CHAPTER 130

NEW STABILITY FORMULA FOR WAVE-DISSIPATING CONCRETE BLOCKS COVERING HORIZONTALLY COMPOSITE BREAKWATERS

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ABSTRACT

A new stability formula for wave-dissipating concrete blocks covering horizontally composite breakwaters is proposed after reviewing the existing stability formulae and verified using experimental data. A method for estimating the expected value of the accumulated damage to wave-dissipating concrete blocks within their lifetime using the Monte Carlo simulation follows thereafter and the practicability of this method for reliability based design of wave-dissipating concrete blocks is shown.

1. INTRODUCTION

Horizontally composite breakwaters (Fig. 1), covered with wave-dissipating concrete blocks, are widely employed in Japan (Takahashi, 1996) due to their proven effectiveness in reducing (i) wave forces acting on caissons, (ii) reflected waves and (iii) overtopping.

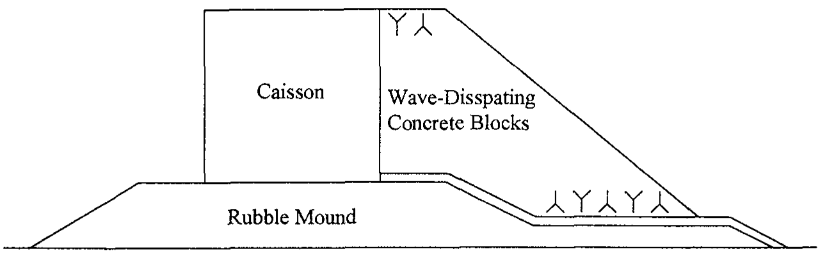

Figure 1 Typical cross section of horizontally composite breakwaters

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The design process used in Japan for horizontally composite breakwaters has strongly relied on the Hudson formula (Hudson, 1959) to provide the design weight of these blocks. Van der Meer (1987, 1988) developed stability formulae for rock/stone and armor unit breakwaters, both of which include parameters not considered by Hudson, i.e., damage level, wave steepness, and wave number.

The formula for armor units, however, is based on a two-layer type rubble-mound breakwater with a high crest level. This results in characteristics such as permeability and run-up/run-down features which are quite different from the case in horizontally composite breakwaters. The stability of wave-dissipating concrete blocks has been investigated by many researchers. For example, Kajima (1994) proposed a stability formula applicable to Tetrapods, as used in the protective covering of vertical seawalls. This formula, however, can not determine a specific design value to correspond to the assurance of no damage.

The above considerations have led to the present study and a new design formula specifically formulated to analyze the stability of wave-dissipating concrete blocks covering horizontally composite breakwaters is proposed herein.

In recent years reliability based design has been introduced to port and coastal structures. Takayama et. al. (1994) developed a probabilistic design for caissons in composite breakwaters in order to estimate the probability of slide and overturn of caissons. As for probabilistic design for wave-dissipating concrete blocks, the total damage level accumulated within the life time of these structures proves very important as well as the probability of occurence of damage. The authors in this study demonstrate a method for probabilistic design for wave-dissipating concrete blocks by Monte Carlo simulation with a new stability fomula.

2. NEW STABILITY FORMULA

2.1 Example of Experimental Data

Various kinds of experimental research for wave-dissipating concrete blocks has been carried out in Japan. The several available data sources shown in Table 1 were carefully examined in this study. The type of strucure is a typical horizontally composite breakwater.

Table 1 Example of experimental data

(Figures in Model)

No.	Port Name	Water Depth (cm)	Sea Bed Slope	$T_{1/3}$ (sec)	$H_{1/3}$ (cm)	Wave Steepness	Mass of Tetrapod (gf)
1	General, Tanimoto (1979)	39.0~48.0	1/100	2.5	16.2~25.9	0.017~0.027	743.3
2	General, Tanimoto (1985)	39.7	1/50	2.46	12.0~23.0	0.013~0.024	296.0, 596.4
3	S	25.0~31.3	1/15	1.95	8.0~15.0	0.013~0.025	91.7, 128.6
4	Mu	29.3	1/50	1.95	11.4~17.1	0.019~0.029	171.7
5	K	24.3	1/50	1.91	12.9~18.6	0.023~0.033	235.0
6	A	37.5~42.0	1/100	1.74~2.37	13.0~20.0	0.015~0.040	449.2
7	Mi	27.2	1/50	1.94	16.8	0.029	297.4

The Tetrapod was applied as it is a typical example of a wave-dissipating concrete block. The scales of the experiments vary from 1/40 to 1/80. The water depth ranges from 25 to 50cm in the model (14 to 20m in the prototype), the seabed slope is from 1/15 to 1/100, and the mass of the blocks is from 90 to 700gf in the model (32 to 80 tf in the prototype). All experiments were carried out under irregular waves of wave height 6.5 to 12m and wave period 11 to 17s in the prototype, and wave steepness 0.013 to 0.04.

