CHAPTER 36

Fifty Years of Wave Growth Curves[1]

Paul C. Liu[2]

Abstract

It has been over fifty years since the public release of the well-known monograph of Navy H.O. 601, prepared by Sverdrup and Munk (1947) during World War II. In celebrating 50th anniversary of the International Coastal Engineering Conference that was started in 1948, it is of interest to revisit the wave growth curves developed in H.O. 601 and compare them with a recent study on wave growth curves (Donelan *et al.* 1992). It is clear that while significant progress has been made over the last 50 years, Sverdrup and Munk's original idea still prevails in light of modern developments.

Introduction

The mid years of the decade of 1990s observe many 50th anniversaries of events that relate to the end of World War II. The end of the war marks the start of two important events that are of interest to coastal and ocean engineers: the public release of the well-known monograph of the Navy H.O. 601 by Sverdrup and Munk (1947) and the beginning of the International Coastal Engineering Conference. These two events practically ushered in the modern era of wind wave studies, and on this Silver Conference of ICCE it is interesting to pay a return visit to Sverdrup and Munk's wave growth curves given in H.O. 601 and to compare them with the

[1]GLERL Contribution No. 1034
[2]NOAA Great Lakes Environmental Research Laboratory, Ann Arbor, MI 48105-1593. Email: liu@glerl.noaa.gov

recent study of wave growth curves given by Donelan *et al.* (1992). This paper summarizes the essentials of the development of the two growth curves 50 years apart to show the differences in general approaches and the significance of the conceptual basis of H.O. 601 that prevails over time and still appears refreshing in light of modern achievements.

The Wave Growth Curves According to Sverdrup and Munk

When Sverdrup and Munk set out to develop techniques for wave forecasting in the early 1940's in support of U.S. amphibious operations during World War II, the state of knowledge at the time was based mainly on empirical relationships. Notable results such as that maximum wave heights are proportional to the square root of the fetch and that wave height is proportional to the wind speed or the square of the wind speed, all dependent on dimensional constants.

Sverdrup and Munk started the initial efforts to realistically model the wind waves by examining the growth of waves on the basis of energy considerations. They formulated the *fetch equation* as

$$\frac{c}{2}\frac{dE}{dx} + \frac{E}{2}\frac{dc}{dx} = R_t + R_v$$

where E is the mean energy per unit surface area, c is the wave phase speed, and R_t and R_v are the energy transferred to the waves at the sea surface due to the tangential stress and normal pressure of wind, respectively.

By formulating the physical considerations of R_t and R_v, incorporating measured information available at the time, and normalizing significant wave height, H, wave phase speed, c, and fetch, x, with wind speed, U, and gravitation acceleration, g, they were able to derive analytical expressions for dimensionless wave height, gH/U^2, and dimensionless wave speed, c/U, (the wave age,) as functions of dimensionless fetch, gx/U^2.

In essence they focused on the two basic parameters, the wave age $\beta = c/U$ and the wave steepness $\delta = H/L$, L is the wave length, where empirical functional relationships between the two can be deduced from the above fetch equation. They then derived three separate expressions for gx/U^2 in terms of β and δ over three divided ranges of β. Wave height relations were then obtained in the dimensionless form from linear theory as

$$\frac{gH}{U^2} = 2\pi\delta\beta^2 = f\left(\frac{gx}{U^2}\right) .$$

Following these developments, the final analytical expressions given in H.O. 601 also included no less than 17 empirical constants. These growth curves are plotted in Figure 1 as thin solid lines along with the historical data originally used.

The developments of Sverdrup and Munk clearly set the stage for modern wind wave prediction and study. Their innovative introduction of the concept of *significant waves* based on statistical considerations is still the most basic and effective parameter used in characterizing wave field parameters. One might conclude that is also part of Sverdrup and Munk's innovations. However, the use of dimensionless parameters was actually suggested to Sverdrup and Munk by Morrough P. O'Brien based on intuitive understanding of fluid mechanics and knowledge of similitude and model laws[2] (Wiegel and Saville, 1996.) The practice of correlating dimensionless parameters gH/U^2 and c/U with respect to gx/U^2 is still widely used today as part of the basic analysis of wind and wave measurement.

[2]The author is indebted to Prof. Wiegel for pointing out this important historical fact to him.

