

CHAPTER 204

Mass Transport and Orbital Velocities with LAGRANGEian Frame of Reference

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Abstract

Measurements of surface elevations and horizontal orbital velocities in regular waves and two component wave groups were performed to prove the experimental validity of periodic wave theories for regular waves, and empirical engineering methods for calculations in irregular waves. Besides the traditional wave theories a LAGRANGEian approach was introduced. With this LAGRANGEian method for waves of finite height, based on the geometry of orbital paths given by STOKES wave theories, it is possible to extend the region of validity of STOKES theories over a wider range of wave parameter. The use of a superposition method with LAGRANGEian approach and the application to two component wave groups, results in the appearance of non linear interaction components in surface elevations and horizontal orbital velocities, which are in fair agreement as well with a 2nd order approach for irregular wave surface elevation as with the measurements of surface elevation and horizontal orbital velocities.

Introduction

Orbital velocities and mass transport velocities are important e.g. for the prediction of forces on coastal structures and the verification of transport models in the near shore region. Traditional wave theories (e.g. STOKES, DSFT-Dean Stream Function Theory etc.) and empirical calculation methods (e.g. modified potential theories) are used for theoretical treatment of regular waves and as a basis for simulation methods for irregular waves (e.g. linear superposition). A point of controversy is the selection of calculation methods to give best results and how to interpret mass transport velocity measurements, especially in wave flumes. Measurements of orbital velocities and mass transport velocities in a wave flume are compared with DSFT and calculations based on STOKES wave theories in different ways of interpretation. A LAGRANGEian approach is introduced by the authors.

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Experimental Set-up

The research program was carried out in the Large Wave Flume (Großer Wellenkanal - GWK, University Hannover / Technical University Braunschweig - Length 324m, width 5m, water depth up to 5m) and the Wave Flume "Schneiderberg" (WKS - Length 120m, width 2m, water depth up to 1,2m) of the Franzius-Institute / University Hannover. The investigations were concentrated on waves in intermediate water depth. The measurements were taken with electromagnetic type velocity probes located in fixed positions below the still water level, and also with a velocity probe fixed to a Movable Instrument Carriage (MIC). The MIC follows the water surface with the velocity probe in a constant position below the moving surface. The waves were generated with a piston type wave maker using modified higher order control signals to reduce free parasitic wave components.

The LAGRANGEian approach

For simplicity the introduced LAGRANGEian approach is explained using the linear description of orbital paths. Figure 1 shows a principal sketch for the LAGRANGEian approach surface elevation. Figure 2 gives a detailed information concerning wave kinematics calculation for a fixed probe position below the deepest wave trough. During the calculation process the centre of the orbital path of the respective water particle will be determined for every time step Δt by linear iteration, using the formulas for η and ζ given by the respective order of the theory (Equations 1, 2 for linear wave theory).

$$\zeta(x_0, z_0, t) = -\frac{H}{2} \cdot \frac{\cosh 2\pi(z_0 + d)/L}{\sinh 2\pi d/L} \cdot \sin 2\pi \left(\frac{x_0}{L} - \frac{t}{T} \right) \quad (1)$$

$$\eta(x_0, z_0, t) = \frac{H}{2} \cdot \frac{\sinh 2\pi(z_0 + d)/L}{\sinh 2\pi d/L} \cdot \cos 2\pi \left(\frac{x_0}{L} - \frac{t}{T} \right) \quad (2)$$

where $\zeta(x, t)$ = horizontal location of water particle;
 $\eta(x, t)$ = vertical location of water particle; L = wave length;
 H = wave height; x_0, z_0 = still water position; T = wave period

The surface elevation results from the particle motion of the most upper water particles. The LAGRANGEian surface elevation for first order particle motion and deep water conditions contains higher harmonic components, which are nearly in the same magnitude, as calculated with STOKES higher order theories. For more shallower water STOKES higher order descriptions of orbital paths are used for the LAGRANGEian method. The calculation procedure is the same as outlined before. As the surface elevation results directly from the water particle movement, the kinematic free surface boundary condition is satisfied.

The orbital velocity is calculated for the actual position of a water particle along its orbital path at the location of the probe for each time step. The velocity below wave crest results from the smaller orbit below the velocity probe, the velocity below wave trough relates to the wider orbital path above the probe position, and in the same way for all time steps between crest and trough. Figure 3 shows the time series of horizontal orbital velocity for a theoretical calculated example of 1st order LAGRANGEian approach in comparison to linear wave theory. Figure 4 is the amplitude spectrum for the difference curve of the both methods.