2.2 Review of Existing Formulae

(1) Hudson's formula

Hudson (1959) caried out an experimental study on 2-layer type armor units under regular waves and proposed his stability formula to calculate a mass of units as shown below.

$$W = \frac{\rho_r H^3}{Ns^3 (Sr-1)^3} \quad (1)$$

where W is the mass of the armor unit, Ns the stability number, $Sr=\rho_r/\rho$, ρ_r the mass density of the unit, ρ the mass density of water, H the wave height.

Then Hudson obtained expression of Ns as a function of the slope angle α and the stability coefficient K_D as shown below.

$$Ns^3 = K_D \cot \alpha \quad (2)$$

Figure 2 shows the relation between the damage D(%) and K_D for the data from Table 1. D(%) denotes the ratio of the number of displaced units to the total number of units. K_D was calculated by Eqs.(1) and (2) using $H_{1/3}$ as H corresponding to the damage D(%). The data is scattered in a wide range and it is difficult to find any certain trend. The reason behind the data scatter is that the damage D(%) is affected by the size of the structure and the duration of wave attack is not considered in the Hudson's formula.

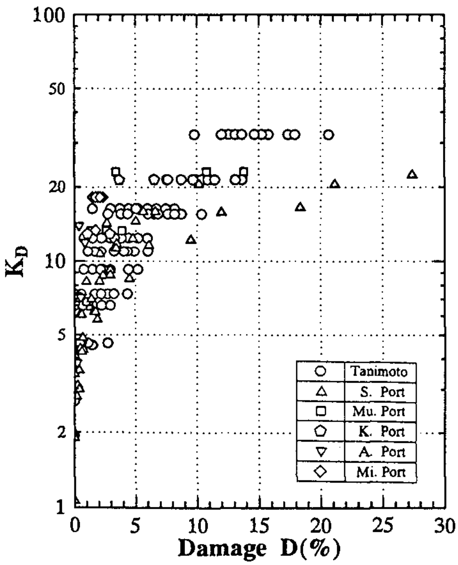

Figure 2 Relation between K_D and damage D(%)

(2) Van der Meer's Expression of Stability Number

Van der Meer (1988) proposed the following expression of stability number for 2-layer Tetrapod armoring on a rubble slope of 1 on 1.5.

$$Ns \approx H_{1/3}/(\Delta Dn) = \left(3.75N_0^{0.5}/N^{0.25} + 0.85\right)Sz^{-0.2} \tag{3}$$

where Δ is Sr-1, Dn the nominal diameter of the unit ($=V^{1/3}$), V the volume of the unit, N_0 the relative damage level defined by van der Meer (1988) as the actual number of displaced units related to the width (along the breakwater alignment) of one nominal diameter Dn , N the number of waves, Sz the wave steepness ($=2\pi H_{1/3}/gTs^2$) and, Ts the wave period.

Figure 3 shows the results from Eq. (3) with the model test data employed. The abscissa is calculated using the right side of Eq.(3) with N_0, N and Sz, and the ordinate is calculated using the left side with $H_{1/3}$, Δ and Dn.

The difference between the model test data and those calculated is not acceptable, even when the slope difference between 1:4/3 of the model data in Table 1 and 1:1.5 of Eq. (3) by van der Meer is considered. This result shows that Eq. (3) derived for the 2-layer type can not be directly used on wave-dissipating concrete blocks covering horizontally composite breakwaters. The reason behind this is considered to be mainly from the difference of permeability as van der Meer (1987, 1988) incorporated permeability into his stability formula for the armor stones.

