CHAPTER 68

An attempt at applying the chaos theory to wave forecasting

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Abstract

In this paper, a prediction method based on the chaos theory is applied to wave forecasting. First, it is investigated whether a time series data of the significant wave is chaotic. The correlation integral and the Lyapunov exponent are calculated for this purpose. In the second part of the paper, the prediction for the time series of the significant wave is attempted on the basis of the chaos theory.

1. Introduction

Wave forecasting information is required to secure port and harbor works. Classifying roughly, two methods have been used for wave forecasting. One is the deterministic method using the energy balance equation. The other is the statistical method using meteorological data - atmospheric pressure and wind speed. However, the former has a problem that the meteorological knowledge, a lot of cost and time are needed. The latter requires much labor to prepare the meteorological data. Recently, a new method based on the chaos theory (deterministic nonlinear prediction method) has been developed. This methodology has been applied to the prediction of stock prices, electricity demand and water supply. An outline of the method is that ; if a time series data has chaotic characteristics, the system which generates the time series can be considered deterministic and nonlinear. The dynamical rule of the system is estimated conversely from

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the time series data. Once the hypothetical dynamical rule has been estimated, the near future of the time series is predicted using the assumed dynamical rule. Two attractive features of this method are that only time series data is needed and there is a possibility of prediction with high accuracy. In this study, it is investigated whether the obtained time series data of the significant wave height has chaotic characteristics. Furthermore the deterministic nonlinear prediction method is applied to the prediction of the significant wave height.

2. Chaotic characteristics of the obtained data

The data of the significant wave height were obtained at the Tottori, Fukui and Miyazaki port in Japan. These data are part of NOWPHAS 1991–1994. Location of the observation points is shown in Figure 1. Length and acquirement rate of the data are indicated in Table 1. Linear interpolation is used for lack of data which is less than continuous 10 times. First of all, it is necessary to verify the chaotic characteristics of the significant wave height data. The method of trajectory reconstruction by delay coordinates (Takens(1981)) is used for that purpose. A set of m-dimensional vectors $\boldsymbol{X}_i$ are made of the time series $\{x_i\}$.

$$
\begin{aligned}
\boldsymbol{X}_1 &= \left(x_1 , x_{1+r} , \cdots , x_{1+(m-1)r} \right) \\
\boldsymbol{X}_2 &= \left(x_2 , x_{2+r} , \cdots , x_{2+(m-1)r} \right) \\
&\vdots \\
\boldsymbol{X}_n &= \left(x_n , x_{n+r} , \cdots , x_{n+(m-1)r} \right)
\end{aligned}
\tag{1}
$$

where, r is delay time. These vectors represent points in the m-dimensional phase space and a trajectory is constructed by connecting these points. Figure 2 illustrates making of 3-dimensional vectors from a time series schematically. Figure 3 is a schematic diagram of a trajectory that is reconstructed in the 3-dimensional phase space. Two methods are proposed to investigate whether a time series is chaotic. One is based on geometric characteristic of the trajectory, and the other is related to dynamical characteristic of it. In this study, the geometric characteristic of the trajectory is examined using the correlation integral method described in 2.1. The dynamical characteristic is examined using the Lyapunov exponent described in 2.2.

2.1 Correlation integral method

If a trajectory reconstructed from a time series

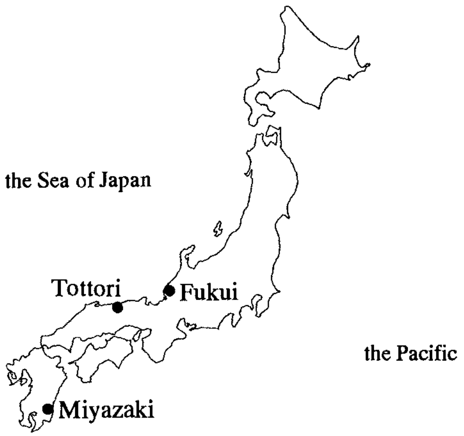


Figure 1. Location of the observation points

Table 1. Length and acquirement rate of the data

point	period	number of data	acquirement rate
Tottori	Jan.'91 ～ Jul.'92 (19 months)	6868	96.8 %
Fukui	Apr.'91 ～ Apr.'94 (37 months)	13512	99.4
Miyazaki	Jan.'92 ～ Nov.'94 (35 months)	12780	98.8

data has fractal structure, there is a possibility that the time series is chaotic. Grassberger and Procaccia(1983) proposed a method to calculate fractal dimension of the trajectory by the correlation integral. The correlation integral is defined by Eq.(2).