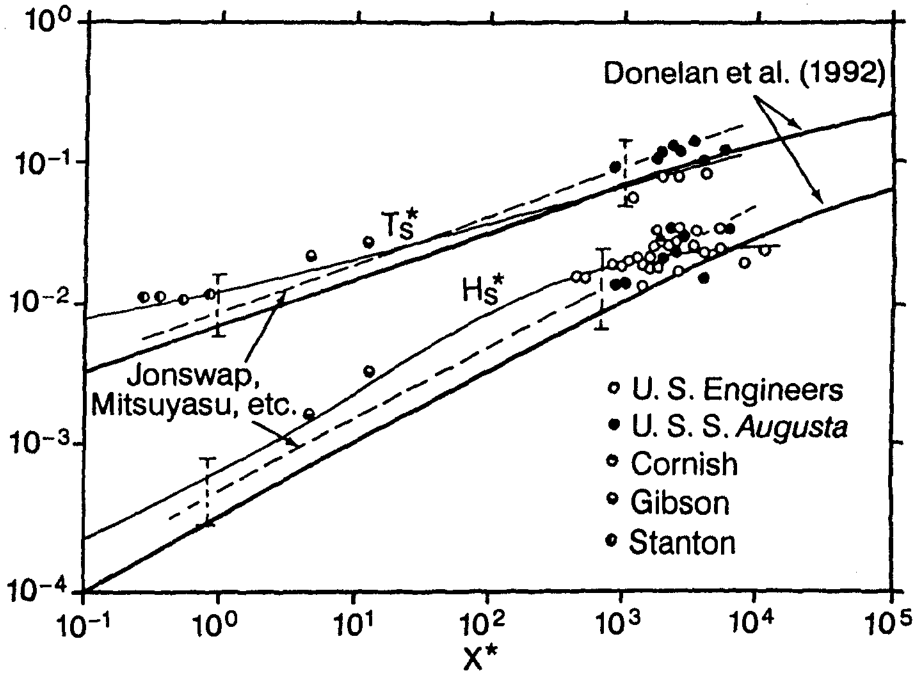

Figure 1. H.O. 601 growth curves and data as compared with later and more recent developments. The original H.O. 601 curves are shown as thin solid lines. The results of JONSWAP, Mitsuyasu, and others are summarized in dashed lines as given in Hasselmann (1982). The results of Donelan *et al.* (1992) are shown as the thick solid curves. Nondimensional significant wave height, H_s^*, and wave period, T_s^*, are plotted as functions of nondimensional fetch, X^*.

An Ode to H.O.601

Sverdrup and Munk prepared the H.O. 601 that established the physical foundations for wind wave predictions and subsequent wave studies. Bretschneider (1952) made further analysis with additional data and developed practical curves that became the widely used SMB (Sverdrup-Munk-Bretschneider) method of wave forecasting. With the advance of spectrum analysis in the 1950's and increasing use of computers in the 1960's, along with new instrumentation and measurement techniques, new approaches have subsequently evolved. But the basic principles implemented in H.O. 601 have been continuously practiced, some maybe unwittingly, by most of the later wind wave analysis and model developments.

Hasselmann (1984) in an article that pays tribute to Walter Munk's 65th birthday, entitled "An Ode to H.O.601," presented a comparison of the H.O. 601 growth curves with modern data of JONSWAP program (Hasselmann *et al.*, 1973) and Mitsuyasu (1969) as shown in Figure 1. He commended the legacy of H.O. 601 and concluded that "The progress in measurement techniques has clearly been commensurate with the progress in wave modeling." More recent modeling developments (*Komen et al.*, 1994) appear to substantiate this effect.

