CHAPTER 33

WAVE ACTIONS ON A VERTICAL CYLINDER IN MULTI–DRIECTIONAL RANDOM WAVES

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Abstract

The wave actions on a vertical cylinder in multi-directional random waves are experimentally studied in this paper. The two-dimensinal (2-D) wave method is extended to calculate the three-dimensional (3-D) wave forces. The variation of various hydrodynamic coefficients with KC number and wave directional spreading are investigated. The three-dimensional wave forces are compared with that of two-domensional waves.

1. INTRODUCTION

The sea waves are three-dimensional (multi-directional) and irregular. So the effects of irregularity and directional spreading of waves should be included in the prediction method of the wave actions on cylinder. The wave actions on a slender rigid cylinder consist of in-line forces and lift forces (transverse forces) and both forces are nonlinear. Moreover, in multi-directional random waves, the in-line forces and the lift forces are mixed each other, it makes the problem more complex. At present the physical model test and the field observation are usually conducted to study it. But these study are rare owing to the complex of technique and the huge expense. Aage et al (1989), Chaplin et al (1993) and Hogedal et al (1994) studied the effects of spectral shape and the directional spreading on the 3-D wave action on a vertical cylinder, and no effect of spectral shape was found. Comparing with that of 2-D waves, for the 3-D waves, the in-line forces were smaller and the transeverse forces were much larger. Moreover, the effect of directional spreading on the drag force is more than that on the inertial force and these effects were specially obvions at the place near and above the still water level. But they only gave a few data points of the ratio between the forces of 3-D waves and 2-D waves. Koterayama and Nakamura (1992) and Chaplin et al. (1993) measured the 3-D wave forces on a platform and in laboratory respectively. They measured the wave forces on cylinder and the orbital velocity of waves simultaneously and found that the 3-D wave forces could be simplified as a 2-D problem and calculated with Morison Equation.

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In this paper, the effects of wave directional spreading and KC number on the wave forces and the hydrodynamic coefficients are emphasized. Moreover the 3-D wave forces are compared with 2-D one.

2. THEORETICAL CONSIDER AND ANALYSIS METHOD

2.1 Sea wave spectrum

The Sea wave is a complex 3-D random process. It is commonly described by the directional spectrum in frequency domain, which is generally expressed as the product of the frequency spectrum, S(f) and the directional spreading function, G (f,θ), i.e.

$$\left.\begin{aligned} S(f,\theta) &= S(f)\cdot G(f,\theta) \\ \int_{-\pi}^{\pi} G(f,\theta)d\theta &= 1 \end{aligned}\right\} \quad (1)$$

There were many studies on the frequency spectrum and many formulas of spectrum have been proposed (Yu, 1992). The effects of the shape of frequency spectrum on wave force coefficients on cylinder in both 2-D and 3-D wave field can be negligible from Yu and Zhang (1989) and Hogedal (1994) respectively. Therefore, only the JONSWAP spectrum ($\gamma=3.3$) is used in test due to its popular application in the world. Concerning the directional spreading function there is not a generally recognized formular, and a simplified Mitsuyasu-type spreading,

$$\left.\begin{aligned} G(\theta) &= G_0(s)\left|\cos\frac{\theta}{2}\right|^{2s} \\ G_0(s) &= \left[\int_{\theta_{min}}^{\theta_{max}} \cos^{2s}\frac{\theta}{2}d\theta\right]^{-1} \end{aligned}\right\} \quad (2)$$

is used. In Eq. (2), the spreading parameter, s is independent on frequency. In this test, s is varied to change the directional spreading, and $s=\infty$ means an unidirectional irreguler wave.

2.2 Effects of directional spreading on wave forces

The unidirectional wave forces on a cylinder consist of in-line forces and transverse forces. According to Morison Equation, the in-line forces consist of drag forces and inertial forces. The transverse forces are equal to lift forces (Yu and Miao, 1989). In the multi-directional irregular waves, the wave actions on a cylinder are rather complex. The waves may be coming from all directions and the in-line force induced by a component wave in a certain direction can be mixed with the lift force induced by the compoent wave in the perpendicular direction. In this paper, the multi-directional wave forces on a cylinder are still divided into the in-line forces, F_x, and the transverse forces, F_y, paralleled and perpendicular to the main wave direction respectively. There are two methods for calculating these forces:

(1)Extend the calculation method from 2-D wave forces to 3-D wave forces. The 3-D wave surface measued at the position of cylinder is treated as a unidirectional wave with the main wave direction. Then the in-line force can be calculated with Moreson Equation, and the transverse force is calculated with lift force equation, but the effects of directional spreading on forces are included in the hydrodynamic coefficients. According to the field observation by Koterayama et al(1992) and the experimental study by Chaplin et al (1993), this approximate treatment was acceptable. Because the effects of directional spreading on drag force are different from that on inertial force and the ratio between drag force and inertial force is dependent on KC number. So the applicability of this method will be further examined and the variation of hydrodynamic coefficients with the directional spreading parameter and KC number will be investigsted in this paper.