$$C^m(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1}^{N} H\left(\varepsilon - \left| X_i - X_j \right|\right) \tag{2}$$

where, $H(t)$ is the Heaviside function, $\left| X_i - X_j \right|$ represents distance between vector X_i and X_j. The correlation integral is calculated for variable ε respectively. If a part of plotted $(\log \varepsilon, \log C^m(\varepsilon))$ is approximately on a straight line,

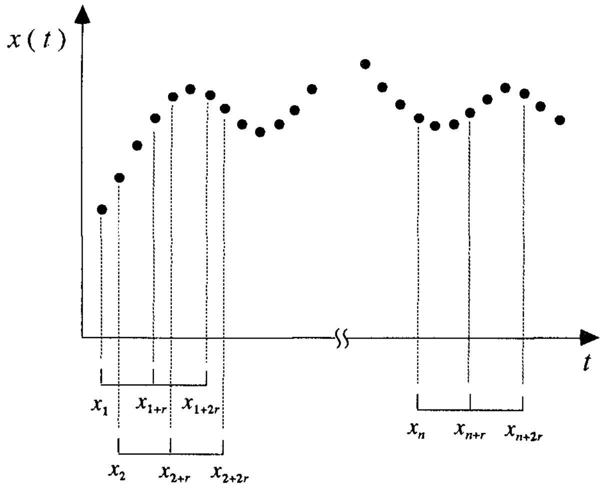


Figure 2. Making of vectors from a time series

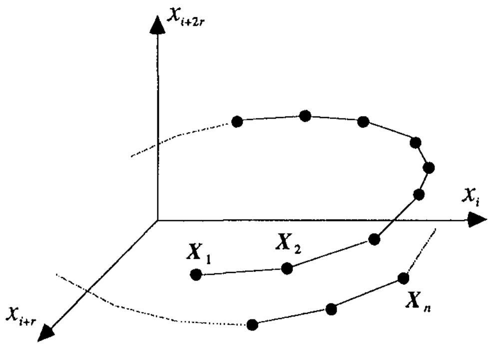


Figure 3. Reconstructed trajectory

the slope is defined as the correlation exponent $\nu(m)$. Furthermore, if the correlation exponent converges to a certain value D_c following the increase of the phase space dimension m, D_c represents the fractal dimension of the reconstructed trajectory. That is, the time series was generated by a deterministic nonlinear system, and it is estimated that the degree of freedom for this system is more than D_c. In this study, the correlation integral is calculated under the following conditions; r=10,20,30 and 40 hours, m=5,6,7,8,9,10,12,14,16,18,20,22,24,26,28,30 and 35. Figure 4.(a), (b) and (c) illustrate the results of the correlation integral. These are the cases for the significant wave height data of Tottori, Fukui and Miyazaki respectively. Figure 5.(a),(b) and (c) show the correlation exponents which were obtained from Figure 4. In the cases of Tottori (Figure 5.(a)) and Miyazaki(Figure 5.(c)), the correlation exponent tends to converge. Therefore, there is a possibility that the significant wave height data of Tottori and Miyazaki are chaotic. In the case of Fukui(Figure 5.(b)), the tendency shown in the others is not distinct. The possibility that the data of Fukui is chaotic can not be found from this result.

2.2 Lyapunov exponent

Trajectory instability is a dynamical characteristic of chaos. It means that a distance between a point on the reconstructed trajectory and its near neighboring point increases exponentially with time development. The Lyapunov exponent is an index to represent change rate of the distance. If the Lyapunov exponent is a positive value, the distance is extended exponentially. Therefore, the trajectory is considered to be unstable. Sano and Sawada(1985) proposed a method to compute the Lyapunov exponent using a time series data. The procedure for this method is as follows. First, a point on the reconstructed trajectory is denoted with $\boldsymbol{X}_t$, and its near neighbors $\boldsymbol{X}_{ki}$ (i=1,···,M) are looked for. Displacement vectors between $\boldsymbol{X}_{ki}$ and $\boldsymbol{X}_t$ are given as ;

$$\boldsymbol{y}_i = \boldsymbol{X}_{ki} - \boldsymbol{X}_t \tag{3}$$

The center point $\boldsymbol{X}_t$ moves to $\boldsymbol{X}_{t+\tau}$ and the points $\boldsymbol{X}_{ki}$ shift to $\boldsymbol{X}_{ki+\tau}$ after the time τ. The displacement vectors z_i are as follows:

$$\boldsymbol{z}_i = \boldsymbol{X}_{ki+\tau} - \boldsymbol{X}_{t+\tau} \tag{4}$$

