CHAPTER 52

THE DIGITAL SIMULATION OF NON-LINEAR RANDOM WAVES

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<u>Abstract</u>

This paper presents the method for the digital simulation of the second-order nonlinear random waves including strongly nonlinear shallow water waves and near breaking waves, which can be considered to be non-Gaussian random process. The method generates time series of the second order nonlinear random waves from the given energy spectrum and bispectrum obtained from the wave records. Numerical examples indicate that the procedure is basically worked well and generated time series of waves having the similar target spectral density function and probability distribution function. The proposed method has a wide range of applicability to problems involving the second-order nonlinear systems where outputs have strongly nonlinear and non-Gaussian characteristics.

Introduction

The time series simulation of nonlinear random waves has many practical applications in many engineering problems. For example, the simulated waves can be used for the analysis of the response of offshore structures excited by irregular waves (Duncan and Drake, 1995) and also can be used as wave-board control signals to generate nonlinear random waves (Klopman and Leeuwen 1990; Yasuda et al. 1994).

For the realistic reproduction of coastal waves, it is necessary to include the effects of the second-order nonlinear waves associated with the sum and difference frequency

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components of linear waves. For weakly nonlinear waves, the second-order random wave theory derived from the perturbation method using potential theory can be used to derive formulas for the generation of second-order random waves (Sharma and Dean, 1979; Klopman and Leeuwen 1990). However, the method cannot be used to generate strongly nonlinear waves in shallow water.

Recently, Ochi and Ahn (1994) developed a method to separate bound harmonic wave components from free linear wave components by applying the concept of the bicoherence spectrum. When various spectral components interact with one another duc to nonlinearities, the resulting components are phase coherent with the primary components. Therefore, those components are not considered to be statistically independent. The bispectrum, which measures the statistical dependence of three spectral components whose sum frequency is zero, may therefore be an useful tool to investigate the nonlinearities of random waves especially in shallow water waves. By applying the concept of a bicoherent spectrum, it is possible to separate the linear free wave spectral component from the spectrum obtained from the measured wave profile record. Then, by using the separated linear spectrum and bispectrum, we can digitally simulate the strongly non-linear random waves which have the similar stochastic characteristic of the target waves.

Digital Generation of Second-Order Nonlinear Random Waves.

In this paper, a method is presented to generate time series of strongly nonlinear waves from the linear energy spectrum and bispectrum of surface elevation obtained from the wave record. For this, we first write surface profile of second-order random waves in finite water depth as follows:

$$y(t) = \operatorname{Re} \sum_{k=1}^{N} c_{k} e^{i(2\pi f_{k}t + \varepsilon_{k})} + \operatorname{Re} \sum_{k=1}^{N} \sum_{l=1}^{N} c_{k} c_{l} \Big[q_{kl} e^{\{2\pi (f_{k} + f_{l})t + (\varepsilon_{k} + \varepsilon_{l})\}} + r_{kl} e^{\{2\pi (f_{k} - f_{r}t) + (\varepsilon_{k} - \varepsilon_{l})\}} \Big]$$
(1)

where f= frequency, ε =phase lag, Q_{kl} = wave-wave sum interaction coefficient associated with $f_k + f_l$, and r_{kl} = wave-wave difference interaction coefficient associated with $f_k - f_l$.

According to Ahn (1993), wave-wave interaction coefficients q_{kl} and r_{kl} can be

represented as follows:

$$q_{kl} \approx \frac{m}{s_k^2 s_l^2} B_s \left(f_k, f_l \right)$$
⁽²⁾

$$r_{kl} \approx \frac{m}{s_k^2 s_l^2} \left\{ B\left(f_l, f_k - f_l\right) - B_s\left(f_k, f_l\right) \right\}$$
(3)

where m = 1 for $k \neq l$, m = 2 for k = l, $s_k^2 = c_k^2/2$, $s_l^2 = c_l^2/2$ represent the discrete linear spectral wave energy components at frequencies f_k and f_l , respectively. $B(f_k, f_l)$ denotes the bispectrum which is formally defined as the Fourier transform of the second-order covariance function (Hasselmann et al., 1963). The bispectrum can also be expressed in terms of Fourier coefficients (Kim and Powers, 1979) as follows:

$$B(f_k, f_l) = E[Y(f_k)Y(f_l) Y^*(f_k + f_l)]$$
(4)

where $y(f_k)$ is the complex Fourier coefficient for frequency f_k and the asterisk denotes the complex conjugate. $B_s(f_k, f_l)$ represents the portion of the bispectrum which is due to wave-wave interactions of sum frequency components.

From Equations (1) through (3) we can generate the time series of nonlinear random waves for given linear energy spectrum and bispectrum.

