CHAPTER 2

A Method for Estimating Standardized Bimodal Directional Spectra

Ryuichi Fujiwara[1] and Masahiko Isobe[2]

Abstract

Random sea waves are described by directional spectra which sometimes have bimodal distributions. Frequency spectra are often expressed in standardized forms which contain several parameters. This makes it easy to accumulate and analyze a large number of data obtained in various fields and to specify incident wave conditions in designing practice. In the present study, a method is proposed for estimating directional spectra in the form of a standardized bimodal distribution. The validity of the method is verified by numerical simulation for a three-component array (water surface elevation and horizontal two components of the water particle velocity). Furthermore, several applications to field data are shown.

Introduction

Many of methods have been proposed for estimating directional spectra : e.g. DFTM (Barber, 1963), parametric method (Longuet-Higgins *et al.*, 1963, Panicker and Borgman, 1974; Mitsuyasu *et al.*, 1975), MLM (Capon, 1969), EMLM (Isobe *et al.*, 1984), MEP (Kobune and Hashimoto, 1986) and BDM (Hashimoto and Kobune, 1988). In the last few years, it has become possible to estimate directional spectra accurately and stably. Degrees of freedom of estimated directional spectra, however, are too large to derive a standard form except for parametric method.

In recent years field observations have been carried out in deep area for

[1] Senior Research Engineer, Hydraulic Engineering Laboratory, Technical Research Institute, Toyo Construction Co. Ltd., 3-17-6 Naruohama, Nishinomiya, Hyogo 663, Japan

[2] Dr. Eng., Professor, Department of Civil Engineering, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan

investigating directional spectra property. To utilize the data in designing practice, it is important to express directional spectra in a standardized form which has a limited number of parameters. This makes it easy to accumulate and analyze a large number of data, and then to specify incident wave conditions in designing coastal and ocean structures as is done for frequency spectra.

In the present study, a method for estimating directional spectra with a standardized bimodal distribution is proposed by taking into consideration, the statistical variability of the Fourier coefficients which have Gaussian distributions.

Theory

(1)Bimodal directional spectrum

A bimodal directional spectrum is expressed in a standard form which is a superposition of two unimodal distribution functions. The unimodal distribution function employed is proposed by Mitsuyasu *et al.*(1975). Then the directional spectrum, $S(f,\theta)$, is expressed by Eq.(1) :

$$S(f,\theta)=\sum_{i=1}^{2}P_i(f)\frac{2^{2S_i(f)-1}\Gamma^2\{S_i(f)+1\}}{\pi\Gamma\{2S_i(f)+1\}}\cos^{2S_i(f)}\left\{\frac{\theta-\theta_i(f)}{2}\right\} \tag{1}$$

where, $P_i(f)$, $\theta_i(f)$ and $S_i(f)$ are, respectively, the power spectrum, the peak wave direction and the directional concentration parameter of each unimodal directional spectrum, Γ the gamma function, f the frequency, and θ the wave direction. To estimate the directional spectrum for a certain frequency f, the unknown parameters are $P_1(f)$, $P_2(f)$, $\theta_1(f)$, $\theta_2(f)$, $S_1(f)$ and $S_2(f)$. For simplicity, we omit "(f)" hereinafter. Next, by the following relationship, we transform the spectrum parameters to P, α, θ_1, θ_2, S_1 and S_2 which are denoted by λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6, respectively:

$$P=\alpha P+(1-\alpha)P=P_1+P_2 \tag{2}$$

A component directional distribution function should have the following two properties: first, it has only one peak and varies smoothly; second, it can express both narrow and broad banded distributions.

(2)Definition of cross-power spectrum

The general relationship between the cross-power spectrum, ϕ_{mn}, of m-th and n-th quantities in the irregular wave field, and the directional spectrum, $S(f,\theta)$, is derived by Isobe *et al* (1984) as:

$$\phi_{mn} = \int_0^{2\pi} H_m(f,\theta)\overline{H_n}(f,\theta)\,\exp\{-ik(x_n - x_m)\}S(f,\theta)d\theta \tag{3}$$

where $\boldsymbol{k}$ is the wave number vector, $\boldsymbol{x}_m$ and $\boldsymbol{x}_n$ the measuring locations, H_m the transfer function from the water surface elevation to the m-th quantity and $\overline{}$ denotes the complex conjugate.

