CHAPTER 72

INFLUENCES OF SPECTRAL SHAPES ON THE STATISTICAL PROPERTIES OF SIMULATED RANDOM WAVES

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Abstract

Random wave fields were generated both numerically using a computer algorithm, and mechanically through a computer-controlled irregular wave generator. Statistical properties of these synthesized random waves are then compared with measured field results. It is shown that, irrespective of the methodology used, wave heights obtained from simulation always have a Rayleigh distribution, and surface elevations are approximately normally distributed, measured field data can, however, show other trend for the former with a large percentage of possibility.

Introduction

Random wave simulations provide a convenient tool for coastal engineers to study the stochastic properties associated with a wave field. Simulation can be carried out either numerically on a digital computer (Hudspeth & Borgman, 1979; Hudspeth & Chen, 1979), or physically through a computer-controlled wave generating device (Svendsen, 1985; Takayama, 1990). Tuah & Hudspeth (1982) compared the numerical methods then available, and a review was given recently by Hughes (1993). Depending upon the desired characteristics to be modeled, simulations can be carried out either in frequency (Hudspeth & Chen, 1979), or in time domain (Mo, 1993). In frequency domain, a white noise spectrum from certain pseudo-random number generator algorithm is filtered through a target spectrum. Surface elevations are then obtained by inverse Fourier transforming the simulated spectral densities.

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For laboratory experiments, a (nonlinear) transfer function connecting the actual strokes of the wave generating device and simulated results must be further implemented.

Although not stated explicitly, it is often assumed that waves thus generated have all the characteristics that are in close agreement with those of actual wave fields (Kimura, 1981; Sobey, 1992). Deterministic properties obtained through analyzing simulated surface elevations are then used to verify theoretical predictions. Kimura (1981), for example, used a two-dimensional Weibull distribution to model the joint distribution of wave heights and periods of mechanically generated random waves. He found that the so-called shape parameter for the marginal distribution of wave heights has a value close to 2. Sobey (1992) studied the distributions of zero-crossing wave heights and periods of numerically simulated random wave fields, and concluded that, wave heights "*differ little from Rayleigh distribution.*" It should be noted that since Rayleigh distribution is a special case of the Weibull distribution with a shape parameter equals to 2, the findings of these two researchers, therefore, all indicate to the same fact that, simulated random waves have heights that are Rayleigh distributed.

It is well known that Longuet-Higgins (1952) derived the Rayleigh model based upon the assumptions of narrow-banded wave fields and normality of the surface fluctuations. Even though this model was often shown to overpredict larger wave heights (Chakrabarti & Cooley, 1977), most researchers, however, do find it rather satisfactory (Dattatri et al., 1979).

For the majority of studies concerning random wave simulations, a theoretical and/or empirical spectrum is often used as target spectrum. Spectra carrying the names such as Bretschneider-Mitsuyasu, Pierson-Moskowitz [P-M], or JONSWAP are the most frequently used ones. Among these spectra, the first two are often considered to represent fully developed seas and can be categorically termed as broad-banded, while the JONSWAP spectral shape is designated for growing wind-seas, and is thus narrow-banded.

Even if simulated random wave fields may have spectral shapes that have close resemblance with targets, it would be justifiable to ask whether other deterministic properties of the simulated wave fields, such as the probability distributions of wave heights and periods, would also bear the same affinity with reality. This assumption can be questioned when one considers that measured spectra are sometimes wider and flatter, and sometimes more energetic and narrower than theoretical predictions. Under these circumstances, it is not clear how close the deterministic statistical properties obtained from simulated random waves would match those of measured ones. Direct comparisons with field data are, as far as the authors are aware of, rather limited. This problem is addressed here in some detail.

In the following, the remaining of this paper is further divided into three parts. In section II a short description of the methodologies applied will be given. Section III presents some of the experimental results, and with a discussion in section IV we conclude this paper.

