CHAPTER 160

COMBINED DIFFRACTION AND TRANSMISSION OF WATER WAVES AROUND A POROUS BREAKWATER GAP

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ABSTRACT

An analytic method for the combined diffraction and transmission around a porous breakwater gap is presented. The method can be summarized as follows. First of all, it is necessary to decompose the incident wave into two components, according to the incidence angle and the permeability of the two breakwaters that form the gap. Then, let the breakwaters be completely transparent to the relevant component of the incident wave and ascertain the transmission field. Next, treat the breakwaters as solid to the other component of the incident wave and solve the diffraction field. Finally, superpose the transmission and the diffraction to give the objective wave motion. The available analytic methods for the diffraction by an aperture on solid wall are comparatively studied since they play the central role in working out the final solution of a real problem with combined diffraction and transmission. Sample computations are carried out for typical cases with both regular and irregular incidence. The computational results show that the phase effects of the porosity of the breakwaters on a combined wave field can be significant.

INTRODUCTION

Being constructed with rocks or concrete blocks in most of the engineering practices, segmented offshore breakwaters for the purpose of shore protection are usually of greater or lesser permeability. For this reason, it has long been a keen

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interest of not only the scientific researchers but also the practicing engineers to have a good understanding on the effects of the permeability of a porous breakwater on the pertinent wave motion.

There exist possibly a few very different approaches to water wave problems with porous breakwaters involved. The one appeared to be relatively dogmatic is to isolate the region occupied by the porous skeleton and consider the details of the flow within the porous medium, which is in general of a complex picture and needs to be treated as viscous and turbulent. A solution of such configured problem also requires the relevant exterior flow to be matched with the flow inside the pores. Since both a straightforward look at the pore water flow and the matching procedure are extremely sophisticated, studies in the past decades, instead of dealing with Navier-Stokes flows, have been directed to modeling the effects of the porous skeleton on the fluid motion. There have been a few effective models based on such consideration established. The one by Sollitt and Cross (1972) and Madsen (1974), which includes both the resistance and the inertia forces exerted on the fluid by the porous skeleton, has indeed been applied to many practical problems.

Even with modeling, treatment of the flow inside a porous medium is still not easy at all. Practical solutions can only be obtained under very simplified conditions. This situation has motivated a more pragmatic approach that is to treat the surface of the porous structure as an absorptive or partial reflective boundary of the problem concerned. With this consideration, however, prescription of the amplitude and phase of the reflected wave relative to the approaching wave is necessary (Chen, 1986; Isaacson and Qu, 1989). Extension of the applicability of this approach thus requires continued efforts to establish highly accurate empirical formulas for the reflection coefficient and the phase trapping property of a structure with given porosity and known hydraulic conditions behind the structure.

The approach adopted in the present study is yet another one. It was originally proposed by Yu and Chwang (1994) and Yu (1995). As a porous breakwater is thin when compared to the local wavelength, the normal component of the seepage velocity through the porous body was noted to be proportional to the difference of the wave function at both sides of the breakwater. The proportionality coefficient is a function of the physical thickness, the porosity as well as the other properties of the porous medium and the wave.

The primary objective of the present study is to provide an analytic method for the combined diffraction and transmission around a porous breakwater gap. We shall try to find a method that takes advantage of the well established techniques for diffraction by a solid aperture. Emphasis will also be paid on the phase effects of the porosity of breakwaters.

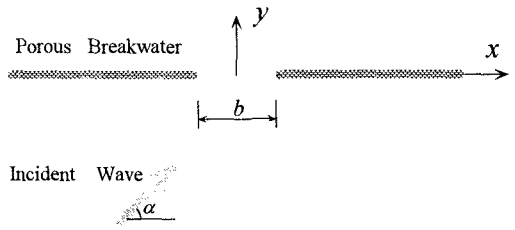

Fig. 1. Definition sketch of a porous breakwater gap.

