CHAPTER 12

Wave Groups in the Surf-Zone: Model & Experiments

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Abstract: Experiments were conducted with regular wave groups incident on a plane beach to analyze the behaviour of the groups in the breaking region and in the surf-zone. The groups were composed of individual cnoidal waves. Emphasis was laid on obtaining measurements in the breaking region and in the surf-zone. It was found that the location of the start of breaking of the individual waves was affected by the groupiness of the waves. The structure of the groups were also seen to be different inside the surf-zone, which changes the forcing for the long wave. The long wave motion is forced at the group frequency and can be resolved into two components, an incident forced wave, which varies along the tank and a free standing wave. The standing wave is generated because the free outgoing long wave is reflected at the wavemaker, where there was no absorption of waves. A conservation model was developed using the kinematic conservation equation and the energy conservation equation. The dispersion relation was used to close the system of equations. Cnoidal theory was used in the shoaling region and bore theory was used in the surf-zone. It was found that the model accurately predicts the group structure and the individual wave location in the shoaling region, but does not do well in the surf-zone.

1. Introduction.

Wave groups have been long recognized as one of the primary driving mechanisms for long wave generation Kostense, 1984; Watson et al. 1994; Longuet-Higgins & Stewart, 1962; Symonds et al, 1982; Schäffer & Svendsen, 1988; Schäffer, 1993; List 1991). However, the lack of comprehensive data in the breaking region and in the surf-zone has precluded the understanding of how the groups develop in that region.

The first part of this study reports experimental results for the development of wave groups in the shoaling region and in the inner surf-zone. The experiments also provide information about the variation of the break point of the individual

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waves in the group. The long wave motion will also be analyzed from the measurements (for further details on the experimental results, see also Svendsen and Veeramony, 1995).

In the second part, we will focus on efforts to model the development of the wave groups on the basis of two conservation equations, namely kinematic conservation and energy conservation. The results of the model will be compared to the measurements described in the experimental part of the study.

2. Experimental set-up.

The experiments were conducted in a wave flume (figure 1) which is 30 m long, 0.75 m wide and 1.0 m deep. The water depth for the experiments was maintained at 0.4 m. The wave flume has a 1:35 beach, the toe of which starts at 11.85 m from the mean position of the piston type wavemaker.



Figure 1: Definition sketch of the experimental setup.

The wave groups were composed of a series of five cnoidal waves, each of which was generated according to the method given by Goring (1978). The individual cnoidal waves were joined at the mean water line to form a group (figure 2). The height of the waves in the group was specified as

$$H_i = H_m (1 + \frac{G}{2} \sin \frac{2\pi i}{n}); \quad i = 1, \dots, 5$$
(1)

where H_i represents the height of the i^{th} wave in the group, H_m the mean wave height, and $G = \Delta H/H_m$ is the variation of the wave height in the group.

In all, seven experiments were conducted. In this paper, discussion will be limited to three of those experiments, the parameters of which are shown in table 1.

Each experiment was repeated many times with different positions of the wave gages. It is therefore important to verify the repeatability of the experiments.

Figure 3 shows the measured wave groups at three different locations for experiment W06. At the reference gage [Figure 3(a)], the variation from wave group to group is seen to be negligible. Inside the breaking region [Figure 3(b)] and inside the surf-zone [Figure 3(c)], there is seen to be some variability which is expected. To an extent, this is caused by the variations in the horizontal position of the waves in the group.



Figure 2: Schematic of the wave group generated at the wavemaker.

Experiment	H_m/h_0	$T_m \sqrt{g/h_0}$	Groupiness
number		•	factor G
W01	0.167	7.43	$\pm 10\%$
W03	0.237	12.38	$\pm 20\%$
W06	0.25	7.92	$\pm 50\%$

Table 1: Wave parameters at the wavemaker. H_m is the mean height of the short waves, T_m is the period of the short waves and h_0 is the water depth at the wavemaker.

To eliminate the effect of initial disturbances and surges in the tank, the data collection for each run was not started until 30 minutes after the start of wave generation.