(3)Definition of likelihood

When the directional spectrum, $S(f,\theta)$, are expressed in a standard form, the expected cross-power spectrum, ϕ_{mn}, becomes a function of prescribed parameters, $\lambda_i\,(i$=1, 6). The maximum likelihood method is employed to determine the most probable values of these parameters. The likelihood, L, is defined by Isobe(1990) as:

$$\begin{aligned} L\left(A^{[j]};\phi\right) &= \left\{p\left(A^{[1]}\right)\times p\left(A^{[2]}\right)\times\cdots\times\left(A^{[j]}\right)\times\cdots\times\left(A^{[J]}\right)\right\}^{1/J} \\ &= \frac{1}{(2\pi\Delta f)^M|\phi|}\exp\left(-\sum_{m=1}^{M}\sum_{n=1}^{M}\phi_{mn}^{-1}\hat{\phi}_{nm}\right) \end{aligned} \tag{4}$$

$$\hat{\phi}_{nm} = \frac{1}{2J\Delta f}\sum_{j=1}^{J}\overline{A}_n^{[j]}A_m^{[j]} \tag{5}$$

where $p(A^{[j]})$ is a joint probability density function of the Fourier coefficients, $A^{[j]}$ the Fourier coefficients of the time series, $\triangle f$ the frequency interval, and $|\phi|$ the determinant of the matrix, ϕ_{mn} and ^ denotes the quantity obtained from measured data.

The likelihood takes the maximum value, $L_{\max}$ when the expected cross-power spectrum corresponding to the assumed directional cross-power spectra agrees with the measured one:

$$L_{\max} = \frac{e^M}{(2\pi\Delta f)^M|\phi|} \tag{6}$$

Therefore, we can define a degree of fitting by computing the ratio, $L/L_{\max}$.

(4) Most probable values of parameters

The most probable values of the parameters λ_i in Eq.(4) are obtained by maximizing the likelihood, L, through the relationship between ϕ_{mn} and λ_i. The most probable values of the parameters are determined so that all the partial derivatives of L with respect to directional spectrum parameters vanish:

$$\frac{\partial L}{\partial \lambda_i} = \sum_{k=1}^{M} \sum_{l=1}^{M} \frac{\partial L}{\partial \phi_{kl}} \frac{\partial \phi_{kl}}{\partial \lambda_i} = 0 \tag{7}$$

Substitution of Eq.(4) with Eq.(7) yields

$$\sum_{k=1}^{M} \sum_{l=1}^{M} \left\{ - \phi_{lk}^{-1} + \sum_{m=1}^{M} \sum_{n=1}^{M} \phi_{\ln}^{-1} \hat{\phi}_{mn} \phi_{mk}^{-1} \right\} \frac{\partial \phi_{kl}}{\partial \lambda_i} = 0 \tag{8}$$

The solution of Eq.(8) are obtained numerically by using the Newton-Raphson method. First, the left-hand side is defined as a function of the parameters λ_i:

$$f_i(\lambda_{i'}) = \sum_{k=1}^{M} \sum_{l=1}^{M} \left\{ - \phi_{lk}^{-1} + \sum_{m=1}^{M} \sum_{n=1}^{M} \phi_{\ln}^{-1} \hat{\phi}_{nm} \phi_{mk}^{-1} \right\} \frac{\partial \phi_{kl}}{\partial \lambda_i} \tag{9}$$

The values of $\lambda_i^{(j+1)}$ at the (j+1)-th iteration of the calculation is expressed in terms of the previous values, λ_i^j,as follows:

$$\lambda_i^{(j+1)} = \lambda_i^{(j)} - \left[\sum_{i=1}^{M} \left[\frac{\partial f_i}{\partial \lambda_{i'}} \right]^{-1} f_{i'} \right]_{\lambda_i = \lambda_i^{(j)}} \tag{10}$$

where $\partial f_i / \partial \lambda_{i'}$ is expressed by Eq.(11) :