The methodology

Waves were also generated mechanically in a large wave flume of the Department of Harbour and River Engineering in National Taiwan Ocean University. The wave flume has a dimension of 100 × 3.0 × 3.0 (length × width × height, in meters). Two series of experiments were conducted. For the first series of experiments only five wave gauges were installed. As the results of these experiments were not quite satisfactory, a second series of experiments with eight wave gauges was carried out. Variations of surface elevations along the flume were measured by capacitance wave gauges. Locations of the measuring stations are: 3.0, 12, 22, 32, and 42 meters for the first series of experiment, and 3.0, 12.0, 32.0, 42.0, 71.5, 73.4, 75.0, and 76.1 meters for the second series, away from the piston wave generator. Figure 1 shows the experimental setups for the first series of experiments.

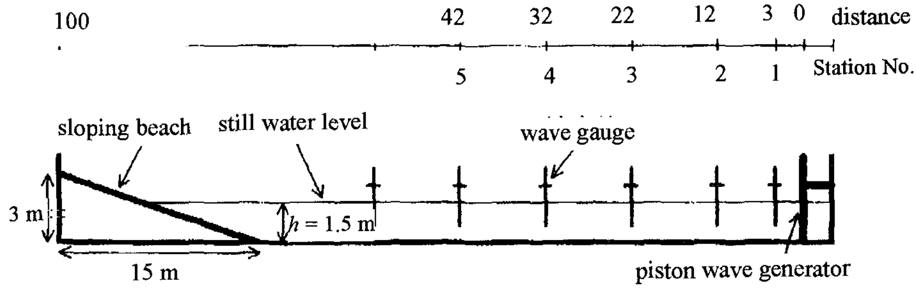

Fig. 1 Schematic representation of wave generation facilities.

Throughout the experiments, water depth was kept constant at 1.5 meter. As target spectrum, the theoretical five-parameter JONSWAP spectrum is used. Experimental conditions were varied by changing values of the parameters of the target spectrum. A recording time of 330 seconds (5.5 minutes) was used for all experiments so that statistical equilibrium in the wave flume was always achieved. Surface elevations measured by wave gauges and the amplified analog signals were then digitized by an A/D converter of type TEAC DR-F1. The sampling frequency is 50 Hz, so that each data set contains a total of 16,500 sampling points. Table 1 summarizes all the experimental conditions used.

During spectral analyses, wave records were further divided into segments, each containing 1024 data points. The degrees of freedom for the spectral density estimates are 32, with a frequency resolution equal to 0.048828. A total of 71 for the first, and 66 for the second, series of experiments were conducted.

Table 1 Experimental conditions (Symbols for f_p: a = 0.3; b = 0.5, c = 0.6, d = 0.8, and e = 1.0 Hz)

f_p	a, b, c, e	e	a, b, c, d, e	e	a, b, c, d, e	a, b, c, d, e	a, b, c, e	a, b, c, d	a, b, c, d, e	a, b, c, e
γ	1.0	1.0	3.0	3.0	3.3	3.3	3.3	3.3	3.3	3.3
σ_a	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.15	0.15	0.15
σ_b	0.09	0.3	0.09	1.0	0.09	0.3	1.0	0.09	0.3	1.0
f_p	a, b, c, d, e	a, b, c, d, e	a, b, c, d, e	a, b, c, d	e	a, b, c, d	e	a, b, c, d	e	--
γ	3.3	3.3	3.3	5.0	5.0	7.0	7.0	9.0	9	--
σ_a	1.0	1.0	1.0	0.07	0.15	0.07	0.15	0.07	0.15	--
σ_b	0.09	0.3	1.0	0.09	0.09	0.09	0.3	0.09	1.0	--

Field data were recorded with a pressure gauge in a two hour interval from October 1991 to October 1993 in northern Taiwan. The measuring station is located 800 m away from coastline where the mean water depth is 16 m. Total length of the data is 10 minutes with a sampling rate of 2.0 Hz. The field records were first divided into segments each containing 256 points. A cosine squared taper function was used to eliminate the discontinuities at the beginning and end of each record. The FFT calculated spectral densities were then smoothed with a Hanning window. The degrees of freedom and the frequency resolution are, 24 and 0.0078, respectively.

