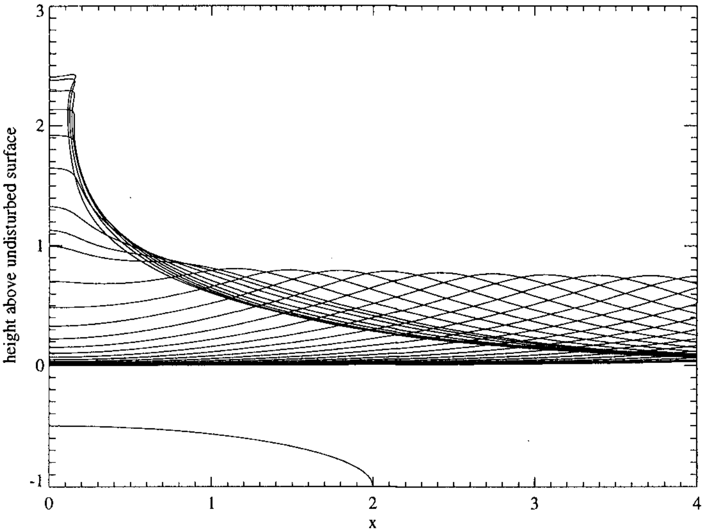


Figure 6: Vertical wall with berm width $2d$, height $0.5d$. $H = 0.75d$

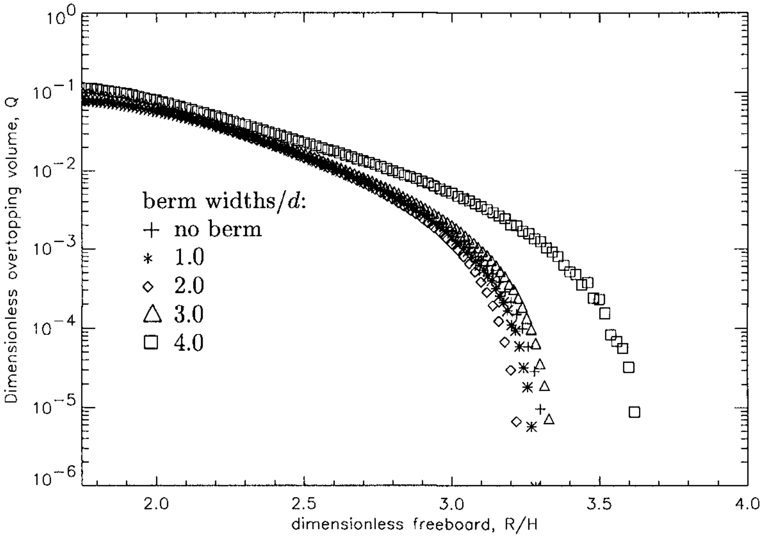


Figure 7: Solitary wave overtopping; berm height $= 0.2d$, $H = 0.7d$.

Comparison to existing experimental data

To the authors' knowledge there are no experimental results for individual overtopping volumes for a vertical breakwater in which the required geometric and hydraulic characteristics are all given. Indeed, the purpose of this study is partly to fill that gap. We can however draw some parallels between the calculations presented here and experimental results. From the knowledge of wave steepnesses (defined in related experimental work as wave height/distance between crests) and the nondimensional wave speed in our model, we can work out a rough overtopping rate. Values for wave heights are given in terms of a significant wave height, defined as the mean height of the 1/3 highest waves. In conversion to approximate rates we use this as equivalent to the height of our test wave. Where a range of significant wave heights is given our test wave is chosen equal to the mean of the range.

The overtopping rate is defined as the overtopping volume divided by the time between overtopping events. The time between overtopping events usually differs from the wave period as only a limited percentage of the incident waves produce overtopping.