$$
\begin{aligned}
\frac{\partial f_i}{\partial \lambda_{i'}} = & \sum_{k=1}^{M}\sum_{l=1}^{M}\left\{-\phi_{lk}^{-1} + \sum_{m=1}^{M}\sum_{n=1}^{M}\phi_{ln}^{-1}\hat{\phi}_{nm}\phi_{mk}^{-1}\right\}\frac{\partial^2 \phi_{kl}}{\partial \lambda_{i'}\partial \lambda_i} \\
& - \sum_{k'=1}^{M}\sum_{l'=1}^{M}\sum_{k=1}^{M}\sum_{l=1}^{M}\frac{\partial \phi_{k'l'}}{\partial \lambda_{i'}}\frac{\partial \phi_{kl}}{\partial \lambda_i} \\
& \times \left[-\phi_{l'k}^{-1}\phi_{lk'}^{-1} + \left\{\begin{aligned} & \phi_{lk'}^{-1}\sum_{m=1}^{M}\sum_{n=1}^{M}\phi_{l'n}^{-1}\hat{\phi}_{nm}\phi_{mk}^{-1} \\ & + \phi_{l'k}^{-1}\sum_{m=1}^{M}\sum_{n=1}^{M}\phi_{ln}^{-1}\hat{\phi}_{nm}\phi_{mk'}^{-1}\end{aligned}\right\}\right]
\end{aligned} \tag{11}
$$

Application to three-component array

A three-component array is composed of the water surface elevation and horizontal two components of the water particle velocity. The main reason for choosing this array is that it is able to detect bimodal directional spectra (Isobe, 1990). Other reasons are: since it is a point sensor, it is easy to implement in the field; and since cross-power spectra of this array are represented theoretically as real values, the theoretical development becomes simple.

By applying Eq.(3) to a three-component array, the cross-power spectra, ϕ_{mn}, have real values as:

$$\phi_{\eta\eta} = P \tag{12}$$

$$\phi_{\eta u} = PH_u\{\alpha m_{11}\cos\theta_1 + (1-\alpha)m_{12}\cos\theta_2\} \tag{13}$$

$$\phi_{\eta v} = PH_u\{\alpha m_{11}\sin\theta_1 + (1-\alpha)m_{12}\sin\theta_2\} \tag{14}$$

$$\phi_{uu} = PH_u^2\{\alpha(1/2 + m_{21}\cos 2\theta_1) + (1-\alpha)(1/2 + m_{21}\cos 2\theta_1)\} \tag{15}$$

$$\phi_{vv} = PH_u^2\{\alpha(1/2 - m_{21}\cos 2\theta_1) + (1-\alpha)(1/2 - m_{22}\cos 2\theta_2)\} \tag{16}$$

$$\phi_{uv} = PH_u^2\{\alpha m_{21}\sin 2\theta_1 + (1-\alpha)m_{22}\sin 2\theta_2\} \tag{17}$$

where

$$m_{11} = S_1/(S_1+1) \tag{18}$$

$$m_{12} = S_2/(S_2+1) \tag{19}$$

$$m_{21} = S_1(S_1 - 1)/\{2(S_1 + 1)(S_1 + 2)\} \tag{20}$$

$$m_{22} = S_2(S_2 - 1)/\{2(S_2 + 1)(S_2 + 2)\} \tag{21}$$

and H_u is the transfer function from a water surface elevation to horizontal component of a water particle velocity. Subscripts, η, u and v represent the water surface elevation and horizontal two components of the water particle velocity, respectively.

Numerical calculation

(1)Procedure

Numerical simulation is performed to verify the validity of the present method. The method to estimate the directional spectrum is independent of the wave frequency, the simulation is performed under a fixed frequency. The procedure is summarized as follows:

① Define a true directional spectrum, $S(f,\theta)$ as Eq.(1).

② For a given data set, compute the cross-power spectra, ϕ_{mn},

using Eq.(12) to Eq.(17)

③ Calculate the estimated directional spectrum parameters using Eq.(10).

④ The estimated directional spectrum is compared with the true one.

(2) Simulated cross-power spectrum

The simulated cross-power spectra for the three-component array are given by in Eq.(12) to Eq.(17). In the present study, we created sets of cross-power spectra by using the Mitsuyasu-type directional distribution function with the directional spectrum parameters given in **Table** 1.

