CHAPTER 4

A METHOD FOR ESTIMATING DIRECTIONAL SPECTRA IN A FIELD OF INCIDENT AND REFLECTED WAVES

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Abstract

A method for estimating the directional spectrum as well as the reflection coefficient in a incident and reflected random wave field is developed for practical use. In the method, the directional spectrum is assumed to be expressed by a circular normal distribution which includes three parameters. Then the parameters are estimated by the maximum likelihood method. The present method is applied to laboratory data, and the parameters are estimated for two kinds of wave gauge arrays. A measure of accuracy of the estimated parameters is also proposed.

Introduction

Recently it has become usual in the design of coastal, harbor and ocean structures to take into account the randomness of sea waves with respect to wave direction as well as to wave frequency. Directional wave spectra can describe the randomness of sea waves as the distribution of wave energy among the wave frequency and direction. However, there are a lot to be studied in the estimation of directional spectra, especially in a field composed of incident and reflected waves. No existing method can estimate the reflection coefficient accurately enough for practical use.

In this paper, we propose a method for estimating the directional spectrum in a short-crested random wave field by introducing a standard directional function which is expressed by the circular normal distribution function (Yokoki *et al.*,

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Figure 1: Definitions of variables

1992, 1994). The parameters in the function are determined by the Maximum Likelihood Method (Isobe, 1990).

Then we apply the method to laboratory data. The estimated results confirm its applicability to the exsiting data.

A method is also proposed for measuring the accuracy of the estimated parameters by means of the root-mean-square values of the errors. We propose the appropriate configurations of wave gauge arrays for estimating the directional spectrum parameters accurately.

Theory

Cross-power spectrum

In a wave field which consists of incident and reflected waves, the cross-power spectrum, $\Phi_{pq}(\sigma)$, between the water surface fluctuations at points p and q is represented as Eq. (1) (ex., Horikawa, 1988; Isobe and Kondo, 1984):

$$\Phi_{pq}(\sigma) = \int_{\boldsymbol{k}} S(\boldsymbol{k}, \sigma) \left\{ \exp(i\boldsymbol{k}\boldsymbol{x}_{p}) + r \exp(i\boldsymbol{k}\boldsymbol{x}_{pr}) \right\} \\
\times \left\{ \exp(-i\boldsymbol{k}\boldsymbol{x}_{q}) + r \exp(-i\boldsymbol{k}\boldsymbol{x}_{qr}) \right\} d\boldsymbol{k}$$
(1)

where $S(\mathbf{k}, \sigma)$ is the directional spectrum, \mathbf{k} the wave number vector, \mathbf{x}_p the measuring position vector, \mathbf{x}_{pr} the point symmetrical to \mathbf{x}_p with respect to the reflective wall (y-axis), and r the reflection coefficient (Fig. 1).

We rewrite Eq. (1) by using the transformation of variables as follows.

$$\begin{aligned} \mathbf{k} &= (k\cos\theta, \quad k\sin\theta \\ \mathbf{x}_{p} - \mathbf{x}_{q} &= (R_{pq}\cos\Theta_{pq}, \quad R_{pq}\sin\Theta_{pq}) \\ \mathbf{x}_{pr} - \mathbf{x}_{qr} &= (R_{pq}\cos(\pi - \Theta_{pq}), \quad R_{pq}\sin(\pi - \Theta_{pq})) \\ \mathbf{x}_{pr} - \mathbf{x}_{q} &= (R_{pqr}\cos\Theta_{pqr}, \quad R_{pqr}\sin\Theta_{pqr}) \\ \mathbf{x}_{pr} - \mathbf{x}_{qr} &= (R_{pqr}\cos(\pi - \Theta_{pqr}), R_{pqr}\sin(\pi - \Theta_{pqr})) \end{aligned}$$

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The definitions of variables are also indicated in Fig. 1. Substituting Eq. (2) into Eq. (1), we obtain the expression of the cross power spectrum, $\Phi_{pq}(f)$:

$$\begin{split} \Phi_{pq}(f) &= \int_{0}^{2\pi} S(f,\theta) \\ &\times \left[\exp\{ikR_{pq}\cos(\theta - \Theta_{pq})\} + r^{2}\exp\{ikR_{pq}\cos(\theta - \pi + \Theta_{pq})\} \\ &+ r\exp\{ikR_{pqr}\cos(\theta - \Theta_{pqr})\} + r\exp\{ikR_{pqr}\cos(\theta - \pi + \Theta_{pqr})\} \right] d\theta(3) \end{split}$$

where f represents the frequency connected with the wave number, $|\mathbf{k}|$, by the dispersion relation, and r the reflection coefficient of the reflective wall.