A series of model test are carried out by Juhl (1995). Results for dimensionless overtopping rates are presented for differing angles of wave attack and wind velocities. Our model is only applicable to the results with zero wind velocity and normal wave attack. Again, we have to be careful when comparing our model to results with overtopping rates. Suppose we assume that only 10% of waves produce overtopping, a likely figure from various experimental results. Our model thus only gives a rough approximation to the experimental data.

Figure 8 compares the rate of overtopping found with zero wind velocity and 0^o angle of wave attack to that predicted by our simple model over its range of validity. The period between overtopping events used to find the rate is derived by assuming the a periodic train of solitary waves which give a steepness equal to the average used in the tests, multiplied by the probability. We see that our model produces results that are of the same order of magnitude over most of the range. However there is a tendency for under-prediction at higher dimensionless free boards. This is explained earlier. The non-dimensional overtopping rate compares well to that of Juhl for dimensionless free boards in the range 2 to 3.

Overtopping with gravity-capillary waves

Surface tension plays an important local role where-ever the surface curvature is sufficiently high. In the calculations above, extremely thin jets are often produced which have a region of very high curvature at the tip. Surface tension can have a significant effect on the shape of the jet at this tip. This in turn might effect the predicted overtopping volumes. There are two ways in which surface tension is likely to effect overtopping - by affecting the profile of the incident wave, and by affecting the resulting jet shape. We ask, at what scale would surface tension become significant in overtopping experiments? The lack of data on scale correction factors to date has been reported as substantially hindering the application of research results to practical engineering analysis or design. Our model permits the inclusion of surface tension in the evolution of the surface. There are, of course, other scale effects not included in our model. These include the break up of jets into water drops and the effects of any wind. There may also be problems including any air entrainment effects.

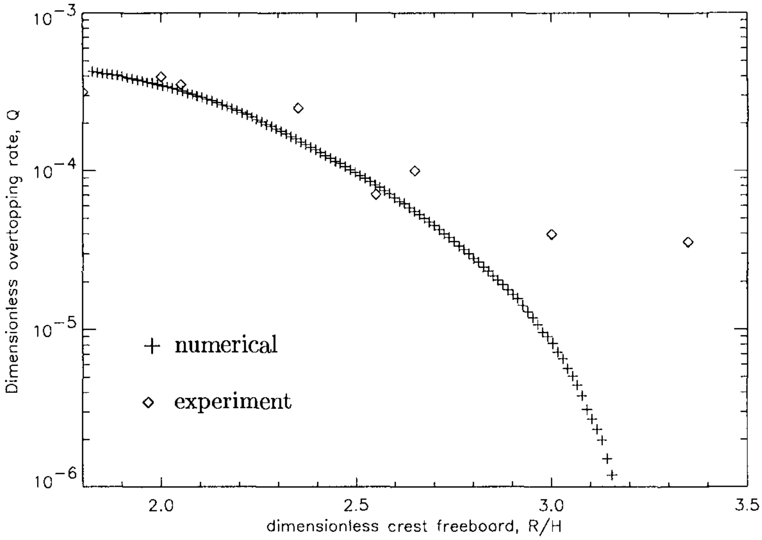


Figure 8: Comparison of estimated overtopping rates with Juhl (1995) (wind speed=0, angle of attack=0^o)

The results are described in terms of the properties of clean water. Hence the description of water depth is used to specify scale of motion and thus the relative strength of surface tension. In our symmetric approximation we implicitly assume a contact angle $\theta = 90^o$. In practice the contact angle may differ from this, and the water is not usually clean.

Run-up of solitary type waves with surface tension

Figure 9 shows the jet at the time of maximum run-up for a solitary waves of $a = 0.7d$ with d =5cm. In Jervis (1996), it was found that for the particular case of a solitary wave with $a = 0.7d$, propagating on depths 5cm and larger no noticeable capillary waves formed. Experimenters might then expect surface tension to play no measurable role in experiments. The main effect of the inclusion of surface tension is to modify the shape of the jet produced in the solitary wave collision. The jet is generally broader and shorter due to surface tension. A decreasing non-dimensional run-up height with increasing surface tension (i.e. decreasing scale) is found. For the d =5cm case for example, the decrease in non-dimensional run-up height is approximately 10%.