The Wave Growth Curves of Donelan *et al.*

The work of Donelan *et al.* (1992) provides an interesting account of the current state of the art of the wave growth curve studies. They explored the fetch-limited wind wave growths, using the wind and wave measurements from a linear array of five towers located along the prevailing wind direction on the eastern shore of Lake St. Clair, and found the differential growth of spectral wave energy between the towers, $\Delta E / \Delta x$, to be a linear function of local wave age, U / c_p, where U is the wind speed, and c_p the phase speed of the waves at the spectral peak as:

$$\frac{c_p^{\ 2}}{2gE}\frac{\Delta E}{\Delta x} = 0.78 \times 10^{-4}\left(\frac{U}{c_p} - 0.83\right).$$

Note that the right-hand side vanishes at $U/c_p = 0.83$, which is the well-established condition for a fully development wave field (Pierson and Moskowitz 1964.) Now combining the differential equation with that of the well-known empirical relation between nondimensional energy, $\varepsilon = g^2 E / U^4$, and wave age substantiated by most of the measurements made from many parts of the world:

$$\varepsilon = 0.0023\left(\frac{U}{c_p}\right)^{-3.2},$$

readily yields the following fetch relations:

$$X^* = 4.0946 \times 10^4 \ln\left[\frac{1}{1 - 5{,}5414\varepsilon^{\frac{1}{3.2}}}\right] - 2.2690 \times 10^5\left[1 + 2.7707\varepsilon^{\frac{1}{3.2}}\right]\varepsilon^{\frac{1}{3.2}},$$

and

$$X^* = 4.0946 \times 10^4 \ln\left[\frac{\dfrac{U}{c_p}}{\dfrac{U}{c_p} - 0.8302}\right] - 3.3992 \times 10^5\left[\frac{U}{c_p} + 0.4151\right]\left[\frac{c_p}{U}\right]^2.$$

where X^* is the nondimensional fetch gx/U^2. The corresponding results of these two equations are also plotted as the thick solid lines in Figure 1 in which $H_s^* = 4g\sqrt{\varepsilon}/U^2$ and $T_s^* = c_p/U$ are used to facilitate the comparison with original H.O. 601 curves. An examination of Figure 1 indicates that the results of Donelan *et al.* (1992) are lower than the H.O. 601 curves, but reach the fully developed stage much later.

In general, however, while data measurement and analytic modeling have made significant progress over the past 50 years, the ingenuity of Sverdrup and Munk's original development with very limited available data not only helped the Allied forces during the North Africa operations that led to WW II victory, but also remained reasonably close to the modern and latest measurements.

Concluding Remarks

For the most part, the wave growth curve developments of Sverdrup and Munk and that of Donelan *et al.* (1992) are basically similar. Both are basically started on available measured results followed by analytical derivations. With limited data Sverdrup and Munk's development required 17 constants for the operation. Donelan *et al.*, however, with much more and better established data and analysis relied on four empirical constants. The current state of the art of wind wave modeling as given in the comprehensive book by Komen (1994) cautioned that "Despite the progress, we still are not able to make wave predictions that always fall within the error bands of the observations." It appears that after 50 years development and progress on wave modeling and wind wave studies, while wave modeling techniques have made significant progress for practical applications, there is ample room of further progress and enhancement remain to be explored.

References

Bretschneider, C. L. (1952), Revised wave forecasting relationships, *Proc.*, 2nd Conf. Coastal Eng, Council Wave Research, Engineering Foundation, Berkeley, CA, 1-5.

Donelan, M. A. *et al.* (1992), On the growth rate of wind-generated waves, *Atmosphere-Ocean*, **30**, 457-478.

Hasselmann, K. (1984), The science and art of wave prediction - An ode to HO 601, in *A Celebration in Geophysics and Oceanography - 1982*, Scripps Institution of Oceanography Reference Series **84-5**, 31-37.

Hasselmann, K. et al. (1973), Measurements of wind-wave growth and swell decay during the Joint North Sea Project (JONSWAP), *Deutsche Hydrogr. Z.*, **12**.

Komen, G. J. et al. (1994), *Dynamics and Modelling of Ocean Waves*, Cambridge University Press, 532p.

Mitsuyasu, H. (1968/1969), On the growth of the spectrum of wind -generated waves, *Rep. Res. Inst. Appl. Mech.*, Kyushu Univ., **16/17**.

Pierson, W. J. and L. Moskowitz (1964), A proposed spectrum form for fully developed wind seasbased on similarity theory of S. A. Kitaigorodskii, *J. Geophys. Res.*, **69**, 5181-5190.

Sverdrup, H. U. and W. H. Munk (1947), *Wind, Sea and Swell: Theory of relations for forecasting*, U. S. Navy Hydrographic Office, H. O. Publ. No. **601**.