(3)Initial values for numerical calculation

For the three-component array, estimated directional spectrum by EMLM (Extended Maximum Likelihood Method) is able to represent the bimodal distribution (Horikawa ed.,1985). The following three parameters are available to estimate in the iteration procedure.

First, directional splitting parameter, γ'/γ, are defined as follows:

Table 1: Values of the directional spectrum parameters

<table>
<tr><th>P
($m^2 \cdot s$)</th><th>S_1</th><th>S_2</th><th>θ_1
(deg.)</th><th>θ_2
(deg.)</th><th>$P_2/P_1=$
$(1-\alpha)/\alpha$</th></tr>
<tr><td rowspan="13">1.0</td><td rowspan="3">100</td><td>100</td><td rowspan="13">0</td><td rowspan="13">10
20
30
60
90
120
150
160
170
180</td><td rowspan="13">1.0
0.5
0.2
0.1</td></tr>
<tr><td>50</td></tr>
<tr><td>10</td></tr>
<tr><td rowspan="3">50</td><td>100</td></tr>
<tr><td>50</td></tr>
<tr><td>10</td></tr>
<tr><td rowspan="4">10</td><td>100</td></tr>
<tr><td>10</td></tr>
<tr><td>5</td></tr>
<tr><td>1</td></tr>
<tr><td rowspan="2">5</td><td>50</td></tr>
<tr><td>10</td></tr>
<tr><td>1</td><td>10</td></tr>
</table>

$$\left(\frac{\gamma'}{\gamma}\right)^2 = \frac{4}{\gamma^2}\left[m_{00}\left(m_{20}m_{02} - m_{11}^2\right) + 2m_{10}m_{01}m_{11} - \left(m_{10}^2 m_{02} + m_{01}^2 m_{20}\right)\right]$$
$$/\left[3m_{00}\left(m_{20} + m_{02}\right)^2 - 4\left(m_{10}^2 + m_{01}^2\right)\left(m_{20} + m_{02}\right) + m_{00}\left\{\left(m_{20} - m_{02}\right)^2 + 4m_{11}^2\right\}\right] \quad (22)$$

where γ is the long crestedness parameter, and m_{pq} are the normalized cross power spectra defined as follows:

$$m_{00} = \phi_{\eta\eta} \quad (23)$$

$$m_{20} = \phi_{uu} / H_u{}^2 \quad (24)$$

$$m_{02} = \phi_{vv} / H_u{}^2 \quad (25)$$

$$m_{10} = \phi_{\eta u} / H_u = \phi_{u\eta} / H_u \quad (26)$$

$$m_{01} = \phi_{\eta v} / H_u = \phi_{v\eta} / H_u \quad (27)$$

$$m_{11} = \phi_{uv} / H_u^2 = \phi_{vu} / H_u^2 \quad (28)$$

if small amplitude wave theory is used to evaluate H_u, errors due to nonlinear effect or inaccurate will be included. Therefore, the following definition of the transfer function is used:

$$H_u = \sqrt{(\phi_{uu} + \phi_{vv})/\phi_{\eta\eta}} \tag{29}$$

Secondly, intersecting angle, Δ, of the two peak directions is defined as follows:

$$\Delta = 2\cos^{-1}\sqrt{\frac{(m_{01}m_{11} - m_{10}m_{02})^2 + (m_{10}m_{11} - m_{01}m_{20})^2}{\left\{(m_{10}{}^2 - m_{01}{}^2) - m_{00}(m_{20} - m_{02})\right\}^2 + 4(m_{10}m_{01} - m_{00}m_{11})^2}} \tag{30}$$

Thirdly, the bias of wave energies in the two directions is defined as follows:

$$r_p = \frac{\tan(\theta_m - \theta_c')}{\tan(\Delta/2)} \tag{31}$$

where θ_m is the mean wave direction, θ_c', the median wave direction defined as follows:

$$\theta_c' = \tan^{-1}\left\{\frac{2(m_{10}m_{01} - m_{00}m_{11})}{(m_{10}^2 - m_{01}^2) - (m_{00}m_{20} - m_{00}m_{02})}\right\} \tag{32}$$