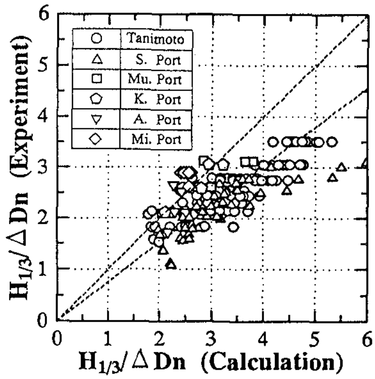

Figure 3 Comparison of van der Meer's formula with model test data

(3) Kajima's Expression of Stability Number

Kajima (1994) proposed the following expression of stability number applicable to Tetrapods for horizontally composite breakwaters.

$$Ns = H_{1/3}/(\Delta Dn) = 8.5(S/N^{0.5})^{0.16}\xi^{-0.5} \tag{4}$$

where S is the deformation level defined by van der Meer (1987) for rubble mound breakwaters as A/Dn^2, A the sectional area of erosion, ξ the surf similarity parameter ($=\tan\alpha/(H_{1/3}/L_0)^{0.5}$, L_0 the offshore wave length).

Kajima et. al. (1993) showed that the deformation parameter S is related to the damage D' (%) corresponding to the 2-layers as follows.

$$S = 0.6\,D' \tag{5}$$

Figure 4 shows the relation between $H_{1/3}/(\Delta Dn)\xi^{0.5}$ and $S/N^{0.5}$.

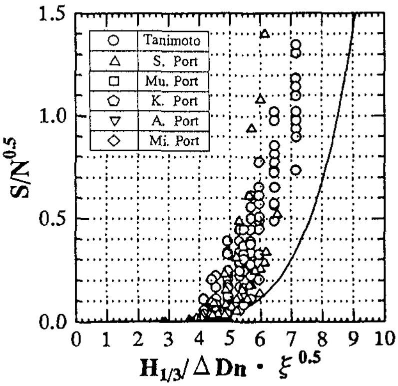

Figure 4 Relation between $H_{1/3}/(\Delta Dn)\xi^{0.5}$ and $S/N^{0.5}$

The solid line indicates the relation derived from Eq.(4). In the model test data, S was calculated from damage D' based on Eq.(5). Eq.(4) was verified by model test data from Kajima et. al. (1993). Eq.(4) however does not agree well with the model test data employed from Table 1.

One of the reasons for this disagreement is considered to be that Eq.(4) is formulated so as to cover a range of relatively high damage level which is usually not seen in the ordinary design of port structures. Also, the water depth of the structure Kajima investigated was greater than those in Table 1 resulting in a bigger cross section. This is considered to affect parameter S calculated through D' from our data.

2.3 Basis of Formulation

To establish a new stability formula, the above mentioned three formulae, i.e. Hudson, van der Meer, and Kajima were comparatively evaluated and various main parameters in the formulae were investigated. These subsequently led to the determination of the basis of a new formula as summarized below:

1) *Type of Formula*: The type of formula desired is for use to obtain the stability number $Ns=H_{1/3}/(\Delta Dn)$ by parameters of damage and number of waves.

2) *Damage*: Damage to wave-dissipating concrete blocks is expressed as N_0 following van der Meer. According to the preliminary investigation for a typical horizontally composite breakwater, the relation between N_0 and D(%) is considered as N_0=0.2 to 0.3 D.

3) *Duration of Wave Attack*: Damage to blocks is considered to be proportional to the root of the duration of wave attack (van der Meer, 1988 and Kajima, 1994). So the number of waves N is incorporated into the formula as the form $N_0/N^{0.5}$.

4) *Wave Steepness* : This parameter is treated somewhat differently among the formulae, with Fig. 5 showing how it respectively varies with respect to wave steepness. Note that the relationship derived by van der Meer and Kajima show opposite behaviors, i.e., increasing wave steepness either decreases (van der Meer) or increases (Kajima) stability. The stability itself however is relatively similar at wave steepness values from 0.02 to 0.04, and this is the normal design condition. In accordance to this discussion, wave steepness is neglected as a parameter.

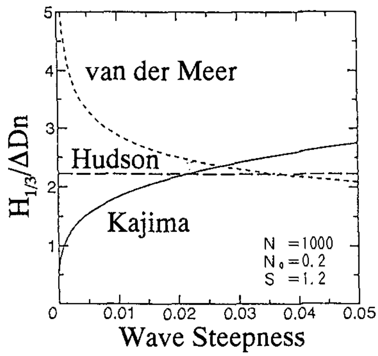

Figure 5 Comparative formula evaluation on the effect of wave steepness on stability

5) *Design for No Damage*: The formula was constructed so as to be able to determine the weight of blocks resulting in no damage.