KC number is usually defined as $KC=U_m \cdot T/D$, where T is the wave period; U_m the maximum horizontal orbital velocity, D the diameter of the cylinder. For irregular waves, the height and period of each wave in a wave train are varied. Here regardless of the directional spreading, the parameters of the significant wave and its maximum horizontal orbital velocity at the wave surface are chosen to define KC number, hereafter referred to as $(KC)_{\frac{1}{3}}$ or KC, and it is in agreement with the engineering practice.

(2) The multi-directional wave surface is decomposed into a set of wave components of definite amplitudes, frequencies, directions of propagation and initial phases. Using the decomposed wave components, more accurate prediction of wave kinematics can be made. Then the directional wave forces on a cylinder can be calculated with Morison Equation in vectorial form. In this way, the characteristics of the 3-D irregular wave forces can be better described. We will discuss it in the near future.

2.3 Forces calculation and force coefficients

(1) The total in-line wave forces acting on a whole cylinder can be calculated with Morison Equation and the drag coefficient, C_D and inertia coefficient, C_M can be determined by the method of least squares in time domain from test data (Yu and Zhang, 1989).

(2) There are some limitation for the application of Morision Equation to 3-D wave forces. So the single force coefficient method is also used to calculate the in-line forces, F_x, transverse forces F_y and their resultant forces, F_R.

$$\left.\begin{aligned}(F_x)_p &= \frac{1}{2}(C_{F_x})_p\rho D\int_0^d u_p^2 dz\\ (F_y)_p &= \frac{1}{2}(C_{F_y})_p\rho D\int_0^d u_p^2 dz\\ (F_R)_p &= \frac{1}{2}(C_{F_R})_p\rho D\int_0^d u_p^2 z\end{aligned}\right\} \tag{3}$$

where C_{F_x}, C_{F_y} and C_{F_R} are the coefficients of in-line forces, transverse forces and resultant forces respectively. The subscript p is the index of the statistical characteristics. For example, when $p=\frac{1}{10}$, $(F_x)_{\frac{1}{10}}$, $(F_y)_{\frac{1}{10}}$ and $(F_R)_{\frac{1}{10}}$ are the average peak values of the highest one tenth in-line forces, transverse forces and resultant forces respectively. $U_{\frac{1}{10}}$ is the maximum horizontal orbital velocity of wave which height is $H_{\frac{1}{10}}$. The coefficients $(C_{F_x})_p$, $(C_{F_y})_p$ and $(C_{F_R})_p$ can be determined from measured wave forces with Eq. (3). With this method, only the characteristic value of the wave forces can be calculated.

2.4 Orbital velocity of waves

For calculating the wave forces on the cylinder, the orbital velocity and the acceleration should be known. It is difficult to measure the horizontal velocities along the water depth simultaneously, so they are usually calculated from wave surface with a suitable wave theory. For the multi-directional irregular waves there is not a generally recognized available wave theory. In the field observation ($d=15m$, $H_{\frac{1}{3}}/$

d⩽0. 19) from Koterayama et al(1992) and the model test with multi-directional irregular waves (d=2. 0m, $H_{\frac{1}{3}}$/d⩽0. 15) by Chaplin et al(1993), the wave surfaces and the orbital velocities were measured simultaneously. It is found that the measured horizontal orbital velocities in 3-D waves conform to that directly calculated from the wave surface with the 2-D linear wave theory. In our experimental study on the unidirectional wave action on pile array (Yu and Shi 1994, d=1. 0m, $H_{\frac{1}{3}}$/d⩽0. 34), the linear wave theory and the Stokes wave theory of second order were used and compared each other. It was found that the results obtained with linear theory are better than another. Moreover, it is considered that the linear wave theory is suitable for the simulation of orbital velocity, acceleration and the wave forces on pile in irregular waves with linear summation method. So the linear wave theory is used in this test ($H_{\frac{1}{3}}$/d⩽0. 297).

3. EXPERIMENT

The experiments were conducted in the State Key Labratory of Coastal and Offshore Engineering, Dalian University of Technology, China. The wave basin is 55m long, 34m wide and 1. 3m deep. The multi-directional wavemaker consists of 30 independent segments of 0. 8m wide. The wave absorbers were placed along the basin walls to prevent wave reflection from the walls.

DS-30 multi-point wave gages were used to measure the wave height. The top of the model cylinder was fixed onto a supporting frame, so the cylinder model worked as a cantilever beam under the wave action and the force meter was set between the cylinder and the support frame. The force meters were used to measure both the total in-line forces and the total transverse forces simultaneously. The sensitivity, linearity and stability of this meter are very good. The natural frequercies of the meter system in both directions in still water are 7. 3~8. 2Hz. A VAX-Ⅱ computer was used for controlling wavemaker. Both the data acquisition and processing of wave surfaces and wave forces were conducted with a computer IBM −386.