In the present study, we assume the directional spectrum, $S(f, \theta)$, is expressed by using the circular normal distribution function, so that we can obtain the cross power spectrum analytically and the numerical calculation becomes faster. Thus, the directional spectrum, $S(f, \theta)$, is expressed by Eq. (4):

$$S(f,\theta) = P(f)\frac{1}{2\pi I_0(a)} \exp\{a\cos(\theta - \theta_0)\}\tag{4}$$

where I_0 is the modified Bessel function of zero order.

Then the relationship between the cross-power spectrum and the parameters is written as follows:

$$\Phi_{pq}(f) = \begin{cases} \varphi(-a,\theta_0, R_{pq}, \Theta_{pq} | f) \\ + r^2 \varphi(-a,\theta_0, R_{pq}, \pi - \Theta_{pq} | f) \\ + r \varphi(-a,\theta_0, R_{pqr}, \Theta_{pqr} | f) \\ + r \varphi(-a,\theta_0, R_{pqr}, \pi - \Theta_{pqr} | f) \end{cases} \times (1 + \delta_{pq} \varepsilon_p) P(f)$$
(5)

$$\varphi(a,\theta_0,R,\Theta|f) = \frac{1}{2\pi I_0(a)} \int_0^{2\pi} \exp\{-ikR\cos(\theta-\Theta)\} \exp\{a\cos(\theta-\theta_0)\}d\theta$$
(6)

where R and Θ , respectively, indicate the length and the angle of the vector $\boldsymbol{x}_p - \boldsymbol{x}_q$, and R_r and Θ_r those of the vector $\boldsymbol{x}_{pr} - \boldsymbol{x}_q$.

Finally, the cross-power spectrum, $\Phi_{pq}(f)$, becomes a function of five parameters: the degree of directional concentration, a; the peak wave direction, θ_o ; the reflection coefficient, r; the frequency spectrum, P(f), of the incident wave; and the ratio, ε_p , of the noise component to the power at the point p.

Most probable values of parameters

The five directional spectrum parameters including the reflection coefficient should be determined so that the cross-power spectrum, $\Phi_{pq}(f)$, in Eq. (5) would approximate the observed cross-power spectrum, $\hat{\Phi}_{pq}(f)$. In the present study, the parameters are determined by the Maximum Likelihood Method (MLM). The likelihood L is defined by Isobe (1990):

$$L = \frac{1}{(2\pi\Delta f)^{M}|\Phi|} \exp\left(-\sum_{p=1}^{M} \sum_{q=1}^{M} \Phi_{pq}^{-1} \widehat{\Phi}_{qp}\right)$$
(7)

where Δf is the interval of the frequency, M the number of measuring points, and $|\Phi|$ the determinant of the cross-power spectrum matrix, Φ_{pq} .

The maximum likelihood method implies that the most probable values of the parameters, λ_i , are the solutions of the equation:

$$\frac{\partial L}{\partial \lambda_i} = \sum_{j=1}^M \sum_{l=1}^M \frac{\partial L}{\partial \Phi_{jl}} \frac{\partial \Phi_{jl}}{\partial \lambda_i} = 0$$
(8)

Equation (8) is expressed as a nonlinear simultaneous equation through some algebraic calculation (Yokoki *et al.*, 1994) as follows:

$$\sum_{j=1}^{M} \sum_{l=1}^{M} \left\{ -\Phi_{lj}^{-1} + \sum_{p=1}^{M} \sum_{q=1}^{M} \Phi_{lq}^{-1} \widehat{\Phi}_{qp} \Phi_{pj}^{-1} \right\} \frac{\partial \Phi_{jl}}{\partial \lambda_{i}} = 0$$
(9)

The directional spectrum parameters, λ_i , which satisfy Eq. (9) for all i ($i = 1 \sim 7$) are the most probable values.

The modified Marquardt method (ex. Fletcher, 1971) is adopted instead of the Newton-Raphson method to solve Eq. (9) iteratively, and thus the solutions are obtained more robustly and quickly.

In the modified Marquardt method, the values of $\lambda_i^{(k+1)}$ at the (k+1)-th iteration is expressed in terms of the previous values, $\lambda_i^{(k)}$, as follows:

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} - \left[\boldsymbol{A} + \boldsymbol{\mu}^{(k)} \left\{ \boldsymbol{E} + \operatorname{diag}(\boldsymbol{A}) \right\} \right]^{-1} \boldsymbol{f}^{(k)}$$
(10)

where λ is the vector expression of λ_i , $A = [A_{ij}] = [\partial f_i / \partial \lambda_j]$, $f = [f_i]$, f_i is the left hand side of Eq. (9), diag(A) the diagonal matrix of A, and E the unit matrix.

If Eq. (9) is strongly nonlinear, the parameter μ increases during in the iterations, and finally Eq. (10) reduces as follows:

$$\lambda_i^{(k+1)} \approx \lambda_i^{(k)} - \left(1 + \frac{\partial f_i}{\partial \lambda_i}\right)^{-1} f_i^{(k)} \tag{11}$$

If μ is small, Eq. (9) reduces to the same form as the Newton-Raphson method.