6) *Type of Block*: Tetrapods with a 1 : 4/3 slope.

2.4 New Stability Formula

Detailed analysis of the experimental data in Table 1 was carried out on the above mentioned basis and the resultant stability formula was expressed as follows.

$$Ns = H_{1/3}/(\Delta Dn) = 2.32(N_0/N^{0.5})^{0.2} + 1.33 \tag{6}$$

Figure 6 compares the results from Eq. (6) with the model test data employed, and good agreement is clearly indicated. Figure 7 also shows another evaluation where the proposed stability formula shows markedly better agreement with experimental data when compared with Figure 3.

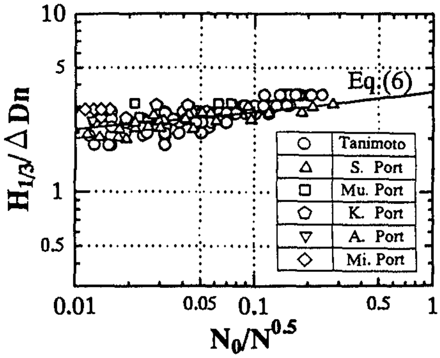

Figure 6 Comparison of proposed new formula with experimetal data

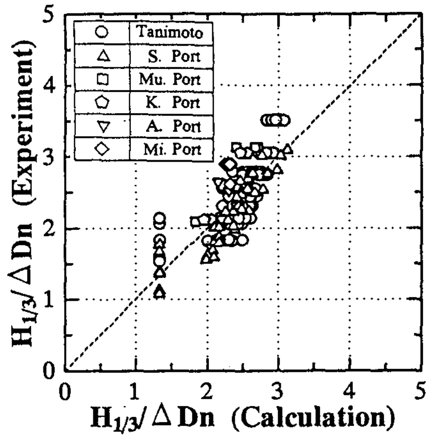

Figure 7 Comparison of proposed new formula with experimetal data

Figure 8 compares the mass of the Tetrapod using the new formula having N_0=0.1, 0.3, 0.6, 1.0 and N=1000 with that using Hudson's formula using K_D=8.3 and cotα=4/3. As shown in the figure the new formula, using N_0=0.3 and N=1000, corresponds to conventional design using Huson's formula. Figure 9 shows the mass variation against N_0 and N with the wave height of $H_{1/3}$=8.0m fixed. As shown in the figure, the change in mass against N_0 up to 0.5 and N up to 1000 is significant.

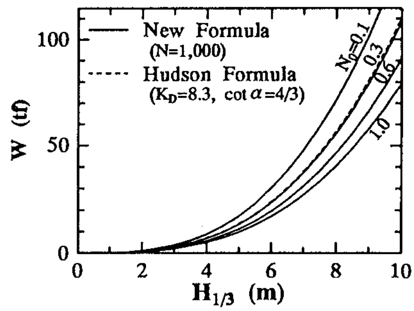

Figure 8 Relation between W and $H_{1/3}$

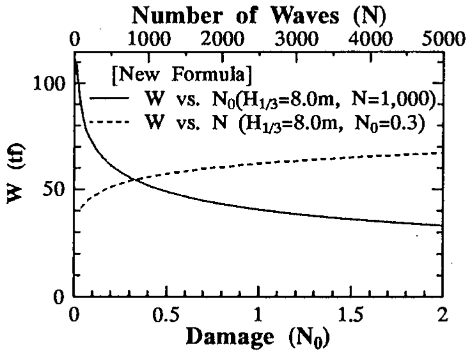

Figure 9 Mass change with N_0 and N

3. APPROACH TO PROBABILISTIC DESIGN

3.1 General Method

Reliability based design is a design methods which incorporate uncertainty and statistical deviation of design parameters. It is considered to be requisite for port and coast related structures from the view popint of economic deisign especially for wave-dissipating concrete blocks. For the design of wave-dissipating concrete blocks, damage level accumulated within the lifetime of the structure is considered to be necessary. Therefore the process of damage accumulation in structures within their lifetime should be taken into consideration.