Numerical simulations were carried out using the two algorithms originally proposed by Tuah & Hudspeth (1982), with an extra term representing nonlinear correction as given by Duncan & Drake (1995). As target spectra, both measured and theoretical spectra were used. Every target spectrum was simulated 205 times and all the parameters obtained were averaged to account for statistical variability. All analyses were carried out on an IBM 586 compatible personal computer.

Traditional zero-crossing methods were applied for all "wave records." Characteristics of numerical simulations are compared with those obtained from measurements, either from laboratory, or field. Since the program for generating measured spectra in wave flume still needs some adjustments, only field spectra with shapes that can be fitted into the standard JONSWAP form were used for comparison. Probability distributions of wave heights, and surface elevations from all these three sources were compared with each other. Three models, namely, the Rayleigh, the Gaussian, and the two-parameter Weibull, were used to test the distribution of wave heights. Surface elevations were fitted using a Gaussian and a Gram-Charlier series expansion.

<u>The results</u>

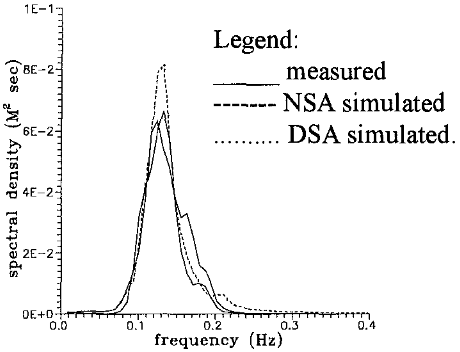

Fig. 2 Comparison of measured and simulated spectral densities: _____ measured; ----- NSA simulated and .... DSA simulated. Field data taken at 17.00, on February 4, 1992.

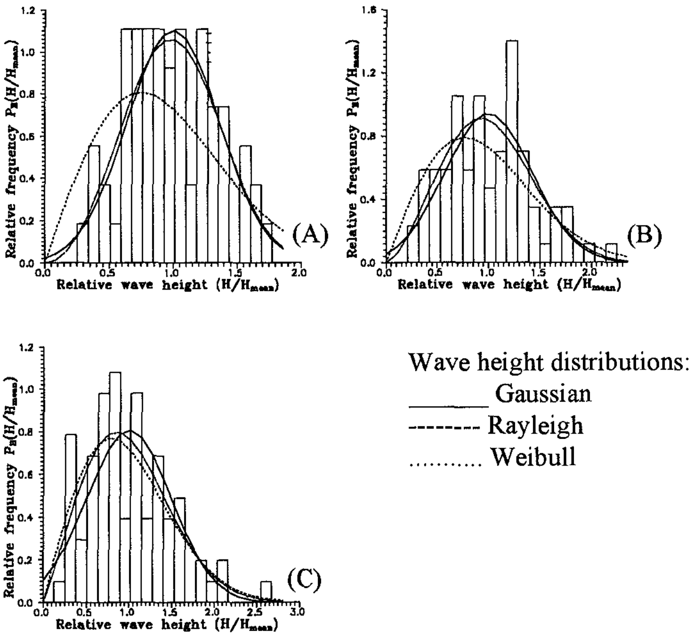

Fig. 3 Wave height distributions from measured (Fig. 3A), NSA simulated (Fig. 3B) and DSA simulated (Fig. 3C). Actual data source same as Fig. 2.

Even though spectral densities of target spectra can be modelled rather satisfactorily through the present algorithms, deterministic properties of a wave field can not be reproduced truthfully. Figure 3 compares measured wave height distribution with the 100th numerically simulated results. It should be mentioned that this choice is purely arbitrary. Three curves, representing, respectively, the Gaussian, the Rayleigh, and the Weibull distributions, were also plotted along with the histograms for comparison. As can be seen, simulated wave heights have distributions that are very close to the theoretical Rayleigh (Figs. 3B & 3C), whereas that of measured results can be approximated with Gaussian or the Weibull distribution (Fig. 3A), either one can fit the data better than the Rayleigh.