3. Wave breaking.

One of the important questions is how the wave groupiness affects the individual wave breaking. There is no one method for the prediction of wave height or water depth at breaking, even for monochromatic waves. All current methods for predicting wave breaking are based on empirical formulations extracted from existing data. However, it is known that both the wave height and the wave height to water depth ratio have a maximum at or near the breaking point. In this analysis, we use the maximum of the wave height to water depth ratio as the definition of the break point, which can be expressed mathematically as

$$\left(\frac{H}{h}\right)_{b} \equiv \left(\frac{H}{h}\right)_{max}.$$
(2)



Figure 3: The variability of the wave profiles at three different positions in the tank: (a), at the reference gage, x = 4.0 m; (b), inside breaking region, t x = 20.9 m; and (c), inside surf-zone, at x = 23.1 m, for experiment W06. Breaking occurs between x = 19.2 m and x = 21.4 m.

The wave heights, wave periods and phase speed obtained from the data, using the zero-upcrossing analysis was used to find other parameters such as the wave length and the slope parameter $S \equiv h_x L/h$, defined as the change in water depth over one wavelength L and h_x is the bottom slope.

The experimental data is compared with similar results obtained for regular waves by Svendsen & Hansen (1976). They found that the H/h ratio at breaking was very well predicted by the local value of S. Svendsen (1987) suggested the following empirical formula as a fit to the experimental data:

$$\left(\frac{H}{h}\right)_b = 1.9 \left(\frac{S_b}{1 - 2S_b}\right)^{\frac{1}{2}} \tag{3}$$

where the subscript b denotes the value at breaking. Hansen (1990) gives a simpler approximation to the data for the range 0.25 < S < 1 as

$$\left(\frac{H}{h}\right)_b = 1.05S^{0.20} \tag{4}$$

Figure 4(a) shows the result for W01, which has a groupiness of $\pm 10\%$. It is seen that, for this case, the two empirical formulae predict the wave breaking height very accurately. Note that wave 3, which is the highest wave at the wavemaker, has the lowest ratio of $(H/h)_b$, and wave 1, the smallest wave at the wavemaker has the highest ratio of $(H/h)_b$.

Figure 4(b) shows the results for W03 (groupiness of $\pm 20\%$) and figure 4(c) shows the results for W06 (groupiness of $\pm 50\%$). The empirical formulae again



Figure 4: Variation of break point for W01, W03 and W06. The curves shown are from Svendsen, 1987 (_____) and Hansen, 1990 (_ _ _). The points shown are for the individual waves in the group, wave 1 (*), wave 2 (o), wave 3 (•), wave 4 (x) and wave 5 (+).

predict the breaking height accurately although the spread in the breaking region is larger. The two smaller waves at the wavemaker have the largest $(H/h)_b$ ratio.

The variation in breaker height combined with variations in the position of the breaking determines the height of each individual wave in the surf-zone. The resulting surf-zone wave motion generally shows a shift in groupiness as demonstrated below.

4. Structure of the wave groups.

This variation in the start of breaking implies that the structure of the groups change as the waves propagate shoreward. To illustrate this, we look at the phase averaged wave groups at different locations in the tank, from the shoaling region through to the inner surf-zone.

First, we look at the shoaling region (figure 5). The vertical axis shows the x-location and the horizontal axis shows the time. The solid line is the phase-averaged η , the filled circles are the zero-upcrossing locations of the individual waves and the broken line show the location of the waves if they were traveling at speed \sqrt{gh} .

Wave 3 is the highest wave at the toe of the beach and the form of the group is essentially unchanged in the shoaling region. The individual waves are seen to travel slightly slower that the shallow water wave speed (\sqrt{gh}) .

Figure 6 shows the groups around the breaking region. Up until breaking, wave 3 is the highest wave in the group. As the individual waves start to break, they lose energy and the wave height decreases rapidly. The highest wave, which



Figure 5: Development of the form of the wave groups for W06 in the shoaling region. x = 11.85 m is at the toe of the beach.

breaks at the largest depth (although, as seen before, the ratio H/h is not the largest), loses energy rapidly enough that, at the end of the breaking region, it is no longer the largest wave in the group. Wave 2 is seen to be the largest wave in the group at that location. Also, the wave speed after breaking is seen to be larger than \sqrt{gh} , which is expected.