The water depth was kept at 0. 5m. The JONSWAP spectrum (γ=3. 3) and the simplified Mitsuyasu type directional spreading, Eq.(2) were used to simulate the multi-directional waves with spreading indices from $s=\infty$, unidirectional wave, to $s=2\sim5$ and the main wave direction is $\theta=0°$. For each group of s, there were several wave heights and periods. Each of cylinder models with diametrers of 2, 4 and 6cm was placed at the center line of basin and was 7m away from the wavemaker.

The single direction per frequency model (Yu et al, 1991) was used to generate the directional waves. The wave gage array consisted of four gages was used to measure the directional spectra. The wave surface measured with gage No. 5 was used to approximately represent that at the place of cylinder. The sampling of data was done at the interval of 0. 05sec, and the data length was 204. 8sec. , 4096 points. The datum signals were lowpass filtered with a cut-off frequency of 4HZ.

There are several methods for evaluating directional spectrum, of which the Bayesian approach (Hashomoto et al. 1987) is better from our primary comparison (Liu and Yu, 1993) and therefore it is used in this peper.

4. RESULTS AND ANALYSES

4.1 Waves

The measured wave parameters are shown in Table 1. The surface variation of the multi-directional irregular waves is more complex than that of the unidirectional waves. The distributions of the instantanesous values of wave surface are close to normal distribution but there is a little trend of skewness in wave profiles, with an excess of high crest points and lack of low trough points. The distribution of wave height is close to Rayligh distribution and is close to Gluhoviski distribution along with the relative water depth decreasing.

4.2 Characteristics of wave forces in time domain

Fig. 1 and Fig. 2 are the examples of the simultaneous histories of measured wave surface, total in-line wave forces, total transverse wave forces and total resultant wave forces. It is found that in the same condition of wave parameter and cylinder diameter, the in-line wave force decreases with s decresasing, but its frequency is basically the same as wave's. For the transverse forces, the effects of spreading parameter, s are more obvious. In Figs. 1 and 2, the KC numbers are not large. The transverse force of unidirectional wave, $s=\infty$, is pure lift force, whose value is small and frequency is higher. For the directional waves with $s=15$, the transverse forces maybe consist of the components of in-line forces induced by oblique waves and the lift forces, and the former rapid increases with s decreasing. Its frequency is larger than wave's. The variation in resultant force is similar with that in in-line forces.

The experimental data and fitting curves of the variation of the ratios between transverse and in-line forces, resultant and in-line forces along with KC number show that even if the data points are scattered to some extent, but the variation law is clear and they are synthesized in Fig. 3. Along with s decreasing, the vortex shedding behind cylinder is changed by the gradually increased oblique waves, it makes lift forces decreasing and the oblique wave force increasing. Therefore, the transverse force changes from lift force dominating to component of in-line forces dominating. As $s=15\sim25$, the value of transverse force can be up to more than half of in-line force, when $s=2\sim5$, it can be up to 74% of in-line force. But the peak values of in-line forces and transverse forces are not of usual occurence simultanously, so the resultant forces increase slowly with s decreasing.

4.3 Wave force coefficients

The drag coefficient C_D and inertia coefficient C_M are varied with KC number and directional spreading parameter s as shown in Fig. 4. Both coefficients descrease with s descreasing.

The single force coefficients C_{F_x}, C_{F_y} and C_{F_R} can be determined with Eq. (3) from measured wave force data and they are also varied with KC and s, but their test datum points are less scattered. It can be noticed from Fig. 5 that the in-line forces and the resultant forces decrease and the transverse forces increase with s decreasing. Concerning the transverese forces, the lift forces induced by unidirectional waves ($s=\infty$) vary with KC along a wave type curve, but along with s decreasing it gradually becomes to a progressively decreasing curve similar to that of in-line forces.

4.4 3–D wave forces compare with 2-D wave forces

In Fig. 5, for a given KC number one can get a set of force coefficients from different curves of s, then divids each coefficient by that of $s=\infty$ to get the ratio