Distributions of measured and simulated surface elevations were plotted in Figure 4 for comparison. Also shown in the figure are the theoretical curves representing, respectively, normal distributions (solid lines) and the Edgeworth's form of the type A Gram-Charlier series (dotted lines). As can be seen from Figure 4, all surface elevations have probability distributions that are approximately Gaussian. Except causing small deviations from normality (Figs. 4B & 4C), nonlinear correction does not seem to a profound influence on the simulated results. In the following, since results concerning distributions of surface elevations are rather similar for all the cases studied, figures concerning this topic will not be presented for space reasons.

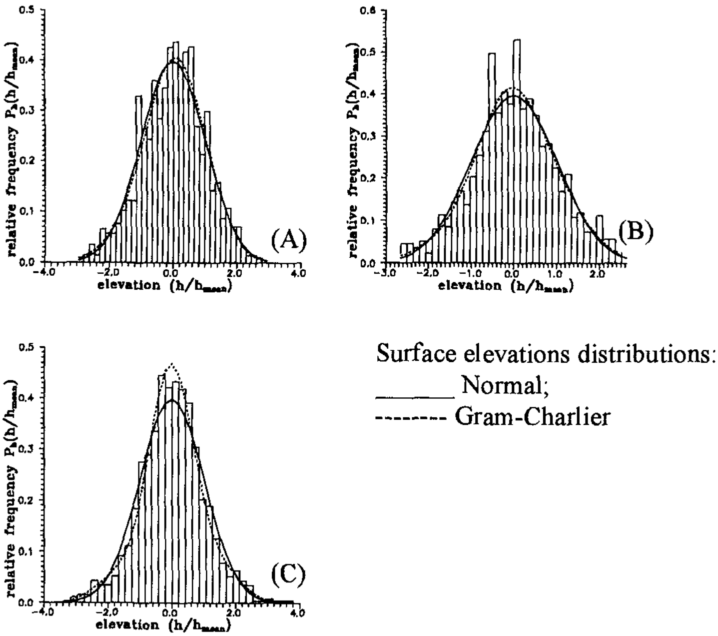

Fig. 4 Comparison of measured and simulated probability distributions for surface elevations. Original data source same as Figure 2.

As mentioned before, experiments were conducted by varying the parameters of the typical five-parameter JONSWAP spectrum. Figure 5 shows the development of the spectra along the wave flume for the first four measuring stations. Conditions used for this example is: peak frequency f_p = 0.5 Hz, peak enhancement factor γ = 1.0, with all other parameters equal to the standard JONSWAP spectrum. Nondimensionalized theoretical spectra are also shown in every figure for comparison. It is seen that, except for the irregularities occurred at approximately 0.75 Hz, the target spectra are reproduced quite satisfactorily.

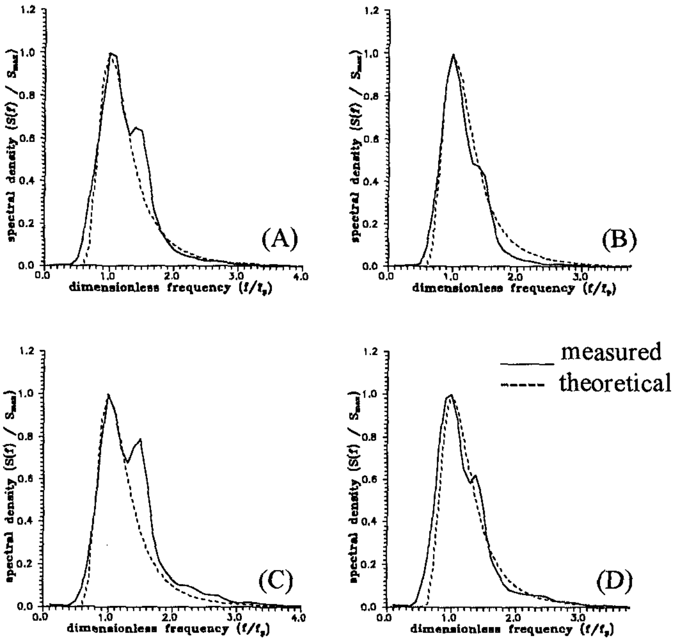

Fig. 5 Measured and theoretical spectra. Experimental conditions: f_p = 0.5 Hz, peak enhancement factor γ = 1.0.