Overtopping volumes with surface tension

Figure 11 shows the dimensionless overtopping predicted by our model for a solitary wave of amplitude $a = 0.7d$. Resulats for depths of 5cm, 10cm, 20cm and 50cm are shown alongside the result for no surface tension. Results for depth 2cm are not shown: at this scale surface tension restricted the formation of a jet too much for our overtopping model to be valid. For small dimensionless free-boards we see that the results coincide. At higher dimensionless free-boards the

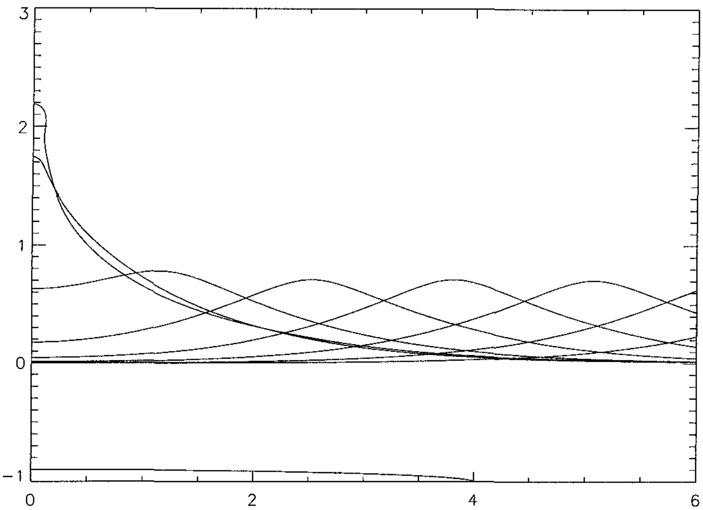


Figure 9: y/d against x/d at time up to maximum run-up for a solitary wave a=$0.7d$, d=5cm, berm width=$4d$, berm height=$0.1d$.

difference in jet shape shown in figure 10 is affecting overtopping volumes. At smaller scale (higher nondimensional surface tension values) the run-up height is less, and the average velocity towards the wall is reduced. Even for the calculation for d=50cm there is a noticeable underprediction.

We can then see that even experiments carried out on depths of the order 20 to 50cm there is an underprediction at the upper limits of dimensionless free-board. For experiments where the incident wave is only 5-10cm relatively large underpredictions of overtopping volume and wave run-up can occur when scaling results back to full scale for use in breakwater design for $R/H > 2.6$. Of course here we are modelling only the non-breaking waves. More violent impacts produce jets beyond the scope of this simple model.

Figure 12 shows dimensionless overtopping for a wave on 5cm depth where the wave is steepened by a berm of width $4d$. As with no surface tension, wave steepening increases the strength of the jet produced. This can be seen in the trend towards a straight line for much of the range on figure 12 for higher berms. Without surface tension, overtopping is recorded for dimensionless crest free-boards of over 3.6, compared to only 2.85 with the corresponding 5cm calculation. It is clear then that extrapolating experimental results from such a small scale can give misleading values at high relative crest free-boards.