Numerical Example Problem

To verify the method proposed, a numerical example is given. We generated 64 records of test waves which involve three primary linear wave components at frequencies f_1, f_2 and f_3 and non-linear wave components generated from interactions of components at frequencies f_1, f_2 and f_3 .

Let's consider a test waves such that

$$y(t) = \operatorname{Re} \sum_{k=1}^{3} c_{k} e^{i(2\pi f_{k}t + \varepsilon_{k})} + \operatorname{Re} \sum_{\kappa=1}^{2} \sum_{l=1}^{2} c_{k} c_{l} \Big[q_{kl} e^{i\{(2\pi f_{k} + 2\pi f_{l})t + (\varepsilon_{\kappa} + \varepsilon_{l})\}} + r_{\kappa l} e^{i\{(2\pi f_{k} + 2\pi f_{l})t + (\varepsilon_{\kappa} - \varepsilon_{l})\}} \\ = c_{1} \cos(2\pi f_{1}t + \varepsilon_{1}) + c_{2} \cos(2\pi f_{2}t + \varepsilon_{2}) \\ + c_{1}^{2} r_{11} \cos(4\pi f_{1}t + 2\varepsilon_{1}) + c_{2}^{2} r_{22} \cos(4\pi f_{2}t + 2\varepsilon_{2}) \\ + 2c_{1}c_{2}r_{12} \cos((2\pi f_{1} + 2\pi f_{2})t + (\varepsilon_{1} + \varepsilon_{2})) \\ + 2c_{1}c_{2}q_{12} \cos((2\pi f_{1} - 2\pi f_{2})t + (\varepsilon_{1} - \varepsilon_{2})) \Big]$$
(5)

where $f_3 = f_1 + f_2$. The phases of each wave were independently taken from a set of uniformly distributed random numbers between 0 and 2π . Fig.1 shows the energy spectrum of test waves y(t). The wave energy in the spectrum corresponds to frequencies from the lowest to the highest frequency $f_2 - f_1$, $f_1, f_2, 2f_1$, $f_3 = f_1 - f_2$, and $2f_2$ respectively. The wave energy at $2f_1$, $2f_2$ and $f_2 - f_1$ is entirely due to self-interactions of f_1 and f_2 and difference interactions of f_1 and f_2 . A portion of the energy at $f_3 = f_1 + f_2$ is due to the sum interaction of waves at f_1 and f_2 and the rest of the energy at f_3 is due to the free linear wave component.

The wave-wave difference interaction coefficient r_{ke} in Eq(3), can be obtained from the bispectrum and corresponding linear energy spectral components. In evaluating the interaction at the frequency $f_1 - f_2$, it is assumed that the spectral energy density at frequencies smaller than the minimum frequency f_s is entirely due to the nonlinear interactions associated with the difference between various combinations of the two frequency component at f_1 and f_2 . Furthermore, noting that $Y(f_k = Y^*(-f_k))$ for real y(t), it can be shown that the bispectrum has the following symmetry relations:

$$B(f_1, f_2) = B(f_2, f_1) = B(f_1, -f_1 - f_2)$$

= $B(f_1, -f_1 - f_2) = B(f_2, -f_1 - f_2) = B(-f_1, -f_2, f_2)$ (6)

By the above symmetry relations and definition of f_s , the difference interaction coefficient r_{ke} , can be obtained in the unique bifrequency space, $B - B_s$ as shown in Fig 2. Fig.3 shows the real part of the bispectrum obtained for y(t) by using MATLAB, higher order spectral analysis toolbox.

The analytically computed bispectral values for y(t) are also shown as follows:

$$\mathbf{B}(f_1, f_1) = E[Y(f_1)Y(f_1)Y^*(2f_1)] = \frac{c_1^4 r_{11}}{8}$$
(7a)

$$\mathbf{B}(f_2, f_2) = E[Y(f_2)Y(f_2)Y^*(2f_2)] = \frac{c_2^4 r_{22}}{8}$$
(7b)

$$\mathbf{B}(f_1, f_2) = E[Y(f_1)Y(f_2)Y^*(f_1 + f_2)] = \frac{c_1^2 c_2^2 r_{12}}{4}$$
(7c)

$$\mathbf{B}(f_2, f_1 - f_2) = E[Y(f_2)Y(f_1 - f_2)Y^*(f_1)] = \frac{c_1^2 c_2^2 q_{12}}{4}$$
(7d)





Fig.2 Domains for computing $B(f_k, f_l)$ and $B(f_l, f_k - f_l)$

$$\mathbf{B}(f_1 + f_2, f_1 - f_2) = E[Y(f_1 + f_2)Y(f_1 - f_2)Y^*(2f_1)] = \frac{c_1^4 c_2^2 r_{11} r_{12} q_{12}}{2}$$
(7e)

$$\mathbf{B}(2f_2, f_1 - f_2) = E[Y(2f_2)Y(f_1 - f_2)Y^*(f_1 + f_2)] = \frac{c_1^2 c_2^4 r_{22} r_{12} q_{12}}{2}$$
(7f)