Figure 6 shows the distributions of wave heights along the wave flume. As is seen from this figure, all the three models used seem can be used quite satisfactorily. In fact, judging from the χ^2 goodness-of-fit test results (Table 2), no definite conclusion can be drawn as which one of the models applied is more appropriate for this case. As can be seen from Table 2, based upon the χ^2 test results, the Gaussian model should sometimes be rejected. However, the numbers of misfits are too small to be conclusive.

Results shown in Figure 6 are not inconsistent with theoretical predictions. As is well known, Longuet-Higgins (1952) derived the Rayleigh distribution for wave heights based upon the assumptions of narrow-bandedness and Gaussianality of the surface elevations. With $\gamma = 1$ of the JONSWAP spectrum, the case shown in Figures 5 & 6 represents, however, Pierson-Moskowitz spectrum for a fully-developed wind-sea. A fully-developed wave field has energies more widely spread over low- and high-frequency component waves, as compared with the case for growing sea. Wave heights for a typical P-M spectrum should, therefore, different from those for JONSWAP.

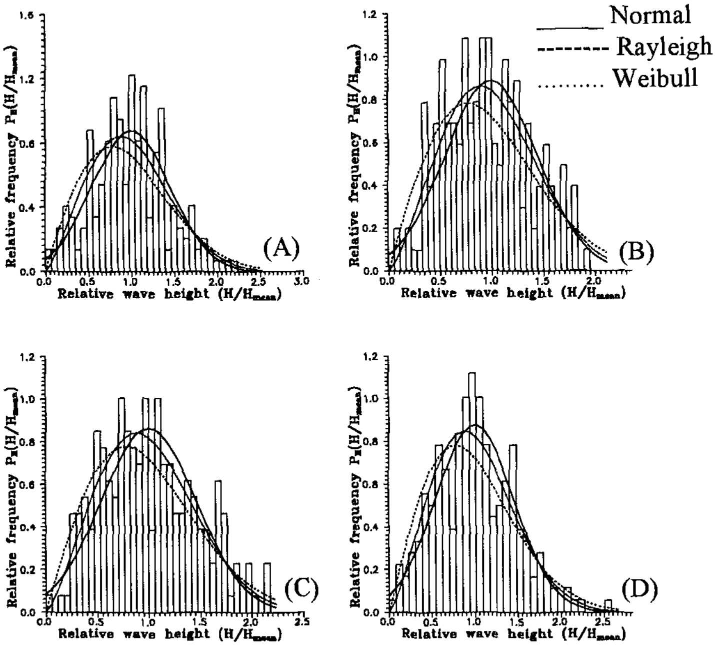

Fig. 6 Distributions of wave heights for the first four stations along the wave flume. Experimental conditions same as Fig. 5.

Table 2 shows values of the χ^2 goodness-of-fit test results for the second series of experiments. For every peak frequency considered, there are occasions where the χ^2 test results of the fitted normal distribution are too large for the theoretical values of the χ^2 distribution based on a 90% confidence interval. This is called misfit here in this paper and the total count of misfits for every peak frequency is shown in the square parentheses in the table. Since for the other two models, namely Rayleigh and Weibull distributions, no misfits occurred, it seems that the normal distribution is probably not a good model for simulated wave heights in a wave flume. However, it seems worth mentioning that the maximum count of misfits is limited to 3, for peak frequency $f_p = 1.0$ Hz at station 3. Considering the total number of experiments for this series, namely 66, it seems also safe to say that this model can not be totally rejected as well. It should, however, be remembered that among other available tests, the χ^2 goodness-of-fit test is not a very stringent one and perhaps other tests should be used before any definite conclusion can be drawn.