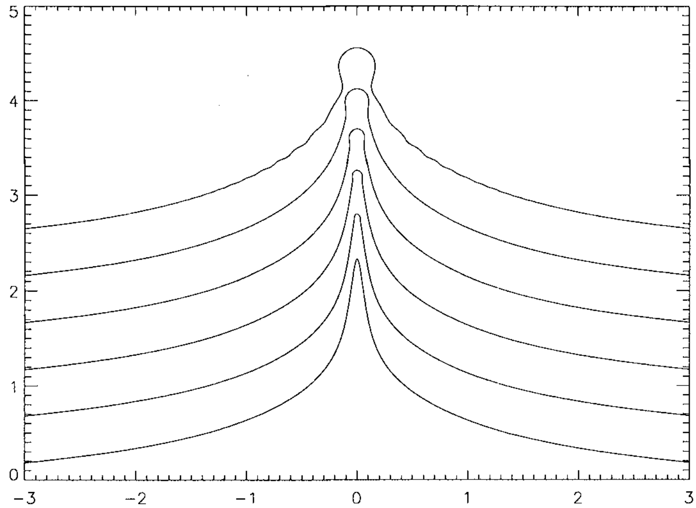


Figure 10: y/d against x/d at time of maximum run-up for a solitary wave $a = 0.7d$, for d=2cm, 5cm, 10cm, 20cm, 50cm and with no surface tension. (Surfaces have been shifted vertically for clarity).

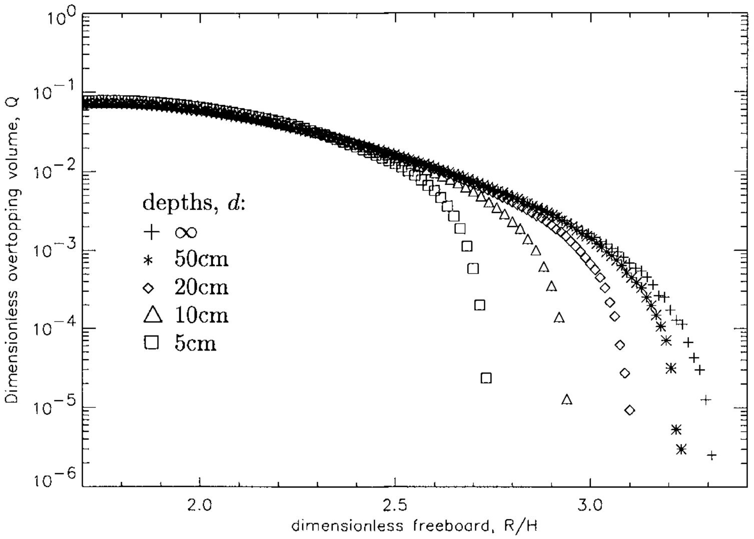


Figure 11: Dimensionless overtopping volume against dimensionless free-board for a solitary wave $a = 0.7d$, d= 5cm, 10cm, 20cm, 50cm and ∞ i.e. no surface tension.

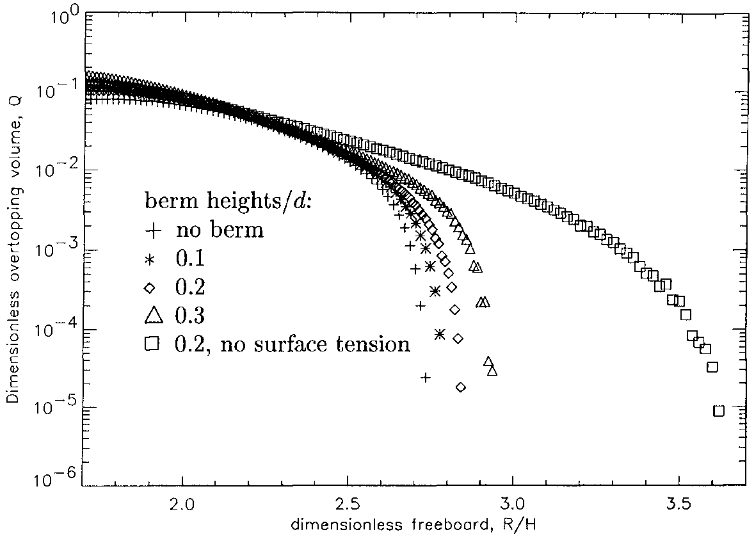


Figure 12: Dimensionless overtopping volume against dimensionless free-board on a berm of width $4d$ for a solitary wave $a = 0.7d$, for d=5cm with surface tension.