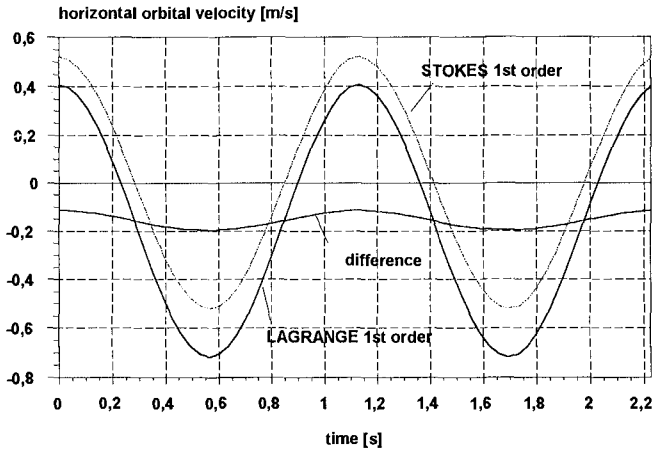


Figure 3. Time series of horizontal orbital velocity
($T=1.13\text{ s}$, $H=0.35\text{ m}$, $d=1.00\text{ m}$, $z=-0.20\text{ m}$)

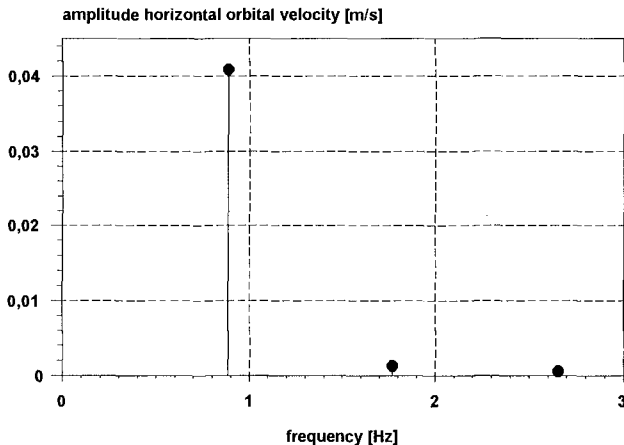


Figure 4. Fourier transformation of difference time series

The Fourier analysis of the theoretical time-series (e.g. motion on orbital paths according to linear theory) results in higher harmonic components and shows a slight decrease of the basic component. In addition a negative dc-value (mean velocity) for the horizontal orbital velocity appears. This negative mean velocity is not to be interpreted as a constant velocity in the opposite direction of wave travel. The LAGRANGEian time series has to be corrected to zero mean, which is in fact the superposition of the mass transport velocity. This again gives a clear argument, that linear wave theory already contains mass transport.

The LAGRANGEian approach can be transferred to irregular waves, by using the superposition method for the particle motion on orbital paths of the linear components of a spectrum, however, effects of mass transport and back flow have to be determined separately. Figure 5 shows an amplitude spectrum of a wave group for a theoretical calculated deep water example.

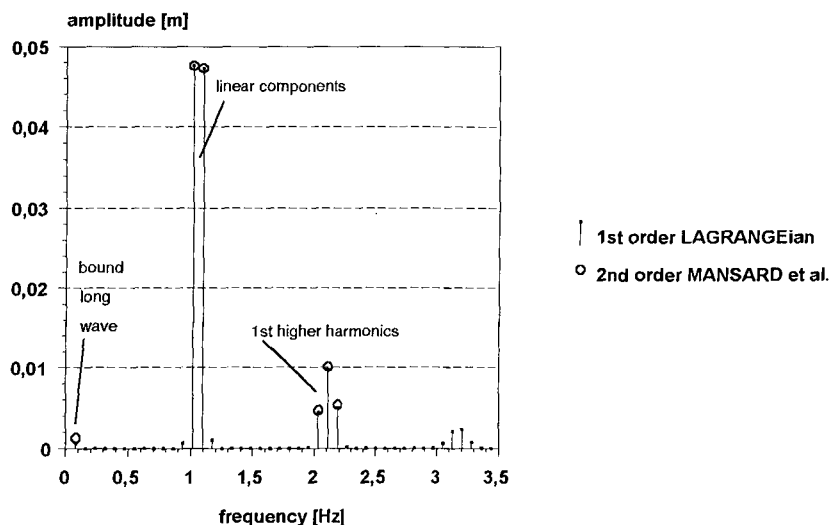


Figure 5. Amplitude spectrum of surface elevation
1st order LAGRANGEian approach - 2nd order MANSARD et al. (1986)
($f_1=1.01\text{Hz}$, $f_2=1.09\text{Hz}$, $H_1=H_2=0.1\text{m}$, $d=1.00\text{m}$)