Design for port related structures includes many parameters to be treated probabilistically such as offshore wave height, wave period, tide, wave deformation by refraction, diffraction, shoaling and breaking for wave calculation. After calculation of the waves, the statistical uncertainty of damage to blocks should be taken into account specifically for wave-dissipating concrete blocks.

In this study, the authors demonstrate a procedure for the estimation of the expected value of damage accumulated within the lifetime of horizontally composite breakwaters incorporating statistical deviation of the offshore wave height, tide, shoaling, wave breaking, and damage to wave-dissipating concrete blocks.

The Monte Carlo simulation is used in probabilistic treatment, because of the advantage in handling physical phenomena using many sequential steps like the design process mentioned here. The simulation is considered to be useful tool well supported by the advance of personal computers in recent years.

3.2 Detailed Calculation Method

(1) Offshore Wave Height

Damage to wave-dissipating concrete blocks is generally considered to be caused by large waves comparable to design waves. Therefore the yearly maximum offshore wave height is considered sufficient to be incorporated into the calculation. This offshore wave height H_{0e} is sampled from an extreme distribution of offshore waves. The estimated value of the offshore wave height is considered to include a statistical deviation. An average value H_0 is considered to follow a normal distribution with a bias (mean shift) α_{H0} , a standard deviation σ_{H0} and a deviation coefficient γ_{H0} as expressed as follows (Takayama et. al., 1994)

$$H_0 = (1 + \alpha_{H0})H_{0e}, \qquad \sigma_{H0} = \gamma_{H0}H_{0e} \qquad (7)$$

The sample offshore wave height datum H_{0c} to be used in the calculation is determined by a normalized random number based on Eq.(7).

(2) Tide

The tide is asssumed to change sinusoidally between the High Water Level and the Low Water Level. A sample of tide value η is determined based on this assumption using a uniformly distributed random number as a phase of the sinusoidal curve.

(3) Wave Transformation (Shoaling, Breaking)

The height of wave incident to the structure $H_{1/3e}$ is calculated by the Goda method (1975) and the average value $H_{1/3}$ is expressed in a way similar to the offshore wave height, i.e.

$$H_{1/3} = (1 + \alpha_{H1/3})H_{1/3e}, \qquad \sigma_{H1/3} = \gamma_{H1/3}H_{1/3e} \qquad (8)$$

where $\alpha_{H1/3}$ is the bias, $\sigma_{H1/3}$ the standard deviation and $\gamma_{H1/3}$ the deviation coefficient. The sample wave height $H_{1/3c}$ at the structure is determined by a normalized random number based on Eq.(8) similar to that used for the offshore wave height.

(4) Stability Formula and Damage

As described before, in the reliability based design for wave-dissipating concrete blocks, the total damage level should be taken into account. Therefore the following formula derived from Eq.(6) is employed in order to estimate the damage level.

$$N_0 = \left(\frac{H_{1/3}/\Delta Dn - 1.33}{2.32}\right)^5 N^{0.5} \qquad (9)$$

As for the uncertainty of damage to wave-dissipating concrete blocks, a statistical deviation of the increment of damage is taken into consideration. Tanimoto et. al. (1985) carried out a model test for wave-dissipating blocks and duplicated tests against the same wave condition with 6 to 7 wave height levels providing statistical data for damage increment. Figure 10 shows a sample of the distribution histogram of damage increment ΔN_0, while the solid line indicates the normal distribution function. The increment of damage is considered to be roughly distributed as the normal distribution function. Analysis of the data from Tanimoto et. al. shows that the larger the damage increase, the larger the standard deviation. Figure 11 shows the relation between damage increase ΔN_0 and standard deviation $\sigma_{\Delta N0}$. The solid line indicates their relation as shown below.