Now, the initial values of θ_1, θ_2 and α are given as follows:

$$(\theta_1)_0 = (\theta_c' \pm \delta_1) - (\Delta + \delta_2)/2 \tag{33}$$

$$(\theta_2)_0 = (\theta_c' \pm \delta_1) - (\Delta + \delta_2)/2 \tag{34}$$

$$(\alpha)_0 = (1 - r_p)/2 \tag{35}$$

where $\delta_1 = 10^\circ$, 20° and 30° and $\delta_1 = 0^\circ$, 10°, 20° and 30° are tried to obtain the largest value of L. For S_1 and S_2, initial values are derived under the assumption that degrees of directional concentration are the same. First, S is derived by Eq.(37) from γ_1 which is calculated by Eq.(22) and Eq.(36), and $(S_1)_0$ and $(S_2)_0$ are given as $0.5S$. Also, $(P)_0$ is given as $\phi_{\eta\eta}$.

$$(\gamma'/\gamma)^2 = (1/\gamma)^2 \gamma_1^2 \tag{36}$$

$$\gamma_1 = (2S+1)/(S^2+S+1) \tag{37}$$

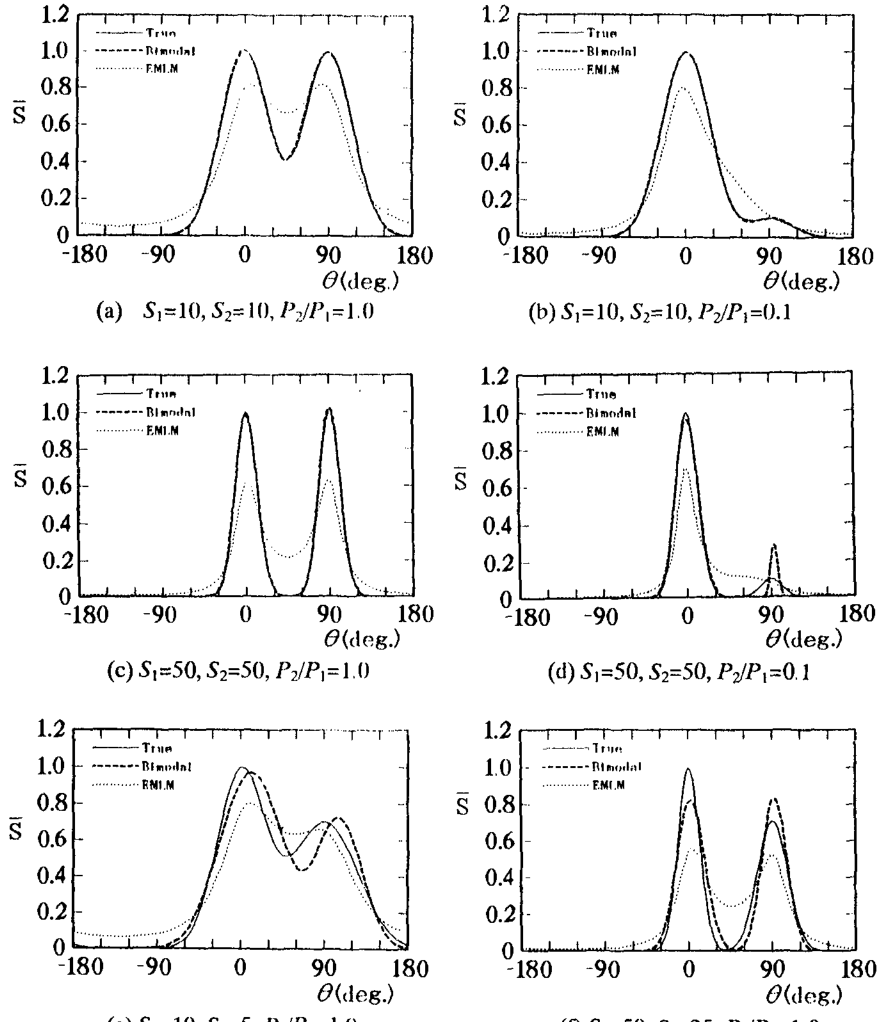
(4)Directional spectra

Before examining the present method, a few remarks should be made concerning the solution by the three-component array. The directional spectrum parameters cannot be determined uniquely by the three-component array, since the number of independent cross-power spectrum is less than that of directional spectrum parameters. Thus, even when the likelihood is equal to $L_{\max}$, the estimated values of the directional spectrum parameters are not always equal to the true values. The directional spectrum for these parameters, however, agrees well with the true directional spectra. Moreover, it is observed that, when both S_1 and S_2 are greater than 5, and α is between 0.3 and 0.7, the estimated values of the directional spectrum parameters almost agree with the true values.