The components of the wave group spectrum according to 2nd order are in the same magnitude for both methods, and for the 1st order LAGRANGEian approach in addition components higher than 2nd order are clearly visible. This non-linear interactions are automatically appearing using the described superposition method for the orbital motion of the two linear components of the surface elevation spectrum. The formulation of η and ζ for the two linear components is simply added during the iteration procedure.

Figure 6 shows a comparison of super harmonics by the LAGRANGEian approach with the transfer function $G^{+nm}h$ for the 2nd order frequencies of the linear components as given by MANSARD et al. (1986). In deep water a 1st order LAGRANGEian approach gives same results as 2nd order theory. For more shallower water a 2nd order description of particle motion is needed for the LAGRANGEian approach to achieve fair agreement with MANSARD's approach.

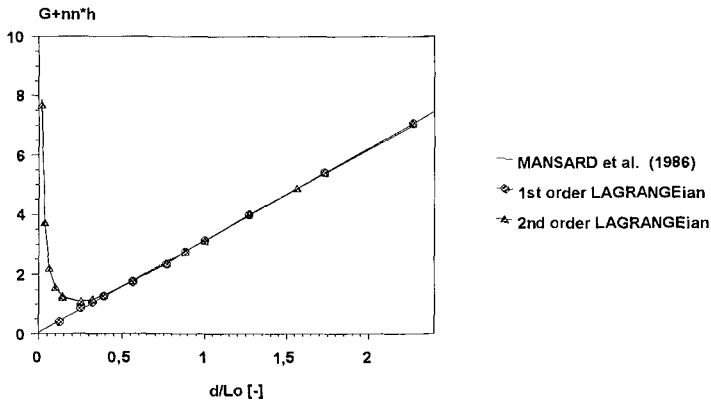


Figure 6. Comparison of 2nd order MANSARD et al. (1986) with LAGRANGEian approach for $f_n + f_n$ and $f_m + f_m$

Figure 7 shows the transfer function $G^{+nm}h$ for the higher harmonic component at frequency $f_m + f_n$.

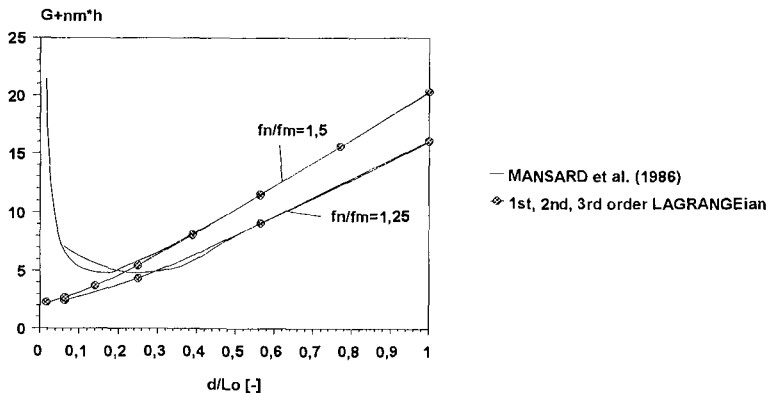


Figure 7. Comparison of 2nd order MANSARD et al. (1986) with LAGRANGEian approach for $f_m + f_n$

The value of the transfer function depends on the relative spacing of the linear components in frequency domain f_n/f_m . For deep water the 1st order LAGRANGEian approach shows fair agreement with the 2nd order theory of MANSARD et al. (1986). In more shallower water the LAGRANGEian approach of higher order is not able to calculate the f_m+f_n higher harmonic amplitude in that order of magnitude, however, differences between LAGRANGEian approach and Mansard's approach are not to big for intermediate water depth. In addition it has not yet been defined clearly to what extend a 2nd order theory is suitable to shallower water conditions. As shown in Fig. 5 LAGRANGEian approach results are of higher than 2nd order.