Table 2 Statistical properties of χ^2 goodness-of-fit test results for wave height distributions. Figures shown are mean values, figures in the parentheses represent standard deviations, and figures in the square parentheses are number of misfits, i.e., the model should be rejected.

frequency (f_p) [Hz] / χ^2 test results		0.5	0.8	1.0
Normal	Station 1	5.47 (6.02) [0]	4.05 (1.68) [2]	5.66 (4.35) [1]
	Station 2	3.22 (0.96) [1]	4.94 (4.47) [1]	5.48 (3.35) [1]
	Station 3	3.13 (0.98) [0]	5.46 (4.36) [1]	6.50 (4.01) [3]
	Station 4	5.81 (4.81) [2]	4.19 (1.85) [1]	6.65 (5.58) [1]
Rayleigh	Station 1	3.68 (1.09) [0]	3.28 (0.83) [0]	3.12 (0.90) [0]
	Station 2	3.24 (0.80) [0]	2.39 (0.60) [0]	2.99 (0.67) [0]
	Station 3	4.79 (1.17) [0]	3.18 (0.75) [0]	2.91 (0.85) [0]
	Station 4	3.39 (0.94) [0]	2.87 (0.64) [0]	3.17 (0.58) [0]
Weibull	Station 1	3.40 (1.60) [0]	2.71 (0.87) [0]	3.15 (2.07) [0]
	Station 2	3.16 (0.99) [0]	2.41 (1.34) [0]	3.33 (2.73) [0]
	Station 3	2.90 (0.85) [0]	3.01 (1.02) [0]	4.31 (4.41) [0]
	Station 4	4.65 (5.03) [0]	2.71 (1.09) [0]	3.26 (1.28) [0]

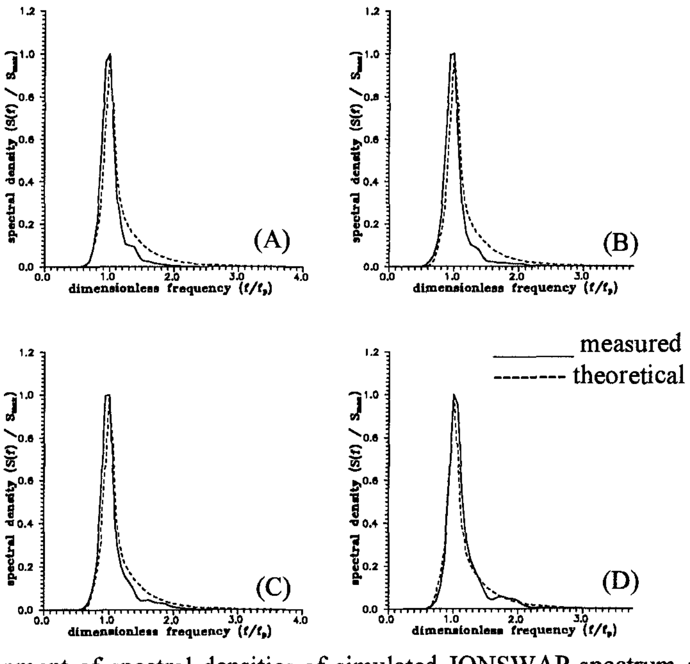

Fig. 7 Development of spectral densities of simulated JONSWAP spectrum along the wave flume. Experimental conditions: f_p = 0.8 Hz, γ = 3.3. Figs. (A)-(D): Measuring stations 1-4.

For the case with narrower band width, Figure 7 shows measured spectra along the flume. It is seen that high frequency components are missing at the first three stations, as compared with theoretical predictions. Whether this will affect the deterministic properties of simulated wave fields is not known at present, but is believed to be small. This is because wave heights from other experiments (not shown here) all have probability distributions that are not markedly different to those shown in Fig. 8. Wave heights for this experiments are rather scattered. Judging by eye examination, strictly speaking, all of the three models applied are not suitable for describing the probability distribution of wave heights in the flume. However, based upon the χ^2 test results, none of these models can also be rejected. It should be stressed that, wave heights shown in this figure are rather special, because in analyzing our experimental results, it is found that wave heights are mostly Rayleigh distributed. This fact is also reflected in Table 2, where all the χ^2 test results of the Rayleigh distribution are smaller than those for the other two models.