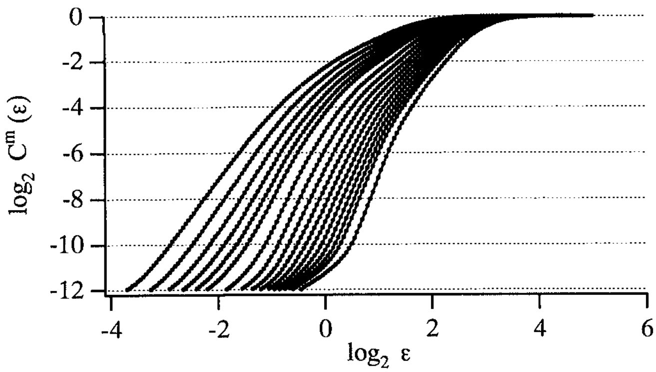
If the absolute values of y_i and z_i are small sufficiently, z_i can be expressed by Eq.(5).

$$\boldsymbol{z}_i = A_t\, \boldsymbol{y}_i \tag{5}$$

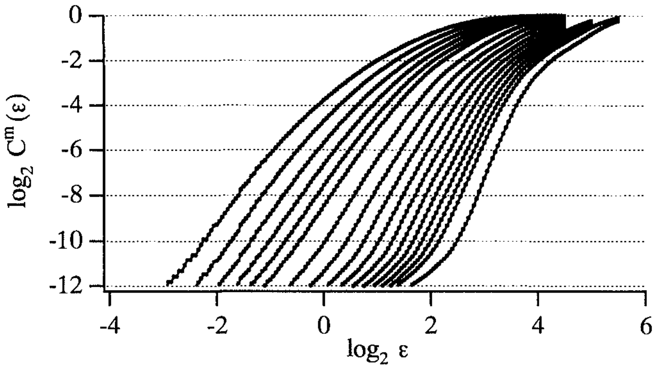
The matrix A_t is given as;

$$A_t\, V = C$$

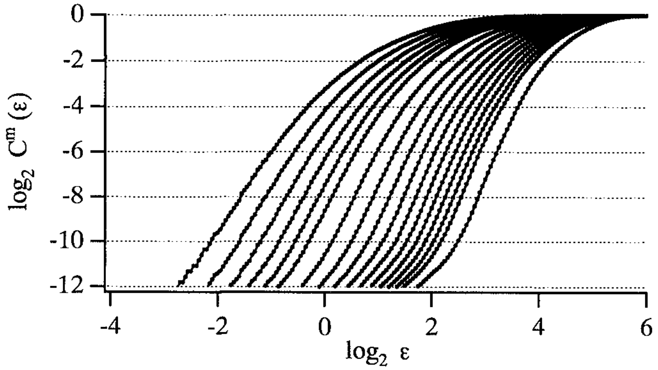
$$v_{kl} = \frac{1}{M}\sum_{i=1}^{M} y_{ik}\, y_{il} \quad , \quad c_{kl} = \frac{1}{M}\sum_{i=1}^{M} z_{ik}\, y_{il} \tag{6}$$



(a) Tottori (r = 10)

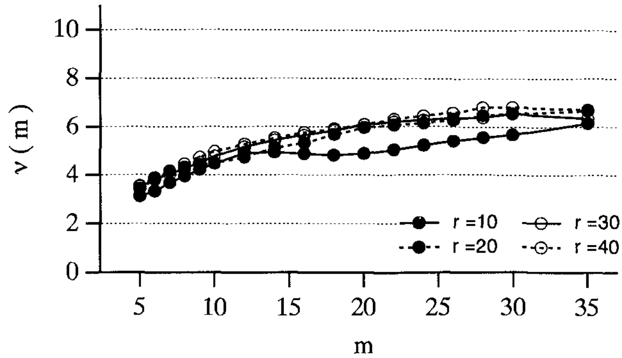


(b) Fukui (r = 10)