Comparison with measurements

Figure 8 shows measured mean velocities and mass transport velocities for one example of regular waves plotted as depth distribution. For the fixed measurements a fair agreement with the EULERian backflow considered as constant over depth can be seen. The analysis is done in a time window without reflecting waves. The mean velocities from the movable probe can be interpreted neither as backflow nor as real mass transport velocity according to STOKES 2nd order theory. Only by a LAGRANGEian analysis of the data, a fair agreement of the data with real mass transport velocity, superposed by a backflow to a zero-net mass transport in the closed wave channel, can be achieved.

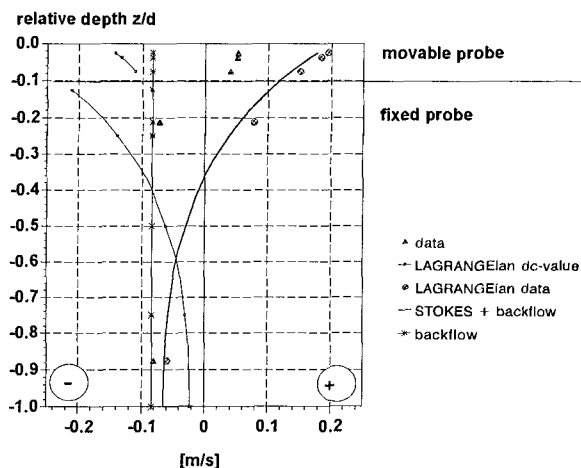


Figure 8. Measured and calculated mass transport velocity and backflow
($T=3.20s$, $H=1.20m$, $d=4.00m$)

The constant EULERian backflow is considered in all theoretical calculations. It is known, that the constant profile of the backflow is only an approximate value, which neglects any influence of viscosity in the boundary layer. The analysis of the data shows that variations of the backflow profile due to the influence of viscosity appear, however, outside the time window chosen for velocity analysis.

Figure 9 shows measured surface elevations in comparison to 5th order DSFT and 3rd order LAGRANGEian approach for a wave in intermediate water depth with $d/L = 0.1$. Small differences are appearing in the crest region, which can be related mainly to a not quite perfect wave generation. It can be assumed, that a certain amount of free wave is superposed on the stable wave, however, both theories show a good overall agreement with the measured time series.

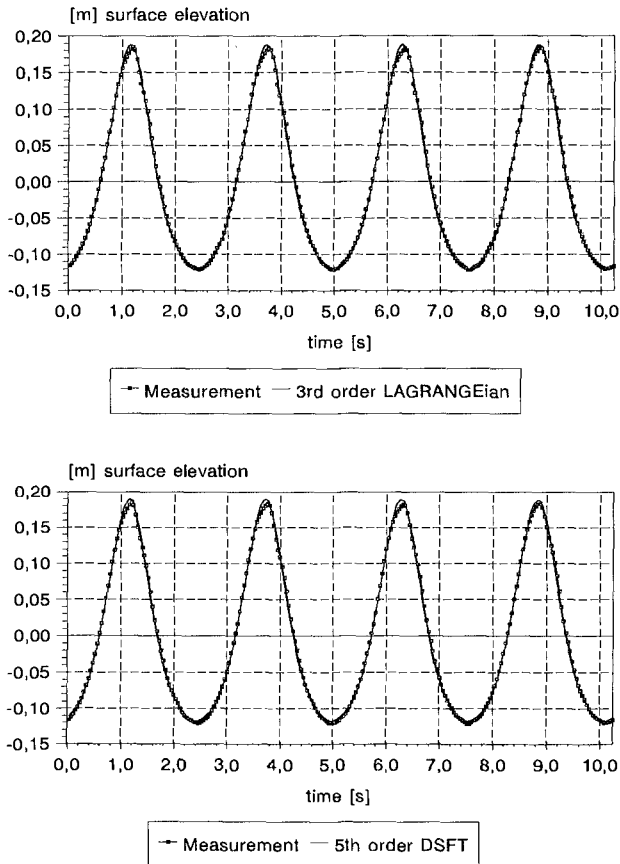


Figure 9. Comparison of measured and calculated surface elevation
($T=2.56s$, $H=0.30m$, $d=1.00m$)

Figure 10 shows comparison of measured and calculated horizontal orbital velocities below the wave crest for the above shown wave case.

