CHAPTER 100

PERFORMANCE OF A RESONATOR DESIGNED BY THE WAVE FILTER THEORY — APPLICABILITY TO A HARBOR

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ABSTRACT

A wave resonator, which has a comparatively wide opening and still keeps effectiveness for a wider range of wave frequency, is newly developed based on the Wave Filter Theory. Applying this resonator both to a long channel and a harbor entrance, the sheltering effect for incoming waves was examined by the numerical and physical experiments. It was confirmed that the Wave Filter theory is very effective to design a wave resonator for required conditions, such as an effective band width of wave frequency.

INTRODUCTION

In the 1950s, Valembois(1953) has already presented an idea of resonators to reduce incoming waves to harbors and canals by installing resonators at the entrance. In this attempt, the shape of a resonant basin is fixed to the definite one; the ratio of a length to a width of the resonant basin is 2:1. It was reported that the effective range of a wave frequency is comparatively narrow. In order to expand the effective range, he has proposed a series of resonators with different dimensions instead of a single resonator. However, in his study, the influence of an opening length between the two facing resonators on the sheltering effect has not been clarified. Consequently, it may be very hard to out find the total geometry of a resonator-type breakwater.

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Recently, Mochizuki et al. (1990) have developed a new method to design a resonator that is effective for a wider frequency range based on the electric filter theory, which is called the Wave Filter Theory. Nakamura et al. (1994) may be very hard to find the total geometry of a resonator-type have conducted a series of experiments on a resonator designed by the theory in a long wave flume. However, applicability of the resonator to a fully three-dimensional wave field, such as harbor tranquility problems, has not been verified sufficiently.

In this study, using the Wave Filter Theory, a resonator that is effective for a wider range of wave frequency is first obtained. Especially, intending the resonator at harbor entrances, a resonator with a wider opening is presumed for passing ships. Experimental verification of this resonator was carried out in a long wave channel to check the principal validity of the Wave Filter Theory. Finally, for protecting a harbor from incoming waves of wider frequency range, the designed resonator is installed at the entrance. Numerical experiments on the wave height distributions in the harbor were carried out to examine the effectiveness of the resonator. Some additional case studies were also given to compare with the result of a resonator case.

(a) Wave resonators in series.

(b) Equivalent electric circuit to the wave resonators.

Fig. 1 Analogy between the wave resonators and the electric filter.
RESONATOR DESIGNED BY THE WAVE FILTER THEORY

Mochizuki et al. (1990) have presented a rational way to determine a geometrical shape of the resonator that is effective for a specified wave frequency range. The basic idea is to consider the analogy between a wave resonator and its equivalent electric filter. In the field of an electric filter, filter actions of various electric filters are well known. Consequently, considering the corresponding parts between the resonator and its equivalent electric filter, we can easily find out the geometrical shape of the resonator.

In the theory, a resonator model as shown in Fig. 1(a) is assumed. The corresponding electric filter to the resonator is given in Fig. 1(b). In the latter figure, \( L_a \) and \( L_b \)=inductance, \( R_0 \)=resistance, and \( C_b \)=capacitance. This equivalent electric circuit is known as a derived m-type low-pass filter. By considering the analogy between the two models and also the frequency characteristics of the electric filter, we can derive design equations of a wave resonator for required conditions.

Here, we made some simplifications for convenience of practical designs. At first, we assume that the long wave approximation can be used. Secondly, a longitudinal length of the first and third resonant basin in Fig. 1(a) is equal to a half of that of the center one, i.e., \( b_2=b_3/2 \). By use of these assumptions, the following equations are given (Mochizuki et al. 1990)

\[
\begin{align*}
    b_3 &= \frac{mb_0}{\sqrt{2(1-m^2)}} \\
    l_2 &= \frac{1}{\pi f_c} \sqrt{\frac{gh(1-m^2)}{2}} \\
    l_1 &= \frac{m \sqrt{gh}}{b_1 \pi f_c b_0}
\end{align*}
\]

(1)

where \( g \)=gravitational acceleration, \( h \)=water depth, and \( m \) is a function of the critical frequency \( f_c \) and the pole frequency \( f_\infty \) of an effective frequency band \( (f_c<f<f_\infty) \) of a wave resonator and is defined by

\[
m = \sqrt{\frac{1 - \left(\frac{f_c}{f_\infty}\right)^2}{1 - \left(\frac{f_c}{f_\infty}\right)^2}} \quad \text{(2)}
\]

The pole frequency \( f_\infty \) is a frequency at which wave transmissions through a resonator become the lowest.
Fig. 2 shows a typical result by use of Eq. (1), which is designed under the required conditions given in Table 1. In the figure, however, only the center part of a series of resonators is adopted. Nakamura et al. (1994) have reported that such a single resonator has similar filter actions to a series of resonators as seen in Fig. 1(a). They also pointed out that the aligned resonators have a sharper cutoff feature of filtering actions than the single resonator. For economical and practical designs, it might be said that a single resonator is much more acceptable than the aligned resonators. Therefore, in the followings, we will use the single resonator as shown in Fig. 2 as a model.