THRORY

Formulation

The physical problem is sketched in Fig. 1. Outside the porous breakwaters we adopt the conventional assumptions that the fluid is inviscid and incompressible, and moreover, the fluid motion is irrotational. The three dimensional wave field can then be represented by a scalar velocity potential ϕ that satisfies the Laplace equation. Let the waves of interest be sinusoidal with respect to time and be of small amplitude. Hence, we can express the velocity potential in the following form with the time and the vertical coordinate separated:

$$\phi(x, y, z, t) = F(x, y) \cosh k(h + z) e^{i\sigma t} \tag{1}$$

where σ is the angular frequency of the wave motion, k is the wave number, h is the water depth, x and y are the horizontal coordinates, z is the vertical coordinate, and t is the time. F, usually called the wave function, is complex-valued with its modulus proportional to the amplitude and its argument equal to the relative phase of the surface oscillation. It can be readily confirmed that Eq. (1) satisfies the impermeable bottom condition and also the free surface condition as far as σ, k and h are related to each other through the dispersion equation:

$$\sigma^2 = gk \tanh kh \tag{2}$$

where g is the gravitational acceleration. By substituting Eq. (1) into the Laplace equation for ϕ we can show that $F(x, y)$ is governed by the following Helmholtz equation:

$$\nabla^2 F + k^2 F = 0 \tag{3}$$

where ∇ is the horizontal gradient operator.

For the problem concerned in the present study the boundary conditions include the incidence condition, the radiation condition at infinity and a partial

transmission condition along the permeable breakwaters. The incident wave, long-crested and propagating in the direction that forms an angle α with the positive x-axis, can be described by

$$F_0 = A_0 e^{ik(x\cos\alpha + y\sin\alpha)} \tag{4}$$

where $A_0 = gH/2\sigma\cosh kh$ and H is the incident wave height. The partial transmission condition at the porous breakwaters can be written as:

$$\frac{\partial F}{\partial y}\Big|_{y=0+} = \frac{\partial F}{\partial y}\Big|_{y=0-} = -ikG(F|_{y=0+} - F|_{y=0-}) \tag{5}$$

where $G = \gamma/k\delta\{f - i[1 + C_m(1-\gamma)/\gamma]\}$ represents the permeability of the breakwaters while, δ is the physical thickness of the breakwaters (geometrically, the thickness of the breakwaters is considered to be zero) and, γ, f and C_m are the porosity, the linearized resistance coefficient and the added-mass coefficient of the porous medium, respectively. $y = 0+$ and $y = 0-$ indicate respectively the downwave and the upwave side of the breakwaters. As the ratio of the physical thickness of the breakwaters to the local wavelength is less than about 0.2, which is right in most of the practical situations, Eq. (5) has been shown to be in good agreement with experimental data (Yu, 1995). Since the wave function and its gradient are proportional to the dynamic pressure and the velocity of the fluid, respectively, in a linear wave theory, Eq. (5) is apparently identical to the Darcy's law which states that the velocity of the fluid flow in a porous medium is directly proportional to the pressure gradient. The actual difference between Eq. (5) and the Darcy's law is that we allowed a phase lag between the velocity and the pressure gradient to represent the inertia effects of the porous flow. This is trivial if the porous medium is closely-packed and the resistance dominates the flow. But it might be important if the porosity of the breakwaters is relatively large.

Reflection and Transmission

Consider the special case where the opening of the porous breakwater gap described in Fig. 1 approaches zero, or the two semi-infinite breakwaters are connected to become a continuous one extending to infinity at both ends. On this particular occasion, we are able to derive an explicit solution for both the reflected and transmitted waves under arbitrary incidence conditions.

Let the reflection and transmission coefficients of an infinite breakwater be defined as the ratios of the amplitudes of the reflected and the transmitted wave to the incident wave. Denoting the reflection and the transmission coefficient by K_r and K_t, respectively, we can formally express the wave field in the reflection region and that in the transmission region by

$$F_r = F_0 + K_r A_0 e^{ik(x\cos\alpha - y\sin\alpha)} \tag{6}$$

$$F_t = K_t A_0 e^{ik(x\cos\alpha + y\sin\alpha)} \tag{7}$$

Both K_r and K_t are assumed to be complex-valued so that information on not only the magnitude but also the phase of the reflected and transmitted waves is included. To satisfy the partial transmission condition (5) at the porous breakwater, we attain the following relations:

$$K_t \sin\alpha = (1 - K_r)\sin\alpha = G(1 + K_r - K_t) \tag{8}$$

which yields

$$K_r = \frac{\sin\alpha}{2G + \sin\alpha} \tag{9}$$

$$K_t = \frac{2G}{2G + \sin\alpha} \tag{10}$$

Back substitution of Eqs. (9) and (10) into Eqs. (6) and (7) gives rise to the final solution of the simple reflection-transmission problem. It must be pointed out that verification of Eqs. (9) and (10) by measured data under general conditions has not been carried out. It is because they were demonstrated to be in satisfactory agreement with laboratory experiments at $\alpha = \pi/2$ or under the normal incidence conditions we give credence to these equations in the present study.