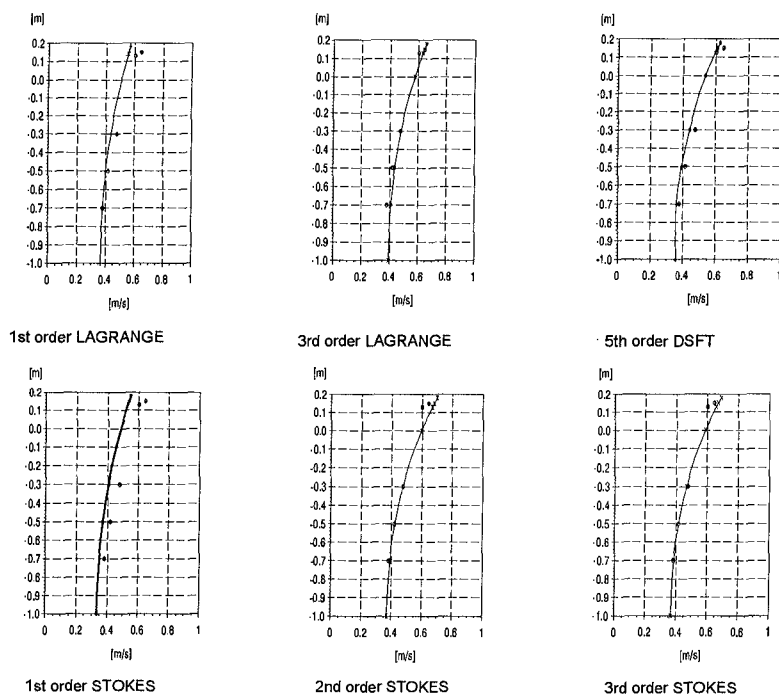


Figure 10. Horizontal orbital velocities below the wave crest
($T=2.56s$, $H=0.30m$, $d=1.00m$)

1st order LAGRANGEian approach and 1st order STOKES theory show a clear underestimation of the measured velocities near the wave crest. Velocities below the deepest wave trough are well predicted using 1st order LAGRANGEian approach. As outlined before, the surface elevation was in fair agreement with 3rd order LAGRANGEian approach and 5th order DSFT, and as the surface elevation is a result of the particle movement on their respective orbits, it is consequent to use this higher order theories also for velocity prediction. Comparison of the data with this approaches show good agreement. 2nd and 3rd order STOKES theory used in EULERian way tend to overpredict the near crest velocities slightly.

The same tendencies also hold for the horizontal orbital velocity below the wave trough, as can be seen in Figure 11. The 3rd order LAGRANGEian approach and the 5th order DSFT fit the measurements, 2nd and 3rd order STOKES theory calculate too high negative velocities, as well as 1st order LAGRANGEian approach and linear wave theory.

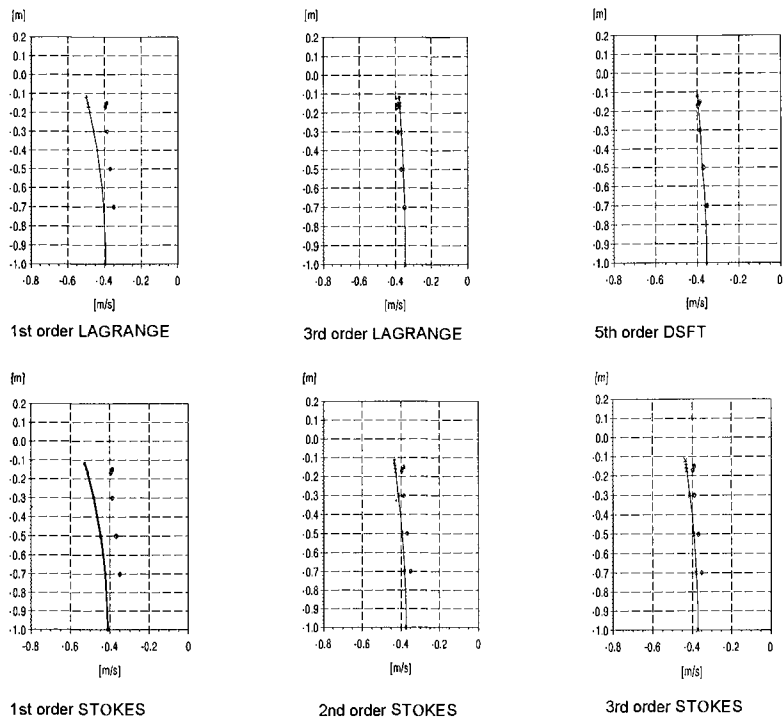


Figure 11. Horizontal orbital velocities below the wave trough
($T=2,56s$, $H=0.30m$, $d=1.00m$)