The following figures show the result of closer examination of the present method. **Figure** 1 shows examples of estimated directional spectrum parameters : $\theta_1 = 0^\circ$, $\theta_2 = 90^\circ$, and $(1-\alpha)/\alpha\,(=P_2/P_1)=1.0$, 0.1. The abscissa represents the wave direction, and the ordinate represents the normalized directional spectrum. Also, the results of EMLM are included in the figure for comparison. The results of estimation agree well with the true directional functions.

The accuracy of the method, however, becomes poor as the difference between S_1 and S_2 becomes large. Nevertheless, the accuracy of this method is higher than that of EMLM. **Figure** 2 shows examples in which S_1/S_2 =0.1, or 10 and $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$

Figure 3 shows examples for small intersection angles or small $(1-\alpha)/\alpha$. The results estimated as unimodal directional spectra are also included in the figure for comparison. These estimated distribution agree well with true values. In these cases, assumption of bimodal spectrum is almost the same as that of unimodal spectrum.

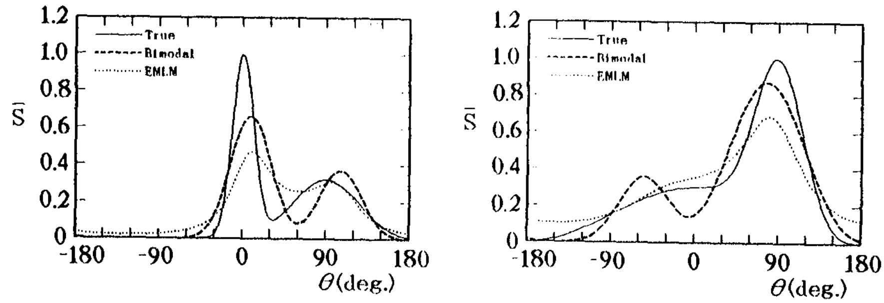


(a) S_1=10, S_2=10, P_2/P_1=1.0
(b) S_1=10, S_2=10, P_2/P_1=0.1
(c) S_1=50, S_2=50, P_2/P_1=1.0
(d) S_1=50, S_2=50, P_2/P_1=0.1
(e) S_1=10, S_2=5, P_2/P_1=1.0
(f) S_1=50, S_2=25, P_2/P_1=1.0

Figure 1 Directional spectra computed by the estimated directional spectrum parameters(1) : $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$

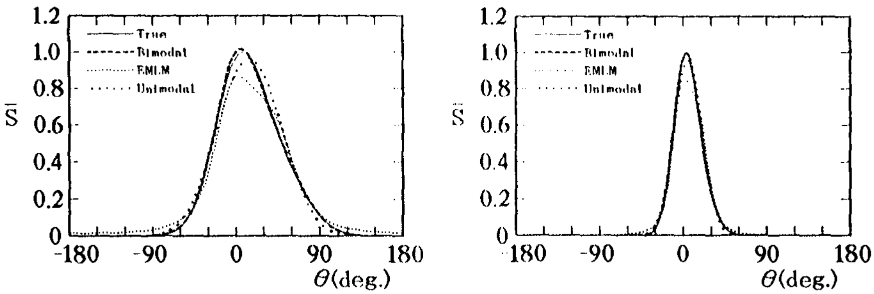
(5)Directional spectrum parameters

Figure 4 shows the accuracy of the estimated directional spectrum parameters. The estimated values of P ,α, θ_1 and θ_2 agree well with true values. The accuracy of estimated S_1 and S_2, however, becomes poor in comparison with the other parameters.