$$\sigma_{\Delta N0} = 0.36\Delta N_0^{0.5} \qquad (10)$$

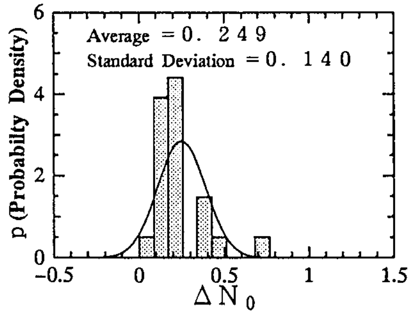

Figure 10 Example of histogram of ΔN_0

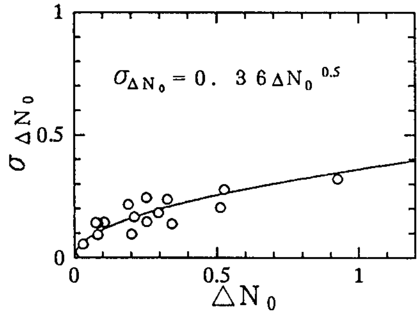

Figure 11 Relation between ΔN_0 and $\sigma_{\Delta N0}$

(5) Damage Accumulation

Damage to wave-dissipating concrete blocks is considered to occur under rough sea conditions with a range of wave height of design level or larger in a cumulative way within the lifetime of the structures. This accumulation of damage is calculated in the following manner. Supposing the wave height at the structure for the i-th year is $H_{1/3c}(i)$ and the total damage up to the (i-1)-th year is $N_0(i-1)$, the equivalent wave number N=N' is determined by the following equation derived from Eq.(6) by substituting $H_{1/3c}(i)$ and $N_0(i-1)$ into $H_{1/3}$ and N_0 respectively.

$$N = \left(\frac{H_{1/3}/\Delta Dn - 1.33}{2.32}\right)^{-10} N_0^2 \qquad (11)$$

The total damage up to the i-th year $N_0(i)$ is calculated by Eq.(9) with N=N'+N(i). The first estimation of ΔN_0 is calculated as $N_0(i)$-$N_0(i-1)$ and the sample data of ΔN_{0c}, taking into consideration the uncertainty of damage increment, is determined by the normalized random number with the standard deviation defined by Eq.(10). The estimated damage up to the i-th year is finally calculated as $N_0(i)=N_0(i-1)+\Delta N_{0c}$.

(6) General Flow of Calculation

The method described above is the procedure for calculating damage up to a certain year and the sample data of total damage accumulated within one lifetime is calculated by repeating this process for corresponding years of the total lifetime. Figure 12 shows the schematic flow of calculation. After obtaining several thousand samples, statistical values such as the expected mean of total damage can be calculated.