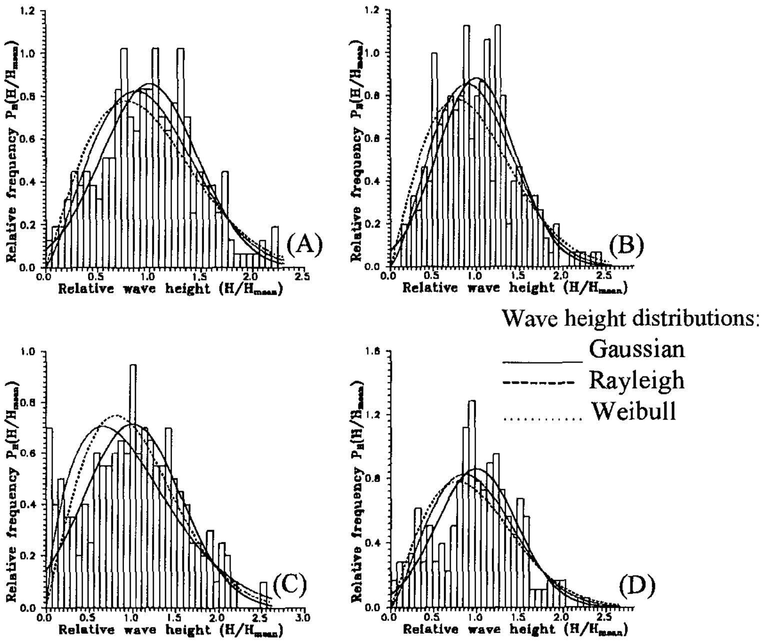

Fig. 8 Distributions of wave heights along the flume. Experimental conditions same as Fig. 7.

Discussions and Conclusions

When the energy of a random wave field is concentrated within a few components (narrow-bandedness), and when the phases of these waves are randomly distributed, then as shown by Longuet-Higgins (1952), wave heights should have a

Rayleigh distribution. Almost all (linear) random wave simulation techniques presently available, utilize these two basic assumptions. Even though a (nonlinear) transfer function is involved in wave flume experiments, present study indicates that the results have little difference as compared with numerical simulations.

In the experiments the JONSWAP spectrum is used. This can be written as:

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right] \gamma^a \qquad \text{...... (1)}$$

where
f_p = the peak frequency;
α = the "Phillips constant" with a mean value of 0.0081;
γ = the peak enhancement factor, having a mean value of 3.3; with

$$a = \exp\left[-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right] \qquad \text{...... (2)}$$

and

$$\sigma = \begin{cases} \sigma_a = 0.07 & \text{for } f \le f_p \\ \sigma_b = 0.09 & \text{for } f > f_p \end{cases} \qquad \text{...... (3)}$$

where the parameters σ_a and σ_b denote deviations from the peak frequency to, respectively, the left and right side of the corresponding Pierson-Moskowitz (P-M) spectrum. As can be seen from Eq. (1), the P-M spectrum is recovered with the peak enhancement factor $\gamma = 1$. It is generally believed that the JONSWAP spectrum represents the case of active wave generation by wind, where energies are concentrated; and the P-M spectrum represents fully developed wind-sea, where energies of the wave field are spread more or less widely over frequency bands.

During second series of the physical experiments, three values, γ = 1.0, 3.3, and 7.0, of the peak enhancement factor were chosen. It is considered that in this way the results should cover all the possible conditions encountered in the reality, i.e., from extremely narrow- to broad-bandedness. Effects of spectral width on wave height distribution have been studied theoretically by Longuet-Higgins (1980), and experimentally by Larsen (1981). They all concluded that spectral width affects the distribution more profoundly than nonlinearity. Our present results indicate, however, no noticeable effects can be found in simulated random wave fields, at least with the present algorithms.

Simulations were also carried out numerically using spectra from measured field data. The results indicate that, irrespective of the underlying target spectral shapes, wave heights obtained from synthesized wave records always have a distribution that is better to be approximated by a Rayleigh than the other two models used. With the inclusion of a term representing nonlinearity in the algorithms, it is found that only the probability distributions of surface elevations is slightly improved, whereas wave heights may still have distributions that deviate largely from reality. In conclusion, it is plausible to say, even though simulated waves may have spectral densities similar to that of the targets, this does not necessarily mean that other characteristics of the wave field are also successfully modelled automatically. Possible modifications of the applied methodologies are needed and are presently under study.

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