Figure 7 shows the development of the structure in the inner surf-zone. The waves groups have evolved such that the smallest wave at the wavemaker is the largest wave here and the wave height is the smallest for wave 4. Note also that as the waves approach the shoreline, wave 4 is captured by wave 5 around x = 25 m. For further details, see Svendsen & Veeramony (1995).

5. Analysis of the long wave motion.

The wavemaker only generates the amplitude modulated short waves. Therefore, the set-down wave associated with the wave groups is generated as the groups propagate shoreward, taking energy out of the short wave motion. No



Figure 6: Development of the form of the wave groups for W06 in the breaking region. Breaking starts between x = 19.20 m and x = 21.60 m.

attempt was made to verify whether the set-down wave has reached an equilibrium value before the group reaches the toe of the beach. On the slope, the transformation of the groups represents a change in the forcing, which implies that the long wave motion changes continuously towards the shoreline.

It was clear, from watching the long period motion of the shoreline, that, apart from viscous effects, the long wave motion was fully reflected from the shore and essentially, sent back out as a free wave. This wave is re-reflected from the wavemaker, and over time, this process creates a standing long wave component. Hence, the total long wave motion in the tank can be analyzed as a forced wave propagating shoreward and a standing free long wave, both at the group frequency. At each gage, therefore, the long wave motion can be represented by an expression of two such components

$$\eta_l(x_j, t) = a(x_j)e^{i(kx_j - \omega t)} + bJ_0\left(\frac{2\omega}{\sqrt{gh}}(l - x_j)\right)e^{-i\omega t}$$
(5)



Figure 7: Wave profile development for experiment W06 in the region near the shoreline where wave 4 in the group ceases to exist. (Vertical scale changed from Fig. 5 & Fig. 6).

where a(x) is the amplitude of the forced wave which includes both the shoaling and the variation caused by the short wave forcing and b is the amplitude of the standing wave component.

The coefficients a and b are determined from the data. The results of this analysis is shown in figure 8. Figure 8(a) shows the total long wave water surface elevation (•), η , at a time when η due to the standing wave is zero (which is essentially the forced wave motion) and also the total long wave water surface elevation when η due to the standing wave is maximum (o). Figure 8(b) shows the amplitude variation of the forced wave, obtained from the data shown in (a). It is seen that, after an initial increase, the energy in the forced wave decreases steadily up to the breaking region (x = 19.2 m to x = 21.6 m). After breaking , energy is fed back into the forced wave which reaches a constant value inside the surf-zone. Figure 8(c) shows the standing wave water surface elevation, calculated from the data (•) and from the linear representation given in equation (5) (——). It can be seen that the standing wave is quite well represented by the zeroth-order





Figure 8: Long wave motion in the tank for W06: (a) water surface elevation of the propagating long wave at time $t = t_0$, (b) amplitude of the propagating long wave as a function of the distance from the wave maker, (c) the standing long wave in the tank at $t = t_0 + T_g/4$ ('•' is data and 'o' is the least-squares fit).

6. Modelling of short wave properties.

In this section, the short wave motion, described in the previous section, is modelled using the kinematic conservation equation and the energy conservation equation. The dispersion relation is used to close the system. Since cnoidal waves are generated by the wavemaker, the same theory is used in the modelling to evaluate the necessary coefficients.

We limit our consideration to a 1-D wave motion and the basic assumption made about the waves is that they are slowly varying in space and time. The effect of the horizontal particle velocity, u_l , of the long wave is assumed equivalent to that of a time varying current.

For such a case, the kinematic conservation equation for the short wave motion can be expressed as

$$\frac{\partial k_s}{\partial t} + \frac{\partial (c_s + u_l) k_s}{\partial x} = 0 \tag{6}$$

where k_s is the wavenumber of the short waves, c_s , the short wave celerity and u_l is the orbital velocity under the long wave. The equation for the evolution of the wave averaged short wave energy density is given by (see Phillips, 1980)

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \left[\mathcal{E}\left(u_l + c_g\right)\right]}{\partial x} + S_{xx} \frac{\partial u_l}{\partial x} = \mathcal{D}$$
(7)

where c_g is the group velocity for the short wave motion, S_{xx} is the radiation stress and \mathcal{D} is the energy dissipation.