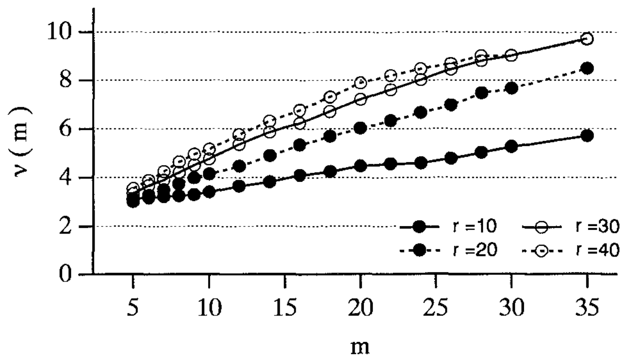


(c) Miyazaki (r = 10)

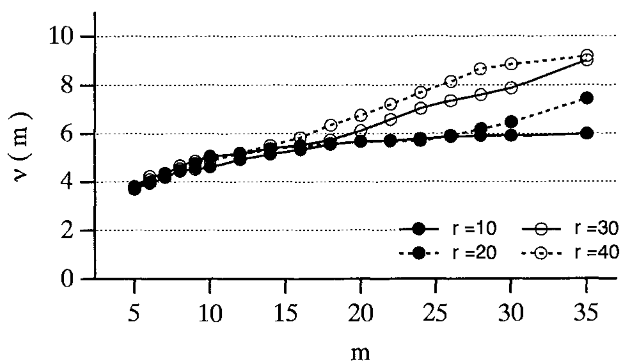
Figure 4. Results of correlation integral



(a) Tottori



(b) Fukui



(c) Miyazaki

Figure 5. Change of correlation exponent

where, y_{ik} :k-th component of y_i, z_{ik} :k-th component of z_i. The matrix A_t is used for computing the Lyapunov exponent. A set of orthonormal system $\left\{u_i(t)\right\}$ (i=1,···,m) is given as an initial condition, and the matrix A_t operates on the orthonormal system. Eq.(7) represents this operation.

$$e_i(t+\tau) = A_t u_i(t) \tag{7}$$

Furthermore, $e_i(t+\tau)$ is orthonormalized by the Gram-Schmidt method (e.g., Shimada and Nagashima(1979));

$$e_i^{'}(t+\tau) = e_i(t+\tau) - \sum_{j=1}^{i-1} \left\langle e_i(t+\tau),\ u_j(t+\tau) \right\rangle u_j(t+\tau) \tag{8}$$

$$u_i(t+\tau) = \frac{e_i^{'}(t+\tau)}{\left| e_i^{'}(t+\tau) \right|} \tag{9}$$

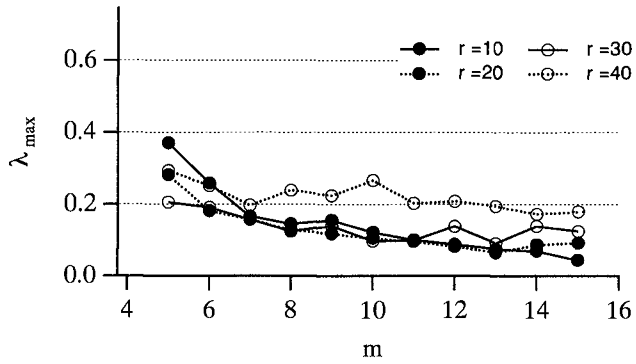
where, <,> denotes the inner product. In the next step, $A_{t+\tau}$, $e_i(t+2\tau)$, $e_i^{'}(t+2\tau)$ and $u_i(t+2\tau)$ are computed. $\left\{e_i^{'}(t)\right\}$, which expresses a set of $e_i^{'}$, is obtained by repeating this procedure. The Lyapunov exponent λ_i(i=1,···,m) is given by Eq.(10).

$$\lambda_i = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} \log \left| \left\{ e_i^{'}(t) \right\} \right| \tag{10}$$