Diffraction

In both the reflection and transmission regions we decompose the wave function F into two parts: the transmitted wave F_t and the diffracted wave F_d. That is, we let

$$F = F_t + F_d \tag{11}$$

where F_t is given by Eq. (7) with K_t defined by Eq. (10). For F_t is known we need only to fix F_d to finally determine F.

Since both F and F_t satisfy the Helmholtz equation, which is linear and homogeneous, F_d should do likewise to ensure Eq. (11). That is,

$$\nabla^2 F_d + k^2 F_d = 0 \tag{12}$$

At $x \to \pm\infty$, the effects of the breakwater gap on the wave field die off and, therefore, F approaches the relevant solution for a continuous breakwater. This can be written as

$$F_d \to 2K_r A_0 e^{ikx\cos\alpha}\cos(ky\sin\alpha) \qquad \text{at } y \leq 0 \text{ and } x \to \pm\infty \tag{13}$$

$$F_d \to 0 \qquad \text{at } y \geq 0 \text{ and } x \to \pm\infty \tag{14}$$

Along the porous breakwater, Eq. (5) leads to

$$\frac{\partial F_d}{\partial y} = 0 \qquad \text{at } y = 0 \text{ and } |x| > \frac{b}{2} \tag{15}$$

Eqs. (12), (13), (14) and (15) are in fact the formulation for diffraction by an impermeable breakwater gap provided the incident wave is given by

$$F_2 = K_r A_0 e^{ik(x\cos\alpha + y\sin\alpha)} \tag{16}$$

which is the original incident wave expressed by Eq. (4) with a phase shift (since K_r is complex-valued) and an amplitude reduction (since $|K_r| < 1$).

It becomes now very obvious that the wave field around a porous breakwater gap can be treated as the superposition of a transmission and a diffraction by impermeable structures. Namely, we can split the incident wave (F_0) into two component waves ($F_1 = K_t A_0 e^{ik(x\cos\alpha + y\sin\alpha)}$ and $F_2 = K_r A_0 e^{ik(x\cos\alpha + y\sin\alpha)}$). To one of these components (F_1) the breakwater is completely transparent and transmission of wave into the region behind the breakwater is totally free. To the other component wave (F_2), on the other hand, the breakwater works like a solid wall. A complete reflection in front of it will occur and transmission of the wave energy into the region behind the breakwater is only by the diffraction process. Since both K_r and K_t are complex-valued in general, it should be emphasized that split of the incident wave is not simply a proportional division of the incident wave energy or amplitude. The phase effects of the porosity of the breakwater must be correctly considered.

A different way to view the combined diffraction and transmission around a porous breakwater gap can be explained as follows. Instead of Eq. (11) we let

$$F = K_r F_0 + K_t F_{0d} \tag{17}$$

where F_0 is intepreted as the wave field due to transparent breakwaters and F_{0d} as the wave field due to solid breakwaters (the relevant incidence is F_0 in either case). Considering the fact

$$K_r + K_t = 1 \tag{18}$$

we can treat a porous breakwater as the proportional combination of a transparent one and a solid one. The resulted wave field associated with the porous breakwater can also be interpreted as the similarly proportioned combination of the relevant wave field as the breakwater is transparent and solid. Again, it is important to notice that the proportional factors are complex-valued, so disintegration of breakwaters depends on not only the porosity of the breakwaters but also the properties of the wave.

ANALYTIC METHODS FOR DIFFRACTION

By the concept described in the previous section, the solution of wave motion around a porous breakwater gap can be obtained as long as the principle for decomposition of the incident wave or disintegration of the breakwaters in addition to an effective solution method for the diffraction by solid breakwater gaps are available. The theoretical elements which underlie the decomposition and disintegration course for given incidence condition and the porous properties of breakwaters has been discussed in details. In the following we shall give a brief summary of the useful methods for the diffraction by solid breakwater gaps with emphasis on their limitations when recently available numerical recipes are employed.