The plan view of the bispectrum is drawn in Fig.4 to identify the bispectrum in the unique bifrequency space. As can be seen in the figure and from Eq.(7d), the difference interaction coefficient q_{12} is obtained from $B(f_2, f_1 - f_2)$. Since bispectral values $B(f_1 + f_2, f_1 - f_2)$ and $B(2f_2, f_1 - f_2)$ in Eq.(7a,b) are order of magnitude smaller than $B(f_2, f_1 - f_2)$, we may neglect the terms in evaluating difference interaction coefficient.

The energy spectral component at frequency f_3 is due to free linear wave component $c_3 \cos(2\pi f_3 + \varepsilon_3)$ as well as the nonlinear sum interaction of waves at f_1 and f_2 . The separation of the nonlinear energy from the total spectrum can be achieved approximately by applying the method proposed by Kim and Power(1979) and Ahn(1993) as following:

$$S(f_m) = S_L(f_m) + \sum_{f_m = f_k + f_l} b^2(f_k, f_l) \ S(f_m)$$
(8)

where the bicoherence squared spectrum is defined as

$$b^{2}(f_{k},f_{l}) = \frac{|B(f_{k},f_{l})|^{2}}{E[|Y(k_{k})Y(f_{l})|^{2}]E[|Y(f_{m})|^{2}]}$$
(9)

Fig.5 shows the bicoherence squared spectrum. The computed bicoherence for the sum interaction is $b^2(f_1, f_2) = 0.14$ which implies that only 14% of the energy at f_3 is due to the nonlinear sum interaction of the waves at f_1 and f_2 . The computed bicoherences for the self interactions and difference interaction are $b^2(f_1, f_1) = 0.9969$ $b^2(f_2, f_2) = 0.9989$ and $b^2(f_2, f_1 - f_2) = 0.9980$, respectively, which implies that the energy at $2f_1$, $2f_2$ and $f_1 - f_2$ are entirely due to the nonlinear interactions.



Fig.3 Real part of bispectrum



Fig.4 Plan view of bispectrum

Digital simulation of non-linear waves

The time series and the histogram of y(t) in Eq.(4) are shown in Fig.6 and Fig.7. respectively. In these figures, the wave profile shows a definite excess of high crests and shallow troughs, which is a typical feature of nonlinear waves (non-Gaussian random In the previous section, we computed the linear energy spectrum and bispectum process). of y(t). We now demonstrate the simulation of time series of the second-order random waves from the given linear energy spectrum and bispectrum. The digitally simulated waves should have similar stochastic characteristics such as the spectral density function and the probability distributions. Fig.8, shows the digitally simulated waves by the method proposed. Fig.9 and Fig.10 show the energy spectral density and the histogram of the simulated waves. The digitally simulated waves have good agreement with the target spectral density function and the probability density function of v(t) (compare Fig.9 and Fig.10 with Fig.1 and Fig.6). Therefore the simulated waves is proved to satisfy both the target spectral density and probability distribution function. The following table shows the statistical values of the target time series and the simulated time series of waves. The agreement between them is satisfactory.

	Target time series $y(t)$	Simulated time series
Variance	2.48	2.83
Skewness	1.11	1.05
Kurtosis	4.17	3.71

Conclusions

A method to digitally generate the time series of second-order nonlinear random waves applicable to strongly nonlinear shallow water waves is developed. For the given bispectrum and the bicoherence spectrum, the linear spectrum is separated from the measured spectrum. Then, the wave-wave interaction coefficients associated with various pairs of sum and difference frequency components can be evaluated from the bispectrum. Time series of the second-order nonlinear waves are then digitally simulated using linear spectral components and wave-wave interaction coefficients derived from the bispectrum. The simulated waves have the similar spectral density function and probability distribution



Fig.5 Bicoherence squared spectrum

Original Time Series



Time in seconds



 $\begin{bmatrix} 0.35 \\ 0.3 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0.05 \\$

Histogram of wave displacement

Fig.7 Histogram constructed from test waves y(t)







Fig.9 The energy spectrum of the simulated waves

Histogram of wave displacement



Fig.10 Histogram constructed from the simulated waves

function. A numerical example is used to verify the method developed. The simulated time series of waves closely reproduce the target spectral density function and probability distribution function. It is noted that the proposed method can be applied to wide range of outputs produced by the quadratic nonlinear system. The method could be applied to many practical engineering fields.

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