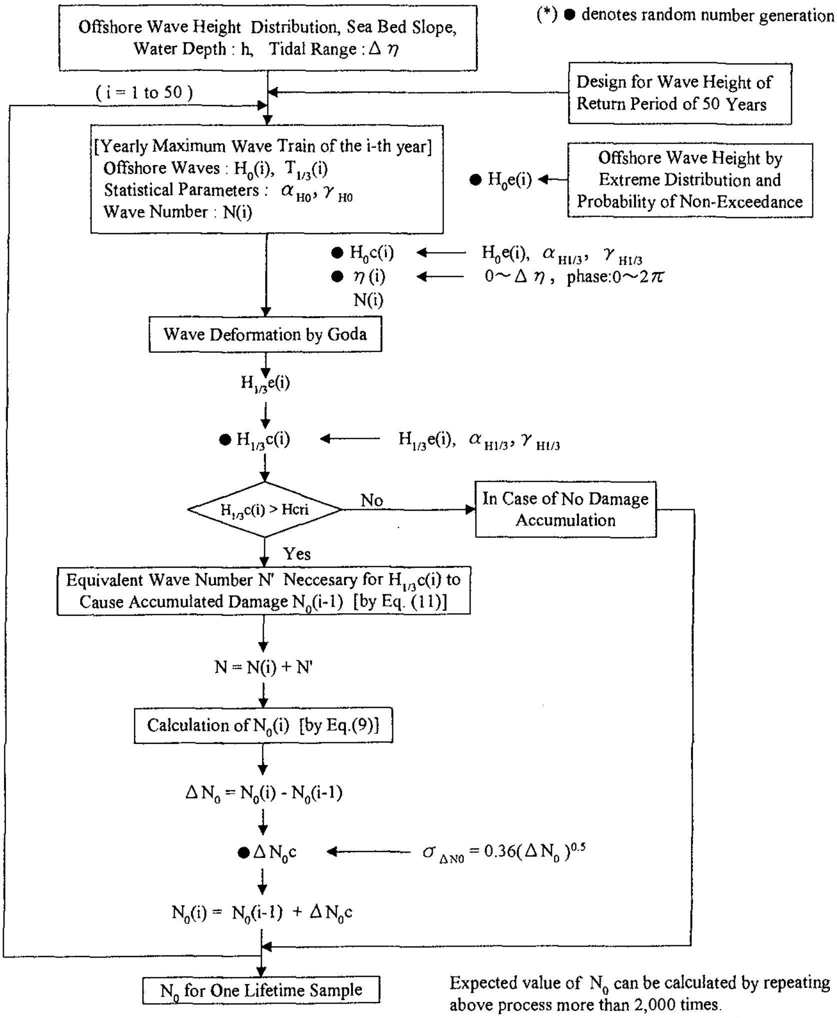

Figure 12 General calculation flow of one lifetime sample

(7) Calculation Sample of One Lifetime

Figure 13 shows an example of the offshore wave height, wave height at the structure and damage in chronological order. The design wave heights offshore and at the structure taken as a conventional manner with return period of 50 years for this example are H_0=9.2m and $H_{1/3}$=8.7m with a wave period $T_{1/3}$=14.0s. According to the figure, the offshore wave heights in the 33rd and 48th years exceeded the design level. The wave height at the structure exceeds the design level in the 33rd year and the heaviest damage is accrued in the same year. Other smaller incidents of damages were recorded in the 2nd, 19th and 48th years resulting in a total damages of 0.45 accumulated within the lifetime of this one sample.

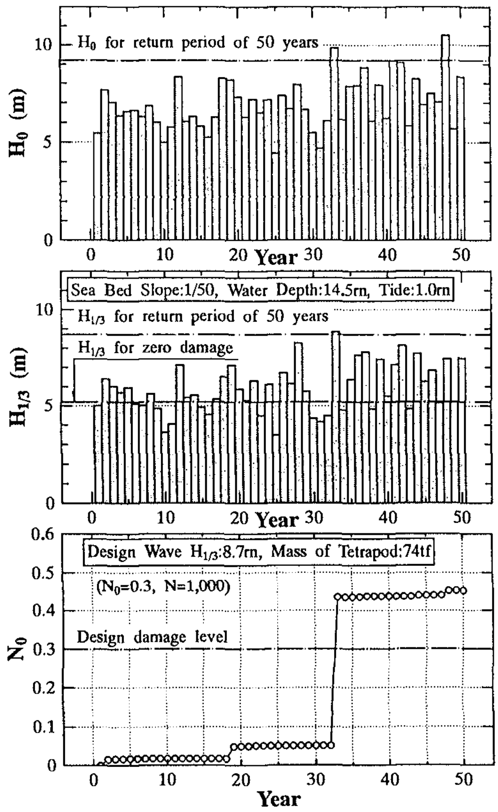

Figure 13 Example of lifetime sample

(8) Number of Lifetime Samples

In the example shown in the next sub-section, 5,000 samples of lifetime were computed for the statistical treatment for estimating total damage. Preliminary analysis, however, showed that 2,000 samples are considered to be enough to obtain a stable statistical result in this method from the practical view point.

3.3 Example of Estimation of Expected Value of Damage

In this sub-section, some examples of calculation based on the method mentioned previously are shown. The conditions of computation are as follows.

a) *Wave Height* : A Weibull distribution with parameters k=2.0, A=2.23 and B=4.78 is assumed to be the extreme function for the offshore wave height (H_0=9.2 for return period of 50 years). The parameters expressing uncertainties for the offshore wave height and wave deformation mentioned in Eqs.(7), (8) were set as $\alpha_{H0} = 0, \gamma_{H0} = 0.1, \alpha_{H1/3} = -0.13, \gamma_{H1/3} = 0.09$ (Takayama et.al., 1994).

b) *Wave Period and Number of Waves* : The wave periods for the examples here were determined so as to correspond to a wave steepness of 0.03 at the offshore. The wave period for the wave height H_0=9.2m (50 year event) is $T_{1/3}$=14.0s accordingly. The number of waves was set as 1,000 for all wave heights in these examples.

c) *Tide* : The tide range of 1.0m (L.W.L. 0.0m, H.W.L. +1.0m) was assumed

d) *Design of Wave-dissipating Concerete Blocks* : The mass of Tetrapods was calculated by Eq.(6) under the criteria of N_0=0.3 and N=1,000 against wave height for the 50 year event.

e) *Sea Bed Slope* : Sea bed slopes of 1/50 and 1/10 were employed.

f) *Water Depth* : Water depths of 7, 9, 11, 13 and 15m at low tide were examined.