Table 1 Measured characteristic value of waves

Wave type		S	m_0(cm^2)	$\bar{H}$	$H_{1/3}$	$H_{1/10}$	H_{max}	$\bar{H}/H_{1/3}$	$H_{1/10}/H_{1/3}$	$H_{max}/H_{1/3}$	$\bar{T}$	T_P	$T_{H1/3}$	$T_{H1/10}$	T_{Hmax}	$T_{H1/3}/\bar{T}$	$T_{H1/10}/\bar{T}$	$T_{Hmax}/\bar{T}$	N
Uni-	4003	∞	3.4	4.61	7.45	9.74	2.82	0.62	1.31	1.72	0.88	0.98	0.93	1.00	0.93	1.06	1.06	1.14	231
	4007	∞	6.9	6.76	10.46	13.45	17.38	0.64	1.29	1.65	1.03	1.18	1.10	1.11	1.08	1.07	1.08	1.05	197
	4001	∞	11.3	8.46	12.94	15.45	17.85	0.65	1.19	1.36	1.25	1.46	1.39	1.39	1.45	1.12	1.12	1.16	161
Directional	5002	60	2.2	3.14	5.47	7.28	8.85	0.57	1.33	1.62	0.90	1.23	0.98	0.97	0.93	1.16	1.14	1.09	239
	5000	45	5.4	5.03	8.83	11.26	14.24	0.57	1.28	1.61	0.89	1.23	1.09	1.11	1.15	1.15	1.17	1.21	214
	5005	35	13.7	8.36	14.21	17.27	19.46	0.59	1.22	1.37	1.15	1.51	1.38	1.34	1.35	1.22	1.19	1.19	180
	5003	40	9.4	6.89	11.78	14.86	17.85	0.58	1.26	1.52	1.11	1.51	1.34	1.34	1.34	1.20	1.20	1.20	180
Directional	6002	20	2.9	3.93	6.58	8.32	11.47	0.60	1.26	1.74	0.81	1.03	1.06	1.08	1.04	1.18	1.20	1.16	225
	6003	25	5.5	5.39	9.29	11.69	13.80	0.58	1.26	1.49	0.93	1.23	1.06	1.11	1.00	1.19	1.25	1.12	227
	6005	15	10.2	7.32	12.04	14.78	18.15	0.61	1.23	1.51	1.09	1.48	1.36	1.35	1.38	1.18	1.17	1.20	176
	6006	15	15.6	9.03	14.82	17.74	19.75	0.61	1.20	1.33	1.16	1.51	1.38	1.37	1.39	1.24	1.23	1.25	183
Directional	3002	5	4.1	4.81	8.05	10.05	12.46	0.60	1.25	1.55	0.85	1.02	0.92	0.94	0.89	1.14	1.16	1.10	251
	3004	5	9.4	6.92	11.78	14.41	18.23	0.59	1.22	1.55	0.95	1.20	1.11	1.09	1.09	1.19	1.17	1.17	218
	3005	4	14.4	8.85	14.29	17.47	21.83	0.60	1.22	1.53	1.13	1.48	1.36	1.35	1.50	1.25	1.24	1.38	186
	3006	2	20.5	9.82	16.59	19.57	23.18	0.59	1.18	1.40	1.12	1.48	1.38	1.35	1.38	1.19	1.16	1.19	176

Note: The unit of wave height is cm, period is second.

N—Number of waves.

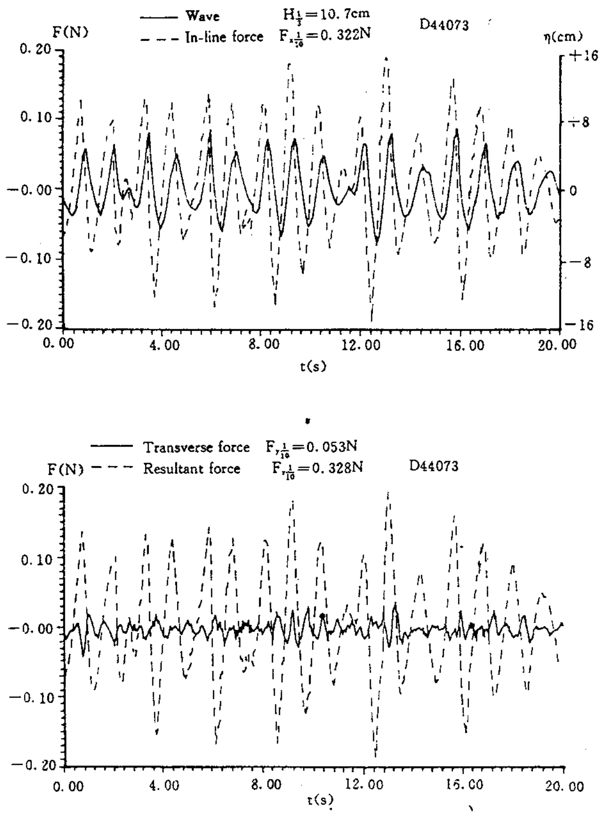

Fig. 1 Measured wave surface and wave force histories $H_{\frac{1}{3}}=10.7cm$, $T_{H_{\frac{1}{3}}}=1.09s$, $D=4cm$, $(KC)_{\frac{1}{3}}=8.9$ Unidirectional wave $(s=\infty)$