(a)S_1=50, S_2= 5, P_2/P_1=1.0 (b)S_1= 1, S_2=10, P_2/P_1=1.0

Figure 2 Directional spectra computed by the estimated directional spectrum parameters(2) : $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$



(a)S_1=10, S_2= 5, P_2/P_1=1.0 (b)S_1= 50, S_2=25, P_2/P_1=1.0

Figure 3 Directional spectra computed by the estimated directional spectrum parameters(3) : $(1-\alpha)/\alpha$ =1.0

Application to field data

Figure 5 shows the arrangement of three-component arrays in the observation field in east coast of Japan facing to the Pacific Ocean. The water depth at the measurement points are about 20m. A tree-component array consists of an ultrasonic wave gage and a two-component electromagnetic current meter, and all the devices are packed together in one container. A pressure-type wave gage is also packed in the same container to increase the reliability of measurement.

Point A is affected by reflected waves from upright caissons; however, phase lags are considered to be random between incident waves and reflected waves because of a fairly large distance from the caisson. Point B, on the other hand, is not affected by reflected waves. **Figure** 6 shows examples of the estimated directional spectra at Points A and B. It is clear that the directional

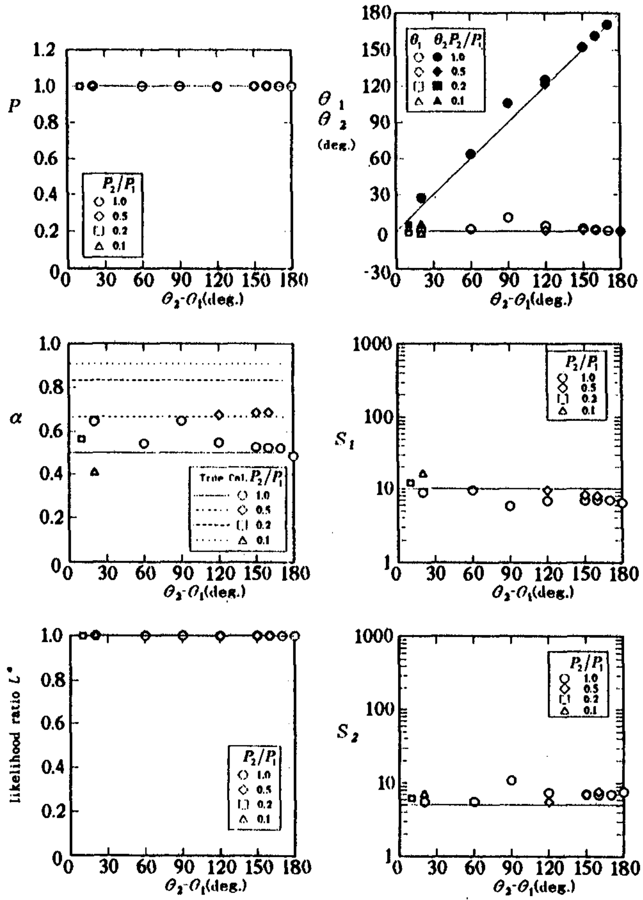


Figure 4 Estimated directional spectrum parameters

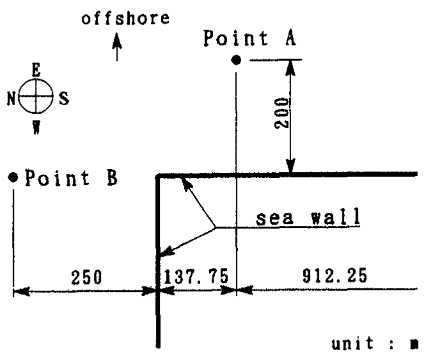


Figure 5 The arrangement of the three-component array in the observation field

spectrum have a bimodal distribution at Point A because of the reflected waves. Therefore, significant wave height, $H'_{1/3}$, of the incident waves was estimated from the wave energy contained in the range of incident wave direction: $H'_{1/3} = H_{1/3}/\sqrt{1+K_R^2}$, $K_R = \sqrt{E_R/E_I}$ ($H_{1/3}$: total significant wave height, E_I and E_R: incident and reflected wave energy). The range of the incident wave direction is defined as **Figure** 6. **Figure** 7 shows the relationship between $H'_{1/3}$ at Point A and $H_{1/3}$ at Point B. Good agreement suggests that the directional spectra are reasonably estimated.