![Figure 2](image)

**Fig. 2** A resonator designed by the Wave Filter Theory.

### Table 1 Design conditions for a resonator

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical frequency $f_c$</td>
<td>1/20 Hz</td>
</tr>
<tr>
<td>Pole frequency $f_p$</td>
<td>1/11 Hz</td>
</tr>
<tr>
<td>Opening length $b_0$</td>
<td>50 m</td>
</tr>
<tr>
<td>Water depth $h$</td>
<td>15 m</td>
</tr>
</tbody>
</table>

**EXPERIMENTAL RESULTS IN A CHANNEL**

**Experimental setup**

In order to check the validity of the Wave Filter Theory, we carried out a model test in a long channel with a width of 1m, a height of 1.2m and a length of 26m. Using the 1/60 model of the resonator shown in Fig. 2, wave transmission and reflection characteristics were examined. In the experiment, as shown in Fig. 3, we built a half of the single resonator in the channel. There are two different types of model structures with the same dimensions, but having different reflection characteristics. One model is made of plywood and shows a high reflective nature. The other model is made of steel nets and pebbles and shows a low reflective nature.

Considering the mirror image effect of side walls of a channel, the wave field in the channel described above is equivalent to the one around an infinite array of identical resonators. In the above case, an allocation pitch length $\lambda$ of arrayed structures is equal to double of a channel width.

According to the recent studies (Darlymple et al. 1990; Nakamura 1994), it has become known that the wave pattern around an arrayed bodies is short-crested, especially under the condition in which a wave length $L$ is less
than a pitch length \( \lambda \) of the array. This is due to the generation of obliquely transmitted and reflected wave trains about the arrayed bodies in addition to the usual scattered waves as seen in a vertical two-dimensional wave field.

In order to be able to analyze the short-crested wave pattern in the channel, we set a linear array of five wave gauges in the transverse direction to measure transmitted waves through a resonator. Wave conditions used in the experiment are as follows; a wave period \( T_m \) ranges from 0.9 sec to 2.6 sec (in prototype scale \( T = 7 \sim 20 \) sec) and a wave height \( H_m \) is fixed at about 4 cm (in prototype scale \( H = 2.4 \) m). Above conditions of a wave period \( T_m \) correspond to the dimensionless parameter \( \lambda / L \) being from 0.5 to 1.8.

![Fig.3 Experimental setup (model scale 1/60).]

![Fig.4 Ratio of a RMS transmitted wave height to an incident wave height.]

**Fig.3** Experimental setup (model scale 1/60).

**Fig.4** Ratio of a RMS transmitted wave height to an incident wave height.
**Wave transmission characteristics**

Fig. 4 shows typical results, where the dimensionless transmitted wave height $K_T$ is plotted as a function of a wave period $T$ in the prototype scale. A parameter $\lambda/L$ is also specified as the second horizontal axis. Where $K_T$ is defined by

$$K_T = \frac{(H_T)_{rms}}{H}$$  \hspace{1cm} (3)

$(H_T)_{rms}$ is a root-mean-square (RMS) value of transmitted wave height along the array direction, which is consistent with the transverse direction of a channel. The RMS value is necessary to account for the short-crested wave pattern in the transmission side of the channel, especially under the condition $\lambda/L > 1$. In other condition where $\lambda/L < 1$, the definition of $K_T$ coincides with that of a well known transmission coefficient.

In the figure, two different experimental results are shown. One is for the resonator consisting of high-reflective walls. Other is for dissipative walls made of steel nets and pebbles. Further, in the figure, the calculated result by the Green's function method is also plotted. Nakamura (1992) has developed a method of numerical analysis to analyze wave transformations about an infinite array of vertical bodies by the wave source distribution method.

From this figure, we can see reasonable agreements between the calculated and experimental results, except near the condition $\lambda/L = 1$. The spike-like variation of $K_T$ is caused by the wave resonance in the transverse direction. It is seen that there is little influence of the different wall types on the wave transmission. As an important point, we can say that the Wave Filter Theory is useful to design a resonator for required conditions, such as a target band width of wave frequency ($f_c < f < f_o$) and a necessary opening length. A small discrepancy of the pole frequency $f_o$ between the Wave Filter Theory and the calculated and experimental results may be due to the assumption of the long wave approximation as described in the Wave Filter Theory.

**Wave height variations in a resonator**

Figs 5 and 6 show typical measurement results of wave height distributions in a resonator, in which the wave height is normalized by the incident wave height. These two figures correspond to the two different wall-type cases, i.e., a high-reflective wall case and a low-reflective wall case, respectively. The condition of a wave period $T$ used in these figures is approximately consistent with that where $K_T$ becomes minimum as seen in Fig. 4.