Results of measurements of horizontal orbital velocities in wave groups and comparison with various engineering methods and with LAGRANGEian approach are shown in Figures 12 to 15. Two wave groups with wave parameter listed in table 1 are considered for this paper.

	f_m [Hz]	f_n [Hz]	H_m [m]	H_n [m]	d [m]
group 1	0,78	0,86	0,07	0,07	1,00
group 2	0,78	0,86	0,12	0,03	1,00

Table 1. Wave parameter of the wave groups

Figure 12 shows a comparison of measured horizontal orbital velocities with the theory as scatter plots for wave group 1. The measurement is done below the deepest wave trough at $z=-0.10m$ with a fixed velocity probe. The Linear Transfer Function Method (LTFM) shows the known tendency of overprediction of the crest velocities.

WHEELER's stretching method gives slightly better results below the wave crest, but there is a small tendency of underprediction of the velocities below the wave troughs. The complementary method shows similar tendencies as the LTFM. The stretching after DEAN and LO (1986) under predicts the crest velocities as well as the velocities below the wave troughs. DONELAN's superposition method (1992) shows good results, and finally a 2nd order LAGRANGEian approach fits the measurement best.

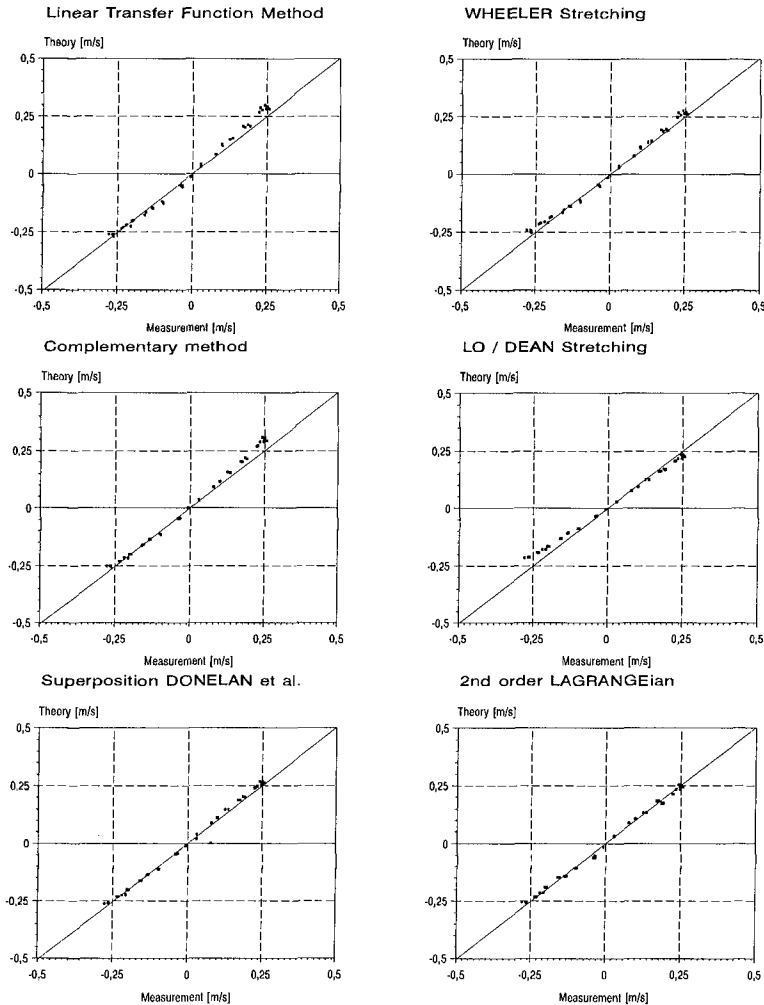


Figure 12. Comparison of measured and calculated horizontal orbital velocity at $z=-0.10\text{m}$ for wave group 1

The tendencies outlined before also hold for wave group 2 in figure 13, however, there are stronger deviations in the results of LTFM and DONELAN's superposition method.