Table 2 summarizes the parameters of the simulation such as the design wave heights and masses of the Tetrapods at each location based on the conditions above in a conventional design for a 50 year event.

Table 2 Design wave height and mass of Tetrapod

Case No.	1	2	3	4	5
Sea Bed Slopes	1/50	1/50	1/50	1/50	1/50
Water Depths (m)	7.0	9.0	11.0	13.0	15.0
Design Wave Heights (m)	5.57	6.64	7.59	8.34	8.81
Mass of Tetrapods (tf)	18.7	31.8	47.6	63.0	74.3
Case No.	6	7	8	9	10
Sea Bed Slopes	1/10	1/10	1/10	1/10	1/10
Water Depths (m)	7.0	9.0	11.0	13.0	15.0
Design Wave Heights (m)	8.16	9.41	10.18	10.31	10.13
Mass of Tetrapods(tf)	59.0	90.5	114.5	118.9	112.9

Figure 14 shows the variation of the expected value of maxH/Hs with water depth, where maxH is the maximum significant wave height at each location within the lifetime, Hs is the design wave height. As shown in Figure 13, the total damage to the wave-dissipating concrete blocks was significantly affected by the maximum wave height within the lifetime. The expected value of maxH/Hs tended to decrease against the water depth in the case of the sea bed slope of 1/50. However, it had a minimum at around 11m for a 1/10 slope.

Figure 15 shows the calculated results of the expected accumulated damage at each location. The expected accumulated damages for these examples are estimated as larger than the design criteria of N_0=0.3. This is considered reasonable because of the possibility of occurence of a wave height larger than the design level. The damage accumulation is accurately taken into account as shown in Figure 13.

From the conventional deterministic design view point, the masses shown in Table 2 are considered to equally assure the damage level of N_0=0.3 against each design wave height with a safety factor of 1.0. The present simulation shows that the expected total damage within the lifetime on the probabilistic basis depends on such parameters as the sea bed slope and water depth. It is also found that the expected value of total damage to wave-dissipating concrete blocks varies corresponding to the variation of maxH/Hs shown in Figure 14.

Figure 15 also shows the effect of mass increase on the damage in the case of a 1/50 slope. It can be roughly said that a 10% to 20% mass increase caused a 30% to 50% decrease in total damage accumulated. This result means that the mass of the wave-dissipating concrete block might be adjusted to account for certain conditions by some percentage higher than that determined by the stability formula when based on the probabilistic design. This would assure the same level of total damage within the lifetime.

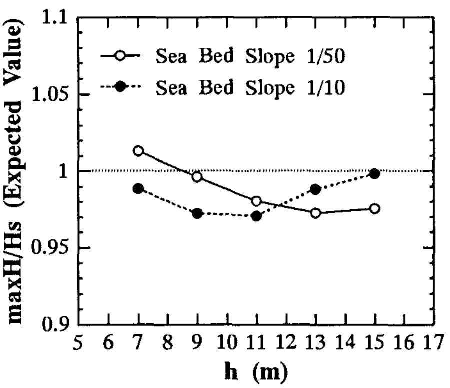

Figure 14 Expected values of maxH/Hs

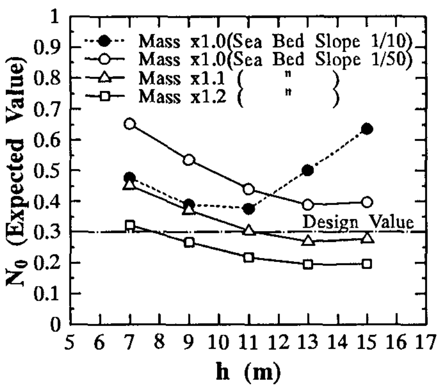

Figure 15 Expected values of N_0

4. CONCLUSIONS

1) A new stability formula for wave-dissipating concrete blocks covering horizontally composite breakwaters was proposed and the applicability of this formula was verified by experimental data.

2) A probabilistic approach to the estimation of total damage to wave dissipating concrete blocks accumulated within a life time by Monte Carlo simulation was proposed and the applicability of this method for reliability based design of wave-dissipating concrete blocks was proved.

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