From these figures, we can see that $K_T$ becomes comparatively small when a node of two-dimensional standing waves appears at the transmission-side mouth of a resonator. It is also seen that the maximum wave height in
the resonator becomes about two times as high as the incident wave height when the wall-type is high reflective. By changing the wall-type of a resonator to the low reflective one, it is possible to reduce the maximum wave height effectively, say by one half. For the practical use of a wave resonator at a harbor entrance, it may be required to reduce the maximum wave height in a resonant basin to the order of an incident wave height for safety operation of ships. Therefore, a resonator consisting of low-reflective walls is highly recommended, especially for the use at a harbor entrance.

**NUMERICAL EXPERIMENT ON THE HARBOR TRANQUILITY**

**Numerical method**

In order to check the applicability of the resonator described above to a harbor, numerical experiments were carried out for a harbor with a typical shape in Japan. The vertical line source Green's function method developed by Isaacson (1978) was mainly adopted. Nakamura et al. (1985) precisely examined singular functions included in the wave source function and modified the numerical method so that a thin-walled structure can be treated as compared to a wave length. In the numerical analysis, reflection characteristics from harbor boundaries can be treated generally by using
partially absorbing boundary conditions (Engquist et al. 1977) instead of well-known no-flux conditions.

Fig. 7 Layout of a harbor and wave reflection characteristics of the boundary.

Fig. 8 A harbor entrance with a resonator.

Fig. 9 A harbor entrance with a detached breakwater.
**Model harbors**

Fig. 7 shows a layout of an original harbor used in the numerical experiment. It is assumed that a water depth $h$ inside and outside a harbor is constant and equal to 15m. Reflection characteristics of harbor boundaries are specified in the figure by using reflection coefficients $C_R$. We presume that the boundary outside the harbor of $C_R=0.4$ is covered with dissipative concrete blocks. Further, the boundary inside the harbor with the same

![Image](image_url)

**Fig. 10** Wave height distributions around the original harbor (T=9sec).

![Image](image_url)

**Fig. 11** Wave height distributions around the harbor with a resonator of high-reflective walls (T=9sec).
Fig. 12 Wave height distributions around the harbor with a resonator of low-reflective walls (T=9sec).

Fig. 13 Wave height distributions around the harbor with a detached breakwater (T=9sec).

$C_R$ is an inclined quay. Other boundaries of $C_R=0.8$ are vertical sea walls. In the experiment, three different models are examined; an original harbor as shown in Fig. 7, a harbor with resonators at harbor entrances (Fig. 8) and a harbor with detached breakwaters outside harbor entrances (Fig. 9). Regarding the resonator case, we have adopted the same shape specified in Fig. 2. Two different reflection characteristics from inside walls of a resonator are again used, a high reflective wall case of $C_R=1.0$ and a low
reflective wall case of $C_R=0.6$, respectively. In all cases, an opening length of each harbor entrance is fixed at 50m.

Inter-comparisons of wave height distributions among different cases described above are shown in Figs.10 to 13 by the use of normalized wave height contours. A normal incidence of plane waves to harbor entrances is presumed. Further, the wave period $T$ is fixed at 9sec, which corresponds to the condition of minimum wave transmission for the case of arrayed resonators as shown in Fig.4.

**Fig. 14** Averaged wave height ratio in a harbor for various types of breakwaters.

**Fig. 15** Averaged wave height ratio in a harbor for obliquely incident waves, 60-degree incidence.
We can see that the resonator with high-reflective walls is the most effective breakwater to reduce wave heights in the harbor. The resonator with low-reflective walls is also very useful as compared to the detached breakwater case. However, it must be noted that the maximum wave height in a resonant basin reaches more than twice as high as an incident wave height for the resonator case with high-reflective walls. Therefore, it is again confirmed that a resonator with low-reflective walls is highly recommended. In this case, the maximum wave height in a resonant basin is reduced to the order of an incident wave height.

Fig. 14 shows the variation of a representative wave height ratio \((K_d)_{av}\) in a harbor with a wave period \(T\) for each case described above. Where \((K_d)_{av}\) is a spatially averaged wave height ratio in a harbor. It can be seen that a resonator designed by the Wave Filter Theory is very effective to reduce incoming waves for a comparatively wide range of wave period. It is also seen that the resonator with low-reflective walls has a wider effective range of wave period than the one with high-reflective walls, especially for longer waves.

Fig. 15 also shows the variation of \((K_d)_{av}\) with \(T\), but under the condition of obliquely incident of waves (incident angle of waves is 60-degree from the x axis). From this figure, it is confirmed that the resonator is still effective for obliquely incident waves to harbor entrances. The reduction of a wave height in a harbor is almost the same for the case of normally incident waves as shown in Fig. 14.

**CONCLUSIONS**

The Wave Filter Theory is very useful to design a resonator for required conditions, such as an effective range of wave frequency \((f_c<f<f_a)\) and a necessary opening length. It was confirmed that the resonator designed by the Wave Filter Theory plays an effective protector for reducing incoming waves to harbors and canals.

A resonator consisting of low-reflective walls is highly recommended for the use at a harbor entrance because of safety operations of ships. It is also possible to expand the effective range of wave frequency of a resonator by the use of low-reflective walls.

**REFERENCES**