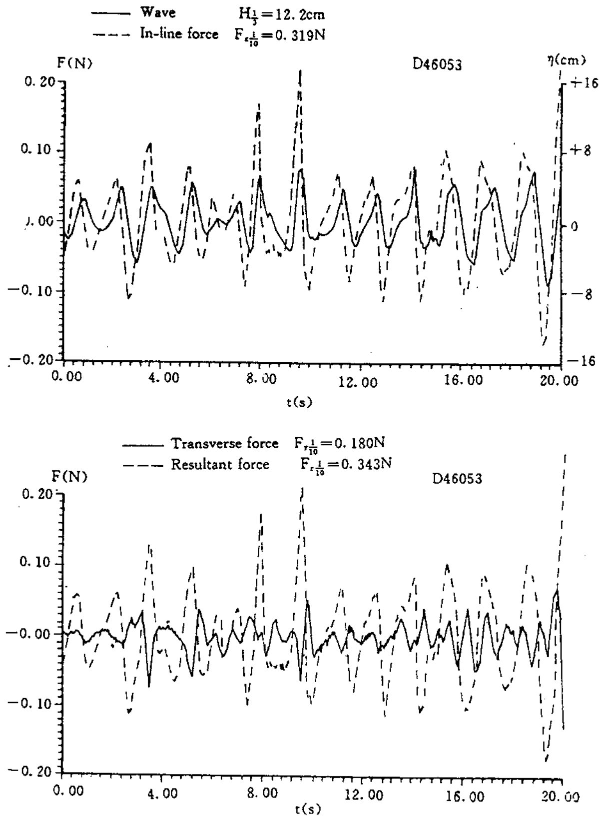

Fig. 2 Measured wave surface and wave force histories $H_{\frac{1}{3}}=12.2cm$, $T_{H_{\frac{1}{3}}}=1.26s$, $D=4cm$, $(KC)_{\frac{1}{3}}=11.5$ Directional wave $(s=15)$

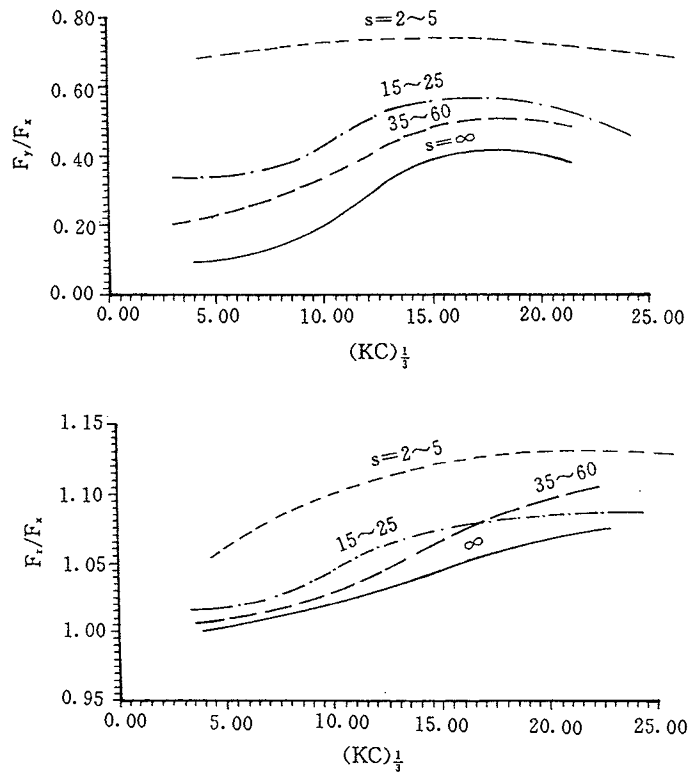

Fig. 3 $F_{y\frac{1}{10}}/F_{x\frac{1}{10}}$, $F_{R\frac{1}{10}}/F_{x\frac{1}{10}}$ versus KC number and s