The dispersion relation is given by cnoidal theory in the shoaling region and bore theory in the surf-zone as

$$\frac{c_s^2}{gh} = \begin{cases} 1 + \frac{H}{mh} \left(2 - m - 3\frac{E(m)}{K(m)}\right) & \text{for } h > h_b \\ 1 + \left(-\frac{3}{2} + 3\delta\right) \frac{H}{h} + \left(\frac{1}{2} - 3\delta + 3\delta^2\right) \left(\frac{H}{h}\right)^2 & (8) \\ + \left(\frac{1}{2}\delta - \frac{3}{2}\delta^2 + \delta^3\right) \left(\frac{H}{h}\right)^3 & \text{for } h \le h_b \end{cases}$$

where m is the elliptic parameter, K(m) is the elliptic integral of the first kind, E(m) is the elliptic integral of the second kind, h_b is the depth at breaking and $\delta \equiv \eta_c/H$ with η_c in the surf-zone given by (Hansen, 1990)

$$\delta = 0.5 + \left[\delta_b - 0.5\right] \left(\frac{h}{h_b}\right)^2 \tag{9}$$

The short wave averaged energy can be written as

$$\mathcal{E}(x,t) = \rho g H^2 B \tag{10}$$

where B is the shape parameter, which can be expressed outside the surf-zone using cnoidal theory. Inside the surf-zone, the empirical formula suggested by (Svendsen, 1984) is used

$$B = \begin{cases} \frac{1}{m^2} \left[\frac{1}{3} \left(3m^2 - 5m + 2 + (4m - 2)\frac{E}{K} \right) - \left(1 - m - \frac{E}{K} \right)^2 \right] & \text{for } h > h_b \\ B_0(B_{0_b}, h_x, \frac{H_0}{L_0}) + \frac{1}{2} \frac{A}{H^2} \frac{c_s}{gT} & \text{for } h \le h_b \end{cases}$$
(11)

where A is the area of the roller, T is the wave period and Hansen's (1990) expression is used for B_0 .

For simplicity, linear theory is used for S_{xx} . The energy dissipation is assumed to be negligible outside the surf-zone and inside the surf-zone can be calculated, using bore theory, as (Svendsen, 1984)

$$\mathcal{D} = -\frac{\rho g H^3}{4hT} \frac{1}{\left(1 + \delta \frac{H}{h}\right) \left(1 + \frac{H}{h} \left(\delta - 1\right)\right)} \tag{12}$$

The equations are solved in conservation form using the MacCormack predictorcorrector scheme. The criteria for breaking used in the model is the one given by equation (3). At breaking, a matching condition is required for each of the governing equations. Continuity in frequency and energy flux are used as matching conditions at the break point.

Thus, we are also assuming that the transition from a regular wave to a bore takes place over an infinitesimally small region. Since the break point varies in WAVE GROUPS

time, the matching conditions have to be evaluated at each time step. The model domain extends from the toe of the beach to the location where $min(h + a_l) = 0$, where a_l is the amplitude of the long wave. The wave envelope, which is the input to the model, is obtained at the toe of the beach using the Hilbert Transform (Melville, 1983). The zero-upcrossing points at the toe of the beach are also input to the model.

7. Comparison between model and data.

Figures 9-11 show the results of the comparison between the model results and the experimental data from W06. In each of these figures, the abscissa shows the time and the ordinate shows the location of the measurement, -- is the measured η , ______ is the predicted envelope of H(x,t), ______ is the predicted speed of the individual waves and • is the upcrossing point of the individual waves.



Figure 9: Comparison between model and W06 in the shoaling region showing η (- -), zero upcrossing point (•), wave location from model (----) and the envelope from model (----).



Figure 10: Comparison between model and W06 shoreward from breaking, showing η (--), zero upcrossing point (•), wave location from model (---) and the envelope from model (---). Breaking region from data is between x = 19.10 m and x = 21.40 m and in model is between x = 19.48 m and x = 22.30 m.