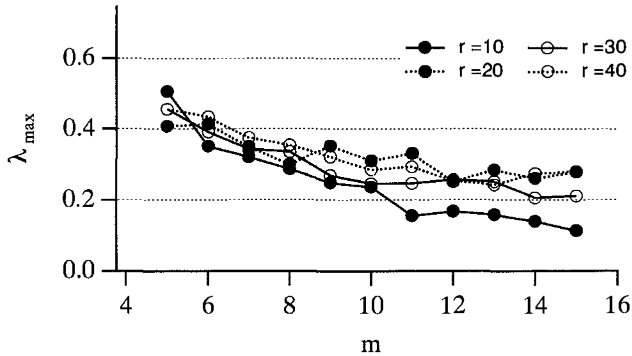
Figure 6.(a),(b) and (c) show the maximum Lyapunov exponent, which is calculated under the following conditions: delay time r is 10,20,30 and 40 hours, phase space dimension m is from 5 to 15 and number of iteration N is 200. Because all of the maximum Lyapunov exponents are positive, the significant wave height data are considered to be chaotic.

3. Deterministic nonlinear prediction method

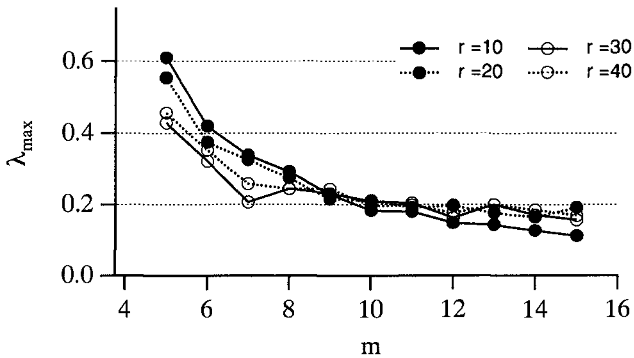
If a time series data is chaotic, the time series is considered to be generated by a deterministic nonlinear dynamical system. However, the dynamical rule of this system is unknown. It is necessary to estimate the dynamical rule conversely from the obtained time series data. In this paper, the reconstructed trajectory is divided into small sections, and a local dynamical rule is estimated for each section. The conception of Farmer and Sidorowich(1987) and the above-mentioned procedure of Sano and Sawada is applied to estimation of the local dynamical rule. In other words, the matrix A_t given by Eq.(6) is an approximation of the local dynamical rule.



(a) Tottori



(b) Fukui



(c) Miyazaki

Figure 6. Maximum Lyapunov exponent

The procedure to predict near future of a time series is as follows:
1) Take the vector which contains the latest data as X_t in Eq.(3). 2) Look for X_{ki}(i=1,2,···,M+1), which are the close vectors to X_t. 3) Permute X_{ki} in the order of increasing distance to X_t. Therefore, X_{k1} is the nearest vector. 4) Set the displacement vectors $y_i = X_{ki} - X_{k1}$ (i=2,··· ,M+1) and $y_p = X_t - X_{k1}$. 5) Obtain the displacement vectors $z_i = X_{ki+\tau} - X_{k1+\tau}$ (i=2,···,M+1). 6) Compute the matrix A_t by Eq.(6). 7) Calculate $z_p = A_t y_p$ and $X_{t+\tau} = X_{k1+\tau} + z_p$.
The m-th component of $X_{t+\tau}$ is the predicted value of the time series. In the prediction of the significant wave height, the following steps are taken furthermore; With changing M in the range of [m+2, 2m+30], compute $X_{t+\tau}$ for each M. From first to (m-1)th (or (m-2)th) component of the predicted $X_{t+\tau}$ corresponds to the observed time series datum. Pick up five $X_{t+\tau}$ whose sum of the square errors between the components and the observed data are small. Define the mean of m-th components as the predicted value of the significant wave height. When all $X_{t+\tau}$ include negative component in the above range of M, the prediction is considered to be impossible.

4. Prediction of the significant wave height

The significant wave height is predicted by the procedure described in the section 3. The conditions under which the prediction is performed are as follows:
Tottori; The delay time r is 20 and 30 hours, the phase space dimension m is 10 and 12.
Fukui and Miyazaki; r is 10 hours and m is 10.
The prediction value is computed every 6 hours, that is, at 0,6,12 and 18 o'clock every day. The following two criteria are used to evaluate the accuracy of the prediction:
Criterion I (Goto et.al.(1993));

$$\begin{aligned} &\left| H_p - H_o \right| \le 0.3\ (m) && \left(H_o \le 1.0\ (m) \right) \\ &\left| H_p - H_o \right| / H_o \le 0.3 && \left(H_o \ge 1.0\ (m) \right) \end{aligned} \qquad (11)$$

where H_o is the observed significant wave height and H_p is the predicted one. The fitting rate is defined as N_r/N_t. N_r is the number of the predicted values which are in the range of Eq.(11), and N_t is the total number of the prediction.
Criterion II; Both the observed and predicted significant wave height are more or less than a set standard wave height of 1 meter.
The results of the prediction are as follows.