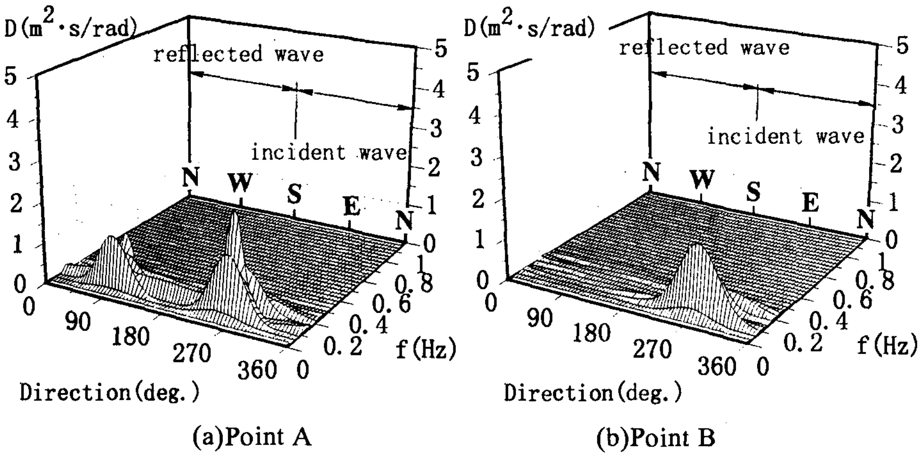


Figure 6 Estimated directional spectra

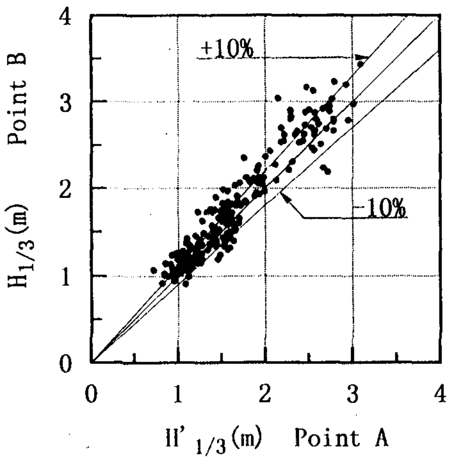


Figure 7 The relationship between the estimated significant wave height at Point A and the significant wave height at Point B

Conclusion

The following conclusions are obtained in this study.

- A method is proposed to estimate a bimodal directional spectrum expressed in a standard form.
- The validity of the method is verified by numerical simulation for a three-component array which is easy to be implemented in the field.
- Application to field data yields reasonable results.

Reference

Barber, N. F. (1963): The directional resolving power of an array of wave detectors, Ocean Wave Spectra, Prentice-Hall, Eaglewood Cliffs, New Jersey, pp.137-150.

Capon, J. (1969): High-resolution frequency-wave number spectrum analysis, Proc. Inst. Electr. Electron. Eng., Vol.57, pp. 1408-1418.

Hashimoto, N. and K. Kobune (1988): Directional spectrum estimation from a Baysian approach, Proc. 21st ICCE, pp. 62-76.

Horikawa, K. ed (1985): Near shore Dynamics and Coastal Process, University of Tokyo Press, pp. 506-524, in Japanese.

Isobe, M., K. Kondo and K. Horikawa (1984): Extension of MLM for estimating directional wave spectrum, Proc. Symp. on Description and Modelling of Directional Seas, pp.A-6-1 - A-6-5.

Isobe, M (1990): Estimation of directional spectrum expressed in standard form, Proc. 22nd ICCE, pp. 647-660.

Kobune, K. and N. Hashimoto (1986): Estimation of directional spectra from the Maximum Entropy Principle, Proc. 5th Int. Offshore Mech. and Arctic Eng. Symp., pp. 80-85.

Longuett-Higgins, M. S., D. E. Cartwright and N. D. Smith (1963): Obserbations of the directional spectrum of sea waves using the motions of a floating buoy, *Ocean Wave* Spectra, Prentic-Hall, Inc., New Jersey, pp. 111-136.

Mitsuyasu, H., F. Tasai, T. Suhara, S. Mizuno, M. Ohkusu, T. Honda and K. Rikiishi (1975): Observations of directional spectrum of ocean wave using a cloverleaf buoy, Jour. of Physical Oceanography, Vol. 5, pp. 750-760.

Panicker, N. M. and L. E. Borgmann (1974): Enhancement of directional wave spectrum estimate, Proc. 14th ICCE, pp. 258-279.