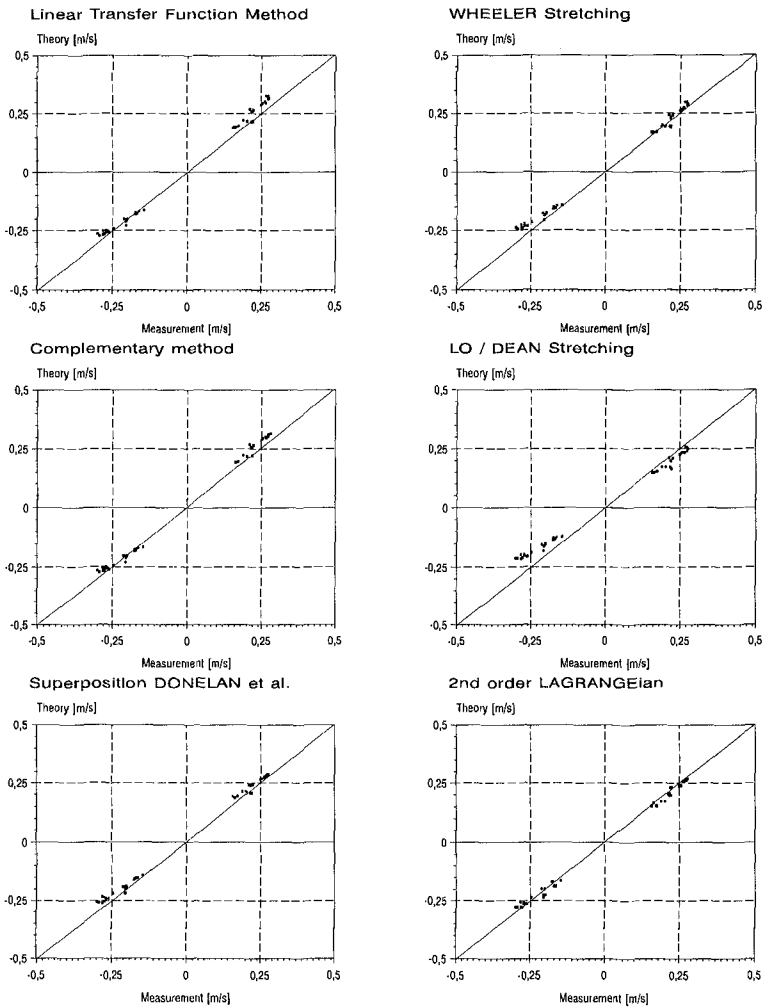


Figure 13. Comparison of measured and calculated horizontal orbital velocity at $z=-0,10\text{m}$ for wave group 2

The reason for the mentioned tendencies becomes more clear looking to the comparison of the calculated and measured velocity components in frequency domain.

Figure 14 gives the measured and calculated values according to 2nd order theory for wave group 1. LTFM calculates too high basic components of the horizontal orbital velocity as well as too high 2nd order super harmonics. WHEELER's stretching counteracts this concerning the linear components, the higher harmonic components, however, are over predicted. DONELAN's superposition method calculates too high linear velocity components, the super harmonics are better predicted than by the two before mentioned methods. Finally, the 2nd order LAGRANGEian approach gives fairly good results concerning the linear components and also concerning the higher harmonic components of the horizontal velocity, which in this example are relatively small. For the velocity at frequency component $f_n - f_m$ it can be stated, that, except DONELAN's superposition method, all methods calculate the velocity in the same magnitude and in disagreement with the measurement. This will not be discussed in detail, since this is state the of actual research work. It is assumed, that the misinterpretation of the bound long velocity component belongs to interaction of mass transport velocity and back-flow in the wave channel and the resulting free long wave. The analysis of the data shows, that the deviations between the theoretically calculated and measured long wave velocity is not the reason for the before mentioned tendencies.