Table 2. Fitting rate (Tottori)

		r = 20 hours		r = 30 hours	
		m = 10	m = 12	m = 10	m = 12
Fitting rate I	Jan. '92	42.7 %	42.7 %	42.7 %	33.1 %
	Apr.	56.7	55.8	38.3	42.5
Fitting rate II	Jan. '92	71.0 %	71.0 %	70.2 %	56.5 %
	Apr.	80.0	86.7	60.8	64.2

Table 3. Fitting rate

		Tottori			Fukui			Miyazaki	
		6-hour	12-hour		6-hour	12-hour		6-hour	12-hour
Fitting rate I	Jan. '92	58.1 %	42.7 %	Jul. '93	79.8 %	66.9 %	Jan. '94	76.6 %	61.3 %
	Apr.	63.3	55.8	Oct.	70.9	48.7	Apr.	80.0	60.8
	Jul.	71.3	55.9	Jan. '94	67.7	40.7	Jul.	78.9	69.1
	–	–	–	Apr.	71.2	56.7	Oct.	79.0	61.3
Fitting rate II	Jan. '92	75.8 %	71.0 %	Jul. '93	85.5 %	80.6 %	Jan. '94	80.6 %	72.6 %
	Apr.	82.5	86.7	Oct.	83.8	72.6	Apr.	84.2	78.3
	Jul.	85.1	89.2	Jan. '94	84.7	70.2	Jul.	81.3	75.6
	–	–	–	Apr.	85.8	79.2	Oct.	85.5	79.8

Tottori: First, the 12-hour prediction for January and April 1992 was performed to compare the accuracy under the conditions of r=20,30 and m=10,12. Table 2 shows the fitting rate I and II, which are based on the criterion I and II respectively. The difference between r=20 and r=30 can be seen, however, the difference between m=10 and m=12 in the case of r=20 is not distinct. The 12-hour prediction for July 1992 and the 6-hour prediction for January, April and July 1992 were obtained under the conditions of r=20 and m=12. The fitting rates are shown in Table 3.

Fukui and Miyazaki: The 6-hour and 12-hour prediction was performed under the conditions of r=10 and m=10. The objects of the prediction are July, October 1993, January and April 1994 in Fukui, and January, April, July and October 1994 in Miyazaki. The data of the past 2 years was used for the prediction. The fitting rates are shown in Table 3.

Figure 7.(a),(b) and (c) illustrate a part of the results, in which the solid line is the observed value and the dotted line with closed circles is the predicted value. In the cases of the 6-hour prediction, the predicted

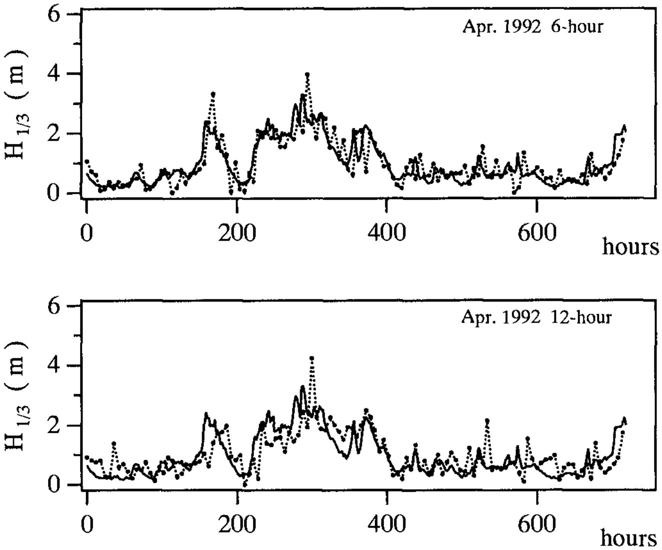


Figure 7.(a) Results of prediction (Tottori)