Further work and conclusion

It appears that this simple theoretical approach gives plausible results for overtopping volumes per wave for non-breaking solitary waves. When strong jets are formed, particularly in calculations were the wave is steepened by interaction with a long berm, it is found that the overtopping volumes have a roughly exponential dependence on run-up for waves of a given height. Such an exponential relationship is similar to experimental results on overtopping rates (Franco, 1994, Juhl and Sloth, 1994, for example). The method predicts the same trends as previously reported in measurement of overtopping rates. A comparison of the results to measured overtopping volumes per wave is needed to validate the method however. Quantitatively, overtopping rates compare well with experimental results of Juhl (1995), allowing for the expected underprediction at high relative crest free boards.

From the results it appears that for the parameter range studied the short high berms decrease overtopping perhaps by interfering with the water motion near the wall, whereas wider berms steepen the incoming wave and make its encounter with the wall more violent and increase overtopping. Wide berms act to steepen the incident wave. In case where the flow was calculated up to the maximum run-up, wide berms tended to increase overtopping volume. Cases where the flow could not be computed up to the time of maximum run-up involved wave steepening often to the point of breaking before impact with the wall. In this case the likely impact pressures can be considerably higher (see Cooker 1990 for example). For the higher berms the shape of the jet can vary significantly, as can be seen in figure 6.

Wind can also play a role in overtopping. A wind force can break jets into spray above wall height. It can thus act to significantly increase or decrease overtopping, according to direction. Such a wind force could easily be incorporated into the simple mathematics of this overtopping model.

Most experiments are carried out at scales where surface tension plays a role in the jet dynamics. The effect of surface tension is generally not taken into account when extrapolating experimental results to full scale. Results here show that for overtopping rates that the effect of surface tension cannot be ignored for high crest free-boards. The action of surface tension on the tip of the jet leads to a considerable reduction in overtopping volume at relative crest free-boards greater than 2.6 for wave on depths up to the order of 20cm, and a corresponding reduction in non-dimensional run-up height. Since our model fits the most common mode of overtopping, we expect experimental results to be affected also, even though they generally include other modes of overtopping not covered here.

Acknowledgements

We acknowledge support from the U.K. Engineering and Physical Sciences Research Council and from the Commission of the European Communities, Directorate General for Science, Research and Development under MAST Contract MAS3-CT95-0041.

References

COOKER, M. & PEREGRINE, D.H (1990) A model for breaking wave impact pressures. *Proc. 22nd Int. Conf. Coastal Eng.* ASCE, Delft. 1473-1486.

COOKER, M., PEREGRINE, D.H., VIDAL, C. & DOLD, J.W. (1990) The interaction between a solitary wave and a submerged semi-circular cylinder. *J.Fluid Mech.* **215**, 1-22.

COOKER, M., WEIDMAN, P.D. & BALE, D.S. (1997) Reflection of a high amplitude solitary wave at a wall. *J.Fluid Mech.* to appear.

DOLD, J.W. & PEREGRINE, D.H. (1986) An efficient boundary integral method for steep unsteady surface waves. *Numerical methods for fluid dynamics II, (ed. Morton, K.W., Baines, M.J.)* Oxford.U.P. 671-679.

FRANCO, L. (1994) Further results of hydraulic model tests on wave overtopping: probability distribution of overtopping volumes per wave. *Proc. 2nd Workshop, Monolithic Coastal Structures Project, E.C. MAST 2, Milan.*

JERVIS, M. (1996) Some effects of surface tension on water waves and water waves at a wall. *PhD. thesis.* University of Bristol.

JUHL, J. & SLOTH, P. (1994) Wave overtopping of breakwater under oblique waves. *24th Int. Conf. on Coastal Eng.*, ASCE, Kobe.

JUHL, J. (1995) Wave overtopping of vertical walls and the influence of wind. *Proc. Monolithic Coastal Structures Project, Final Workshop. E.C. MAST 2 Programme.*

TANAKA, M. (1986) The stability of solitary waves. *Phys. Fluids.* **29** 650-655.