Case No.	$H_{1/3}$	$T_{1/3}$	θ_0	Smax	Array of
	(m)	(s)	(°)	max	wave gauges
4151	0.04	1.25	180	75	Ι
4191	0.05	1.00	180	10	Ι
4152	0.04	1.25	180	75	II
4192	0.05	1.00	180	10	II

Table 1: Incident wave conditions

Application to Laboratory Data

Laboratory data

To verify the validity of the present method, we applied it to experimental data. Figure 2 shows the layout of wave gauges in the multi-directional wave basin, where the water depth is uniformly 50cm, the reflective wall (y-axis) is vertical and no wave absorber is installed. The water surface elevations were measured by two kinds of wave gauge arrays: The array I is linear and normal to the reflective wall, and the array II parallel to the reflective wall.

Multi-directional incident waves were generated by using the Bretschneider-Mitsuyasu spectrum proposed by Mitsuyasu *et al.* (1975). The incident wave conditions and the corresponding wave gauge arrays are listed in Table 1.



Figure 3: Estimated values of parameters (No. 4151)

Estimated parameters

Figure 3 shows the estimated values of the parameters by the array I composed of the wave gauges A, B and C, for which the incident wave condition is No. 4151 in Table 1. Figure 4 shows the estimated values for the incident wave condition No 4191. The dashed lines in the graphs for the power, P(f), indicate the frequency spectrum of the incident waves. From these figures, we can see that the array I estimates the reflection coefficient and the peak wave direction accurately around the peak wave frequency, but not the directional concentration.

Figures 5 and 6 show the estimated values of the paramters by the array II composed the wave gauges F, G and I, for which the incident wave conditions are No. 4152 and 4192 respectively. These figures show that the peak wave direction and the directional concentration are accurately estimated by the array II, though the reflection coefficient cannot be estimated accurately.



Figure 4: Estimated values of parameters (No. 4191)



Figure 5: Estimated values of paramters (No. 4152)



Figure 6: Estimated values of paramters (No. 4192)

θ_0 (°)	S _{max}	r	
$\left\{\begin{array}{c}180\\135\\95\end{array}\right\}$	$\left\{\begin{array}{c}10\\25\\75\end{array}\right\}$	$\left\{\begin{array}{c}0.1\\0.5\\0.9\end{array}\right\}$	

Table 2: Incident wave conditions and reflection coefficient

Accuracy of Estimated Parameters

Definition of accuracy of parameters

In this section, we show how to evaluate the performance of wave gauge arrays for estimating the directional spectrum parameters. We proposed Eq. (12) as a measure of the accuracy of the estimated parameters:

$$\operatorname{Err}(\lambda_i) = \frac{1}{\widetilde{\lambda}_i} \sqrt{\frac{1}{N_{\text{total}}} \sum_{j=1}^{N_{\text{total}}} (\lambda_{i,j} - \overline{\lambda}_i)^2}$$
(12)

where $\lambda_{i,j}$ is the estimated values of the parameter, λ_i in the *j*-th sample, $\overline{\lambda}_i$ is the (given) true value, N_{total} is the total number of samples, and $\tilde{\lambda}_i$ is the value by which the error is normalized.

Equation (12) is regarded as the normalized root-mean-square value of the error. We used Eq. (12) to evaluate the perfomance of the array for the twenty-seven incident wave conditions shown in Table 2. In all cases, the incident directional wave spectra are calculated by using the Bretschneider-Mitsuyasu frequency spectrum, and the directional function proposed by Mitsuyasu *et al.* (1975). The significant wave height, $H_{1/3}$, is 0.1m and the significant wave period, $T_{1/3}$, is 1.0s.

Errors were calculated for the parameters estimated by the various arrays shown in Fig. 7: two kinds of linear arrays, and the three kinds of triangular arrays. The errors are calculated for each array by changing the distance, D_p , from the reflective wall and the interval, D, of the wave gauges.

Calculated accuracy

Figure 8 shows calculated errors in the peak wave direction, θ_0 , the directional concentration, a, and the reflection coefficient, r.

We can see from Fig. 8 that the linear array (1) and the triangular array (3) cannot estimate the peak wave direction accurately, and the linear array (2) cannot estimate the reflection coefficient accurately, either. In other words, the linear array parallel to the reflective wall and the triangular array can estimate



Figure 7: Arrays of wave gauges

the peak wave direction accurately, and the linear array normal to the reflective wall and the triangular array can estimate the reflection coefficient accurately. No particular trends relationship was found for the the directinal concentration.

Concluding Remarks

Major concluding remarks of the present study are as follows:

- 1. A method is derived for estimating the directional spectrum in a field of incident and reflected waves using the cicular normal distribution function.
- 2. The method is proved to be valid for data in laboratory experiments.
- 3. The performance of wave gauge arrays is estimated through the root-meansquare error of the estimated paramters.



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