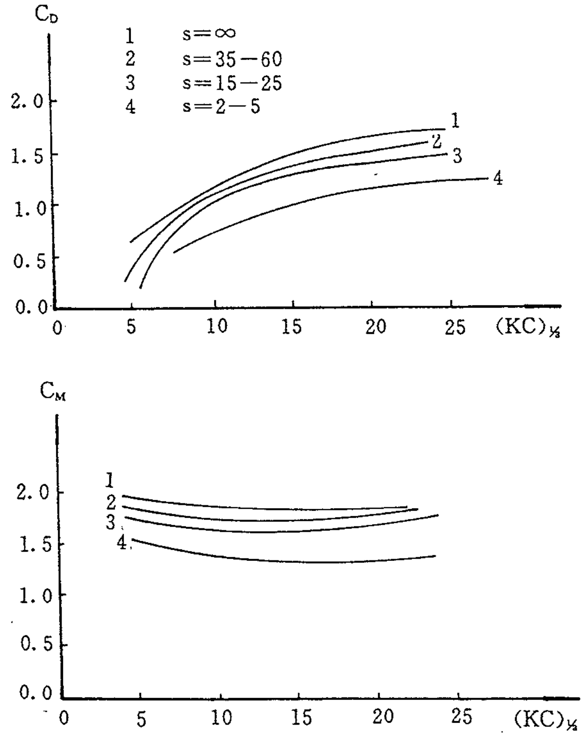

Fig. 4 Variation of C_D and C_M with KC and s

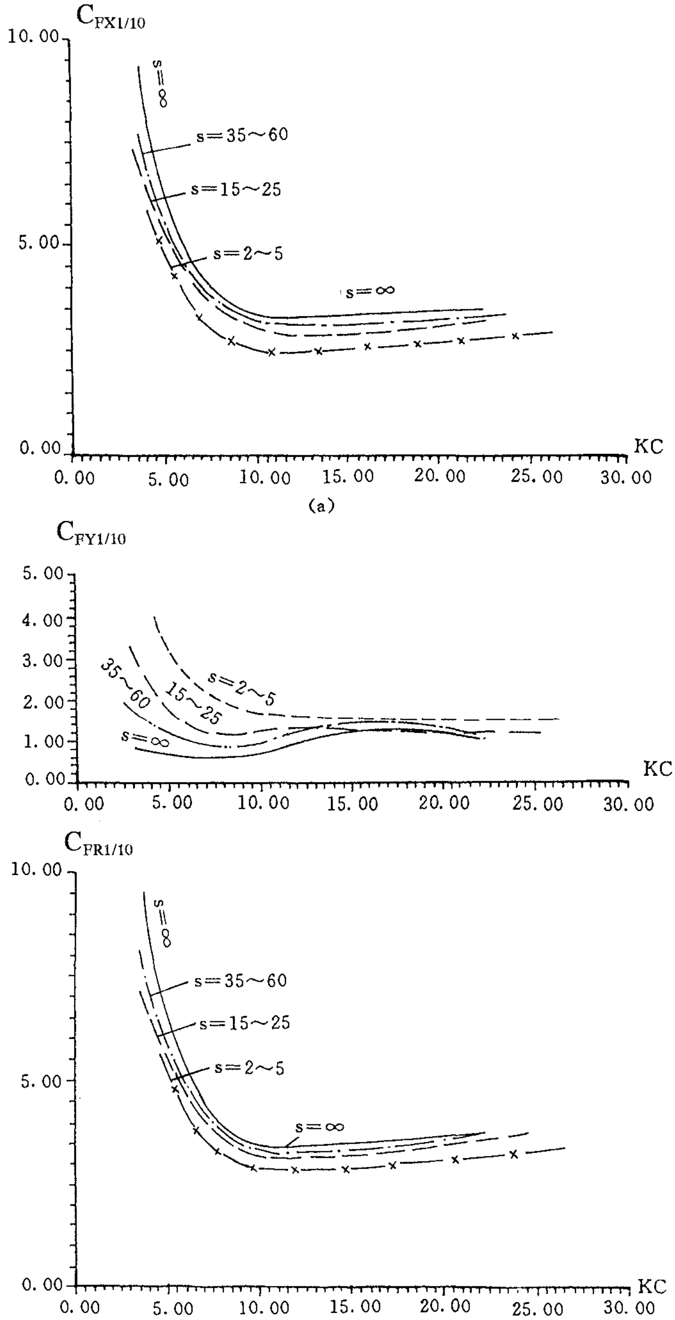

Fig. 5 Single forces coefficient versus KC and s

between 3-D wave force and 2-D one as shown in Table 2. Because the test runs are limited and the test data are scattered to some extent, so these results are preliminary and should be further examined and improved.

Table 2 Ratio between 3-D wave force and 2-D wave force

	$(KC)_{\frac{1}{3}}$ / s	5	10	15	20
In-line wave forces	35—60	0.92	0.92	0.91	0.91
	15—25	0.88	0.88	0.82	0.89
	2—5	0.80	0.73	0.74	0.77
Transverse wave forces	35—60	1.82	1.48	1.21	1.09
	15—25	2.90	2.03	1.09	1.02
	2—5	5.24	2.58	1.30	1.28
Resultant wave forces	35—60	0.89	0.96	0.96	0.96
	15—25	0.83	0.93	0.92	0.94
	2—5	0.80	0.83	0.80	0.83

4.5 Characteristics of wave forces in frequency domain

Fig 6 shows two examples of wave spectrum and wave force spectrum. The spectral shapes of in-line force and resultant force basically conform to that of wave spectrum. But for unidirectional wave the transverse force spectrum is exactly the lift force spectrum and its peak frequently appears at the twice peak frequency of wave. When s is small, the transverse force spectrum have not a twice frequency peak but it is different in shape from wave spectrum.

5. EXAMINATION AND DISCUSSION

(1)For examinating the feasiblity of Morison Equation to calculate the in-line wave forces on cylinder , the values of C_D and C_M obtained from Fig. 4 are substituted into Morison Eq. to calculate the in-line wave force history with measured wave surface history. Then the calculated wave force history is compared with the measured one as Fig. 7 shows. For the unidirectional wave, two histories are conformable. Along with s decreasing the degree of conformabilty decreases to some extent, but two historis are basically conformable. It means that the in-line force of directional waves can be calculated with present method.

(2)Concerning the wave fore coefficients, there are rare data for directional waves. The experimental data obtained by Chaplin et al(1993) are very scattered. Koterayama et al gave the variation of C_D、C_M with KC from field observation, but the value of s is not clear. Their curve of C_D is basically within the range of curves in Fig. 4 and the curve of C_M is close to that of s=15~25 in Fig. 4.