Water wave diffraction by solid breakwater gaps is one of the most classical subjects of coastal and harbor hydrodynamics. The problem has its analogy in both acoustics and electromagnetics. A large number of studies, analytical and numerical, have been carried out in the past half century. Exact solutions of the problem can be obtained by separation of variables in the elliptic coordinates. This elegant method is usually attributed to Morse and Rubenstein (1938) who gave the first outline of application for the diffraction of sound and electromagnetic waves by a slit in the infinite plane. Its effectiveness in solving the relevant water wave problems was widely recognized after Carr and Stelzriede's work (1952). The method is essentially straightforward but it does involve substantial efforts if numerical results are necessitated. The difficulty is owing to the appearance of the unusual Mathieu functions, for which advanced computational programs do not seem to have been readily available until this manuscript is to be finalized (although no sign showed they are more accurate than the present development, the routines in the recent book by Zhang and Jin (1996) are valuable). Even the excellent work by Sobey and Jonsson (1986) still had to partially rely on tables for some characteristic values. In principle, Morse and Rubenstein's method is valid with no restriction. However, accurate evaluation of the modified Mathieu functions of the third kind, which are involved in the solution, is rather difficult if the breakwater gap is wide. Owing to this fact, the exact solution method has not been recommended if the ratio of the opening of the breakwater gap to the local wavelength is larger than 3 (Carr and Stelzriede, 1952). Whether this suggested limit is still reasonable at present and how accurate is Morse and Rubenstein's solution near this limit are, however, not known.

The standard method for the diffraction by a solid breakwater gap with relatively large opening is attributed to Penney and Price (1952). The method is based on the Sommefield solution for the diffraction around the tip of a semi-infinite wall. It was widely promoted in the coastal engineering society (CERE, 1984), because of its relative simplicity from the numerical point of view when

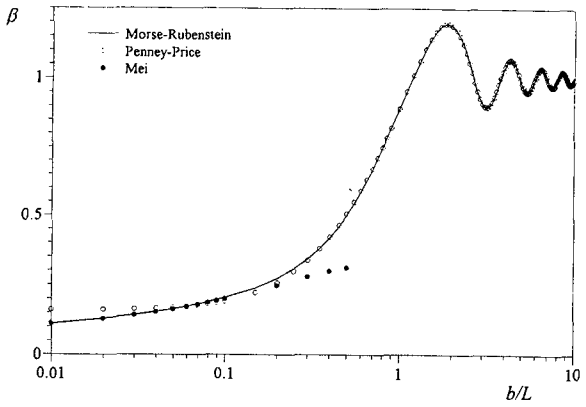

Fig. 2. Computed wave height at $x = 0$ and $y = L$ for varying opening of breakwater gap. Morse and Rubenstein's solution, Penney and Price's solution and Mei's solution are compared.

compared to Morse and Rubenstein's method. One needs only to evaluate the Fresnel integrals for determining the wave motion. Penney and Price's solution is not exact because it assumed that the scattered waves due to one breakwater arm produce no transverse flow at the position of the other arm and this assumption is approximately valid only when the opening of the breakwater gap is larger than a few wave length depending on the accuracy requirement.

Approximate solution for the diffraction by a breakwater gap with small opening is also available (Mei, 1989). This is called Mei's method and is based on matched asymptotic expressions. It assumed that the gap is equivalent to a singularity to a far-field observer. In the near-field, on the other hand, general representation of the wave field can be derived by comformal mapping. Mei's solution is concise but it can not be applied to cases with large ratio of the opening of the breakwater gap to the local wavelength. The limit for given accuracy is yet open to question.

Owing to the fact that practical problems in coastal and harbor engineering may not always fall inside the limits of the approximate solutions for large and small gaps, we developed a highly accurate numerical method for evaluating Morse and Rubenstein's solution in the present study. The characteristic values of the Mathieu functions in the method are determined through finding the eigenvalues of real-symmetric tridiagonal metrices with elements differing widely in order of magnitude. The relevant eigenvalue problems are solved by the QL method with implicit shifts (Press et al., 1989) in double precision. The computer codes were strictly checked by the tables given in Abramowitz and Stegun (1972). The Bessel functions of the various kinds involved in the expansions of the Mathieu functions are also evaluated in double precision, following the