—— ; observed , ···●·· ; predicted

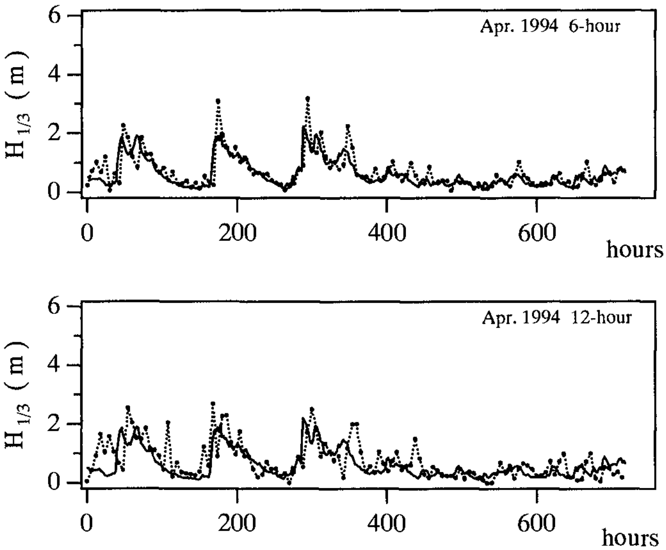


Figure 7.(b) Results of prediction (Fukui)

—— ; observed , ···●·· ; predicted

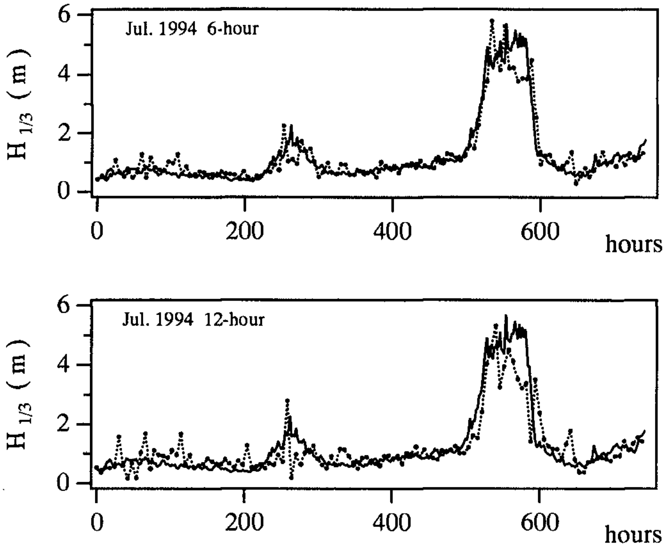


Figure 7.(c) Results of prediction (Miyazaki)

—— ; observed , ···●·· ; predicted

agree comparatively with the observed. However, time lags and fluctuations of the predicted become large in the cases of the 12-hour prediction.

5. Conclusion

In this study, the prediction method based on the chaos theory (deterministic nonlinear prediction method) was applied to the significant wave height data. The data whose length were from 19 months to 37 months were used for the prediction, and a certain of the applicability was found. However, it is necessary to improve the prediction accuracy. To increase the number of the data and to improve the estimation method of the local dynamical rule are mentioned for that purpose.

References

Farmer, J.D. and Sidorowich, J.J.(1987): Predicting Chaotic Time Series, Physical Review Letters, Vol.59, No.8, pp.845-848.

Goto, C., Shibaki, H., Aono, T. and Katayama, T.(1993): Multiple Regression Models Described in Physical Parameters for the Purpose of Wave Forecasting, Proc. of JSCE, No.473/II-24, pp.45-53. (in Japanese)
Grassberger, P. and Procaccia, I.(1983): Characterization of Strange Attractors, Physical Review Letters, Vol.50, No.5, pp.346-349.
NOWPHAS 1991-1994, Coastal Development Institute of Technology, Ports and Harbours Bureau of the Ministry of Transport.
Sano, M. and Sawada, Y.(1985): Measurement of the Lyapunov Spectrum from a Chaotic Time Series, Physical Review Letters, Vol.55, No.10, pp.1082-1085.
Shimada, I. and Nagashima, T.(1979): A Numerical Approach to Ergodic Problem of Dissipative Dynamical Systems, Prog. Theor. Phys., 61, 6, pp.1605-1616.
Takens, F.(1981): Detecting Strange Attractors in Turbulence, in Dynamical Systems and Turbulence, Lecture Notes in Mathematics, 898, pp.366-381, Springer-Verlag.