(3)Ratio between 3-D and 2-D wave forces

Some researchers studied the 3-D wave forces on cylinder mainly by experiments and gave some data concerning the ratio between 3-D and 2-D wave forces, which are collected in Table 3. These data are in comparison with present results. It turns out that the existing data are in the range of present results except the ratio of transverse forces, but in this paper, the variation of ratios with KC and s are given. About the transverse force, it is found from tests that when KC is small, the transverse forces induced by unidirectional wave are very small, but under the

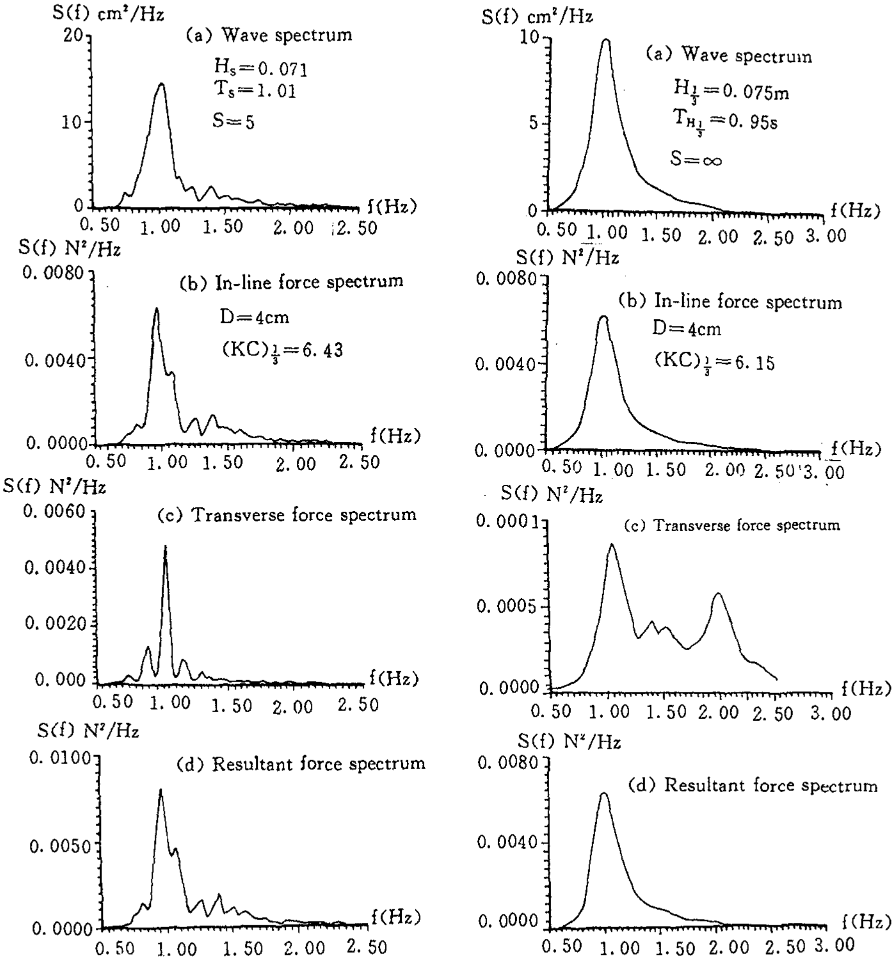

Fig. 6 Measured wave spectrum and force spectra

actions of directional waves with small s, the transverse forces induced by oblique waves are large. Therefore, the ratio of transverse force can be up to about 5.0. It is reasonable.

6 CONCLUSIONS

1 The effects of directional spreading of waves on wave forces acting on a cylinder are obvious and the level of effect is dependent on $(KC)_{1/3}$ and s. Within