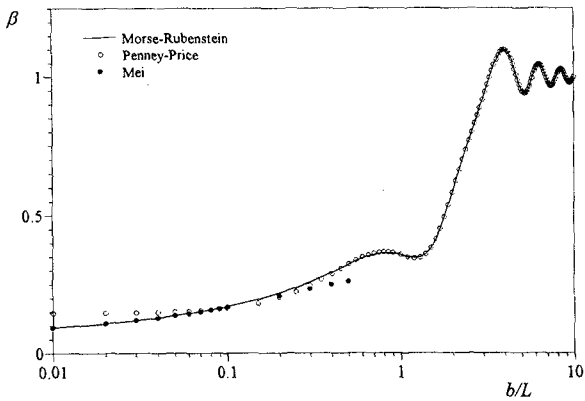

Fig. 3. Computed wave height at $x = L$ and $y = L$ for varying opening of breakwater gap. Morse and Rubenstein's solution, Penney and Price's solution and Mei's solution are compared.

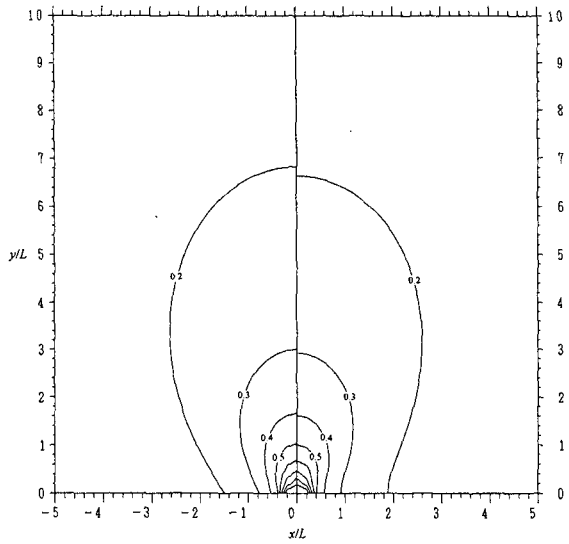

Fig. 4. Comparison of the wave height distribution around the breakwater gap. The let half is Penney and Price's solution. The right half is Morse and Rubenstein's solution. $b/L = 0.5$.

schemes given by Watanabe et al. (1989).

Figs. 2 and 3 are the plots of the relative wave height β (= the ratio of the local wave height to the incident wave height) against the relative opening of the breakwater gap (= the ratio of the opening b to the wavelength L) at two different positions. It can be noticed that the agreement between Morse and Rubenstein's solution and Penney and Price's solution is fairly good if b/L is larger than about

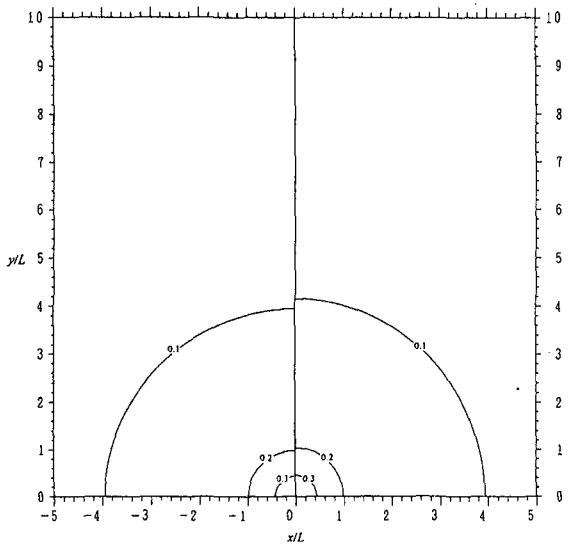

Fig. 5. Comparison of the wave height distribution around the breakwater gap. The left half is Mei's solution. The right half Morse and Rubenstein's solution. $b/L = 0.1$.

0.5. At the other end, Morse and Rubenstein's solution almost coincides with Mei's solution if b/L is less than about 0.1. Accurate evaluation of Morse and Rubenstein's solution has been successfully done for b/L up to more than 10, which is much larger than the number reported previously.

Figs. 4 and 5 also compare Morse and Rubenstein's solution with Penney and Price's solution and Mei's solution. The comparisons are made for the wave height distribution around the breakwater gap at $b/L = 0.5$ and $b/L = 0.1$, respectively. The selected ratios of the opening of the breakwater gap to the wavelength can actually be viewed as the limits of the approximate solutions. The agreement, as can be seen, is fairly good from the engineering point of view.