In this experiment, the groupiness factor at the wavemaker was $\pm 50\%$, the short wave period was 1.6 s and the mean wave height at the wavemaker was 0.1 m. In the shoaling region (figure 9), the wave envelope and and the wave speed are predicted extremely accurately by the model.

The results for the breaking region are shown in figure 10. The prediction of the wave envelope is poor in the breaking region, although the prediction seems to improve as the shoreline is approached. It is seen from figure 10 and 11 that in spite of the fact that the wave speed in the model is represented by the bore velocity, the highly nonlinear kinematics, in particular in the inner parts of the surf-zone, is poorly predicted. This may in part be due to errors in predicting the precise breaking region (model predicts breaking between x = 19.48 m and x = 22.3 m whereas the analysis of the data gives the breaking region to be between x = 19.2 m and x = 21.4 m), in part due to fact that the surf-zone is very wide and the waves near the shoreline are small.



Figure 11: Comparison between model and W06 in the inner surf-zone, showing η (- -), zero upcrossing point (•), wave location from model (----) and the envelope from model (----).

8. Conclusions.

Experiments were conducted to analyze the behaviour of wave groups in the breaking region and in the surf-zone.

It was found that the location of the individual wave break point is affected by the groupiness of the waves. The altered structure of breaking causes the group structure to be completely different inside the surf-zone. The long wave motion is mainly forced at the group frequency. These long waves can be resolved into a forced wave travelling shoreward and a free standing long wave. The amplitude of the incident forced wave varies along the tank, whereas the standing wave is shown largely to agree with a linear representation of a free standing wave system.

The model is seen to predict the behaviour of the group very well until the breaking region. Inside the surf-zone, the breaking waves interact strongly with each other and with the long wave motion, which the model predicts poorly.

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- Abdelrahman, S.M. and Thornton, E.B., 1988. Changes in the short wave amplitude and wave number due to the presence of infragravity waves. *Proceedings of the ASCE Special Conference on Coastal Hydrodynamics*, pp. 458-478.
- Goring, D.G., 1978. Tsunamis-The propagation of long waves onto a shelf. Rep. No. KH-R-38, W.M.Keck Lab. of Hydraulics and Water Res., CalTech, Pasadena, California.
- Hansen, J.B., 1990. Periodic waves in the surf zone : Analysis of experimental data. Coastal Engineering, Vol 14, pp. 19-41.
- Kostense, J.K., 1984. Measurements of surf beat and set-down beneath wave groups. Proceedings of the 19th International Conference on Coastal Engineering, pp. 724-740.
- Longuet-Higgins, M.S. and Stewart, R.W., 1962. Radiation stress and mass transport in gravity waves, with application to surf beats. *Journal of Fluid Mechanics, Vol.* 13, pp. 481-504.
- List, J.H., 1991. Wave groupiness variations in the nearshore. Coastal Engineering, Vol. 15, pp. 475-496.
- Phillips, O.M., 1980. The dynamics of the upper ocean. Cambridge University Press, Cambridge.
- Schäffer, H.A., 1990. Infragravity water waves induced by short-wave groups. Series paper # 50, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark.
- Schäffer, H.A. and Svendsen, I.A., 1988. Surf beat generation on a mild slope beach. Proceedings of the 21st International Conference on Coastal Engineering, pp. 1058-1072.
- Svendsen, I.A., 1987. Analysis of surf zone turbulence. Journal of Geophysical research, Vol. 92., pp. 5115-5124.
- Svendsen, I.A. and Hansen, J.B. 1976. Deformation up to breaking of periodic waves on a beach. Proceedings of the 15th International Conference on Coastal Engineering, pp. 477-496.
- Symonds, G. Huntley, D.A. and Bowen, A.J., 1982. Two-dimensional surf beat: long wave generation by time varying break point. *Journal of Geophysical Research*, *Vol.* 87, pp. 492-498.
- Svendsen, I. A. and Veeramony, J., 1995. Groups of breaking waves: Experiments. Submitted to the ASCE Journal of Waterways, Port, Coastal and Ocean Engineering.
- Watson, G., Barnes, T.C.D. and Peregrine, D.H., 1994. The generation of lowfrequency waves by a single wave group incident on a beach. Proceedings of the 24th International Conference on Coastal Engineering, pp. 776-790.