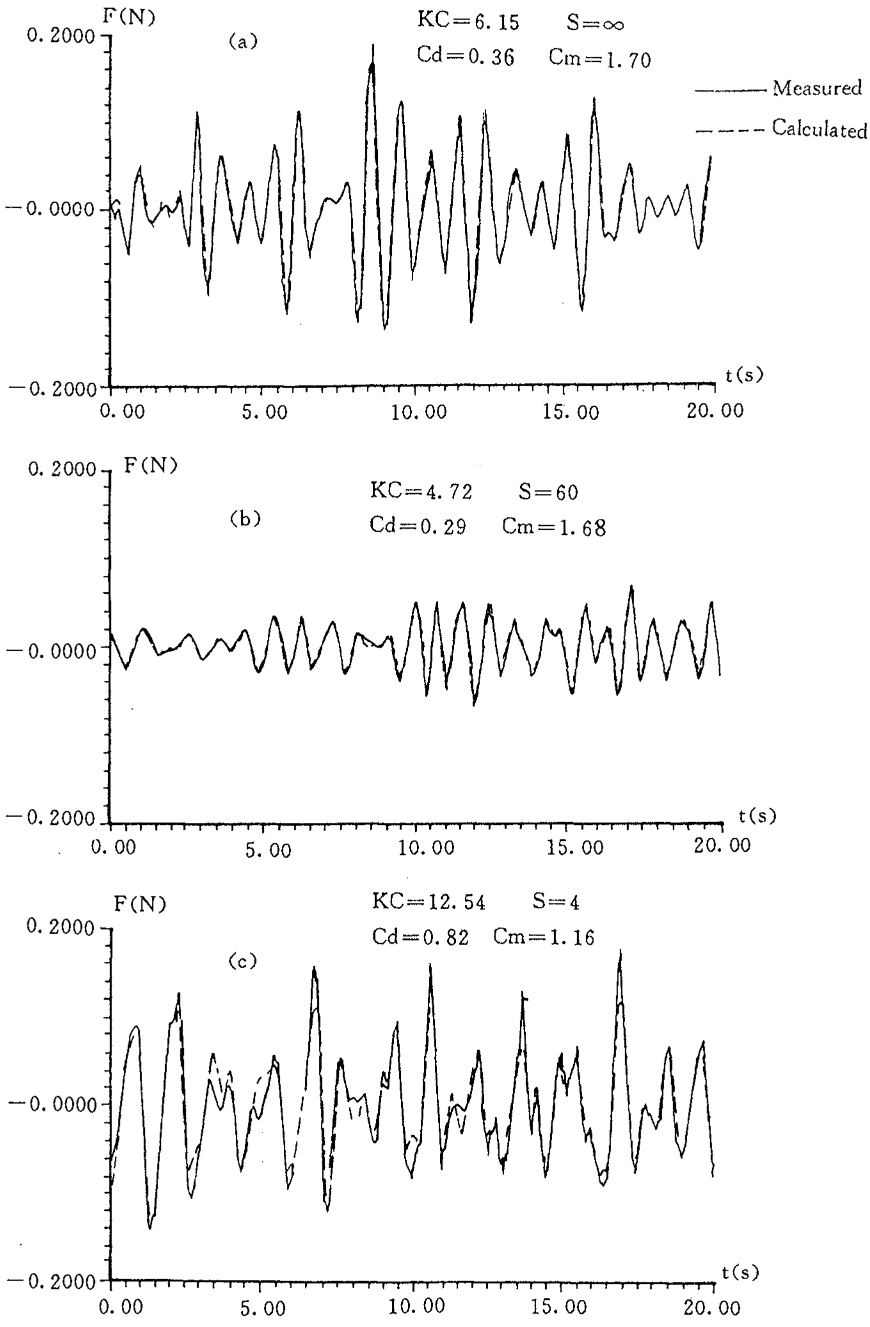

Fig. 7 Comparision between calculated and measured in-line wave force historids

Table 3 The ratios between 3-D and 2-D wave forces

Authors	Directional spreading	KC	Inline forces	Transeverse forces	Resultant forces
Nwogu and Isaacson (1989)	$\sigma_\theta=32°$ $\sigma_\theta=43°$ $\sigma_\theta=65°$				0.95 0.911 0.816
Aage et. al (1989)	North Sea spreading and $\sigma_\theta=29°$	≈20	0.82～0.87	1.4～1.8	
Chaplin et. al (1993)	$n=2, \sigma_\theta=26.5°$ $n=8, \sigma_\theta\cong15°$	2～9	0.77 0.88		
Hogedal et. al (1994)	North Sea spreading $\sigma_\theta=30°$ $\sigma_\theta=43°$	8.8～12.6* 4.22～6.52* 4.22～8.86*	0.875 0.865 0.765	～1.62	0.916 0.874 0.798
This paper (1996) $G(f,\theta)=G_0(s)\cos^{2s}\frac{\theta}{2}$	s=35～60 ($\sigma_\theta=13.6\sim10.4°$)	3.0～20	0.92～0.91	1.82～1.09	0.96～0.89
	s=15～25 ($\sigma_\theta=20.6°\sim16°$)	3.9～20	0.89～0.82	2.90～1.02	0.93～0.83
	s=2～5 ($\sigma_\theta=43°\sim33.6°$)	3.9～20	0.80～0.73	5.24～1.28	0.83～0.80

Note: (1) σ_θ is the standrad deviation of directional spreading function.
(2) * KC number is defined with peak frequency period, T_P and $H_{\frac{1}{3}}$.

the extent of this experiment, the in-line forces of multi-directional waves can decrease to 73—92% of that of unidirectional waves, and the transverse forces can increase to 200% or more.

2 Multi-directional wave forces exerting on a vertical cylinder can be predicated with the methods used for unidirectional wave except s = 2—5, but the effects of directional spreading must be included in their force coefficients.

3 All wave force coefficients of C_D, C_M, C_{F_X}, C_{F_Y} and C_{F_R} are varied with $(KC)_{1/3}$ and s. Among them, the scatters of the coefficients C_{F_X}, C_{F_Y} and C_{F_R} are relatively small.

4 For unidirectional waves (s = ∞), the transverse force acting on cylinder is the pure lift force and its main frequency is frequently double or three times the wave frequency. Along with s dreasing the transverse component forces induced by the oblique waves increase. As the test results show, when s=15—25, the transverse component forces of oblique waves have been dominant.

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