COMBINED DIFFRACTION AND TRANSMMISION

Hereinbelow we examine the combined diffraction and transmission around a porous breakwater gap with numerical examples. The first case under consideration is of an oblique incidence with $\alpha = \pi/4$. The relative opening of the gap is 1.0. For reference, the diffracted wave height distribution as the breakwaters are solid, which can also be treated as a limiting case of the combined diffraction and transmission at $G = 0$, is shown in Fig. 6. The wave pattern we can have is evidently a typical one for pure diffraction which represents the dynamic

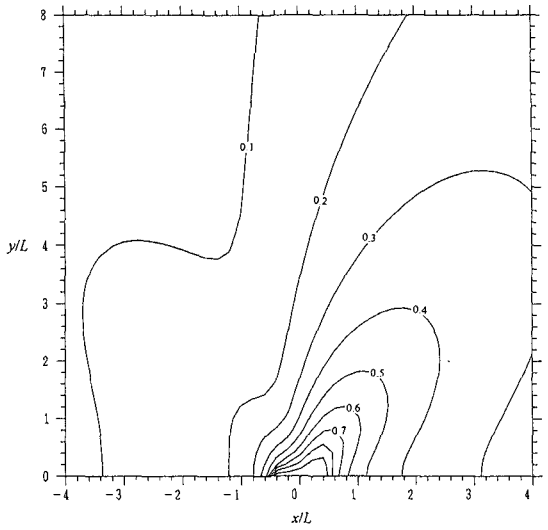

Fig. 6. Wave height distribution around a solid breakwater gap. $b/L = 1.0$ and $\alpha = \pi/4$.

processes of essentially radial waves.

The combined diffraction and transmission around the gap formed by breakwaters with any permeability can be readily assembled from a unidirectional wave and the diffracted wave as depicted in Fig. 6. Fig. 7 presents an example for $G = 0.25 + 0.25i$, that is, $K_r = 0.500 - 0.207i$ and $K_t = 0.500 + 0.207i$. What we can recognize from this figure is that the wave pattern for combined diffraction and transmission is very different from the one for a pure diffraction. This may have to be explained by the phase interactions between the diffraction (an essentially radial wave) and the transmission (a unidirectional wave).

To understand the combined diffraction and transmission of random waves, we turn to a case with normal incidence of the Bretschneider-type spectrum:

$$S(f) = 0.430\overline{H}^2\overline{T}(\overline{T}f)^{-5}\exp[-0.675(\overline{T}f)^{-4}] \tag{19}$$

where $\overline{H}$ is the mean wave height, $\overline{T}$ is the mean wave period and $f = \sigma/2\pi$ is the frequency. Fig. 8 is a comparison of the energy-spectrum of the wave at $x = \overline{L}$ and $y = \overline{L}$ ($\overline{L}$ is the mean wavelength) under various conditions of the porosity of the breakwaters. In the computations, the mean wave period is 10 s and the still water depth is considered to be 5 m. The opening of the breakwater gap equals to the mean wavelength. The permeability of the breakwaters in terms of the mean wavelength is assumed to take different values representing structures with different porosity ($G = 0$ is for solid breakwaters; $G = 0.1$ for densely-packed and resistance dominated breakwaters; and $G = 0.25(1 + i)$ for loosely-packed and

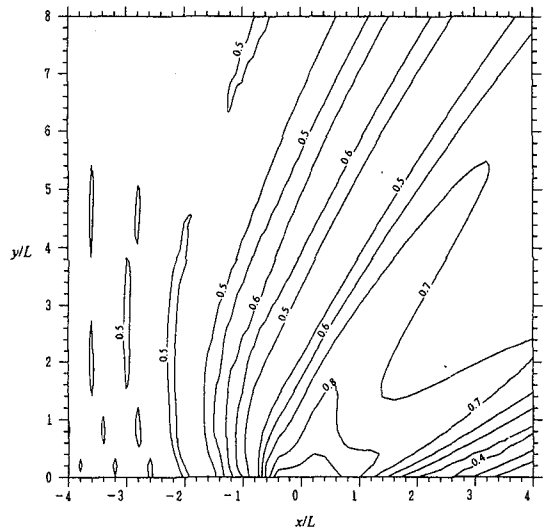

Fig. 7. Wave height distribution around a porous breakwater gap with combined diffraction and transmission. $b/L = 1.0$, $\alpha = \pi/4$ and $G = 0.25(1+i)$.

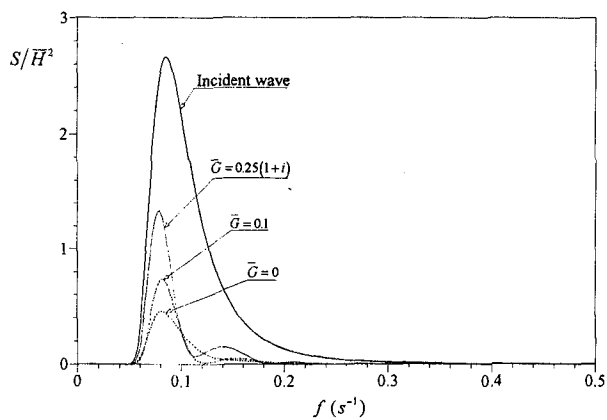

Fig. 8. Spectrum of the combined diffraction and transmission at $x = \overline{L}$ and $y = \overline{L}$ with various permeability of the breakwaters. Incidence is of the Bretschneider-type. $b/\overline{L} = 1.0$.

moderately dissipative breakwaters). The important information included in the figure is that the wave spectrum behind the breakwater undergoes significantly different transformation for different properties of the breakwaters. When the breakwaters are impermeable, the spectrum is of only one peak, like the incident wave. When they are porous, however, the spectrum shows at least two peaks. This implies that the performance of the breakwaters depends closely on the

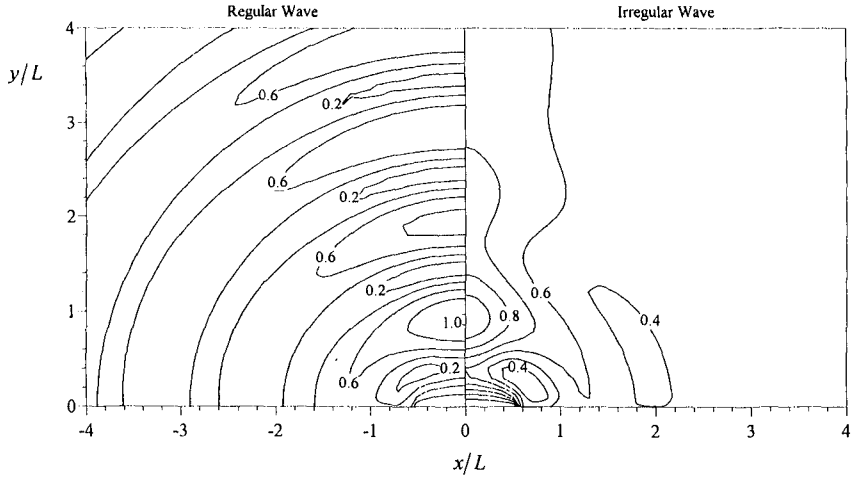

Fig. 9. Comparison of the wave height distribution for regular and irregular waves. Normal incidence is considered. $b/\overline{L} = 1.0$ and $\overline{G} = 0.25(1 + i)$.

wavelength. It should also be noted that as the permeability increases, wave energy behind the breakwater increases, as it should be. But, the increase of the energy related to long waves is particularly remarkable. This indicates that a porous breakwater is less effective to long waves.

Fig. 9 compares the wave height distribution around the porous breakwater gap with $G = 0.25(1 + i)$ when the incident wave is regular and irregular. We are able to observe totally different wave patterns for monochromatic and spectral waves. This is again explained by the effects of phase interactions between diffracted and transmitted waves.

SUMMARY AND CONCLUSIONS

We presented an useful method for the combined diffraction and transmission around porous breakwater gaps. The method can be summarized as follows: (1) we need to decompose the incident wave into two components, according to the incidence angle and the permeability of the breakwaters which form the gap; (2) we assume the breakwaters to be completely transparent to the relevant component of the incident wave and ascertain the transmission field; (3) we treat the breakwaters as solid to the other component of the incident wave and solve the diffraction field; (4) we superpose the transmission and the diffraction to obtain the objective wave. Since they play the central role in working out the final solution of a real problem with combined diffraction and transmission, methods for the diffraction by solid breakwater gaps were comparatively stud-

ied. Limitations of the approximate solutions for large and small breakwater gaps were demonstrated. Sample computations were done for typical cases with combined diffraction and transmission. The computational results showed that the effects of phase interactions between diffracted and transmitted waves, which result from the porosity of the breakwaters, are not insignificant.

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