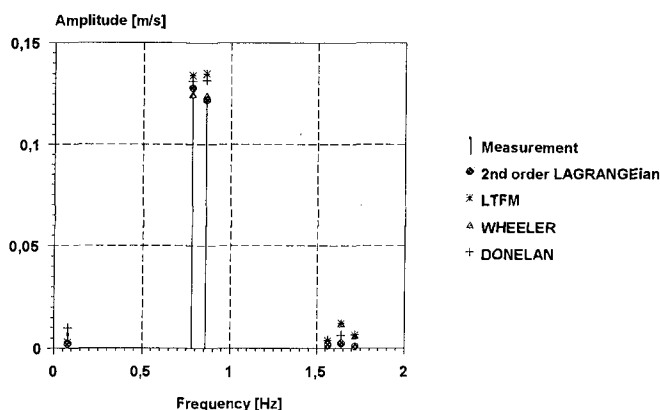


Figure 14. Amplitude spectrum of measured and calculated horizontal orbital velocity according to 2nd order for wave group 1, $z=-0.10\text{m}$

A main reason for the failure of the engineering methods in prediction of orbital velocities in irregular waves is the misinterpretation of the higher harmonic components. In figure 15 measured and calculated profiles of the horizontal orbital velocity below the highest wave crest and the deepest wave trough are plotted for both wave groups. The overprediction of the higher order velocity components using the higher order components of the surface elevation as linear input for the LTFM, increases rapidly above still water level. Complementary method and also DONELAN's superposition method counteracts this tendency, but clear deviations are visible. Only LAGRANGEian approach, which uses the basic components of the surface elevation as input, fits the measurement near the surface. The opposite tendency of underpre-

diction of the trough velocities is for all methods the same. LAGRANGEian approach is in fair agreement with the measured values.

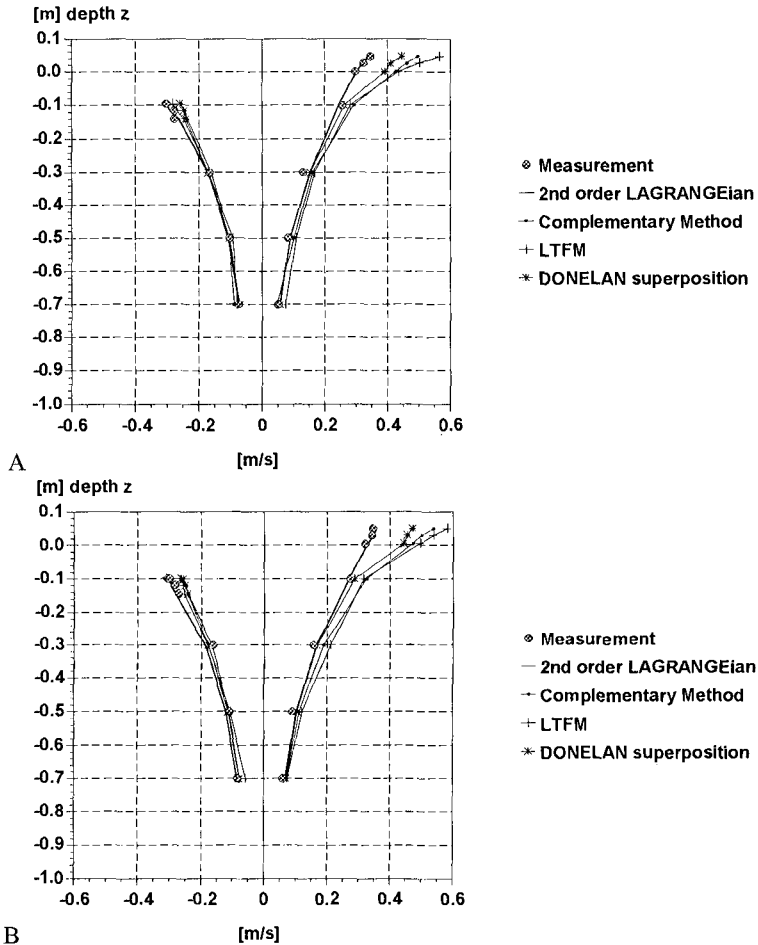


Figure 15. Measured and calculated horizontal orbital velocities below the highest wave crest and below the deepest wave trough (A : wave group 1 | B : wave group 2)

Conclusions

Measurements of surface elevations and horizontal orbital velocities in regular waves and in two component wave groups are compared with various calculation methods. A LAGRANGEian approach, which bases on the geometry of orbital paths given by STOKES 1st, 2nd and 3rd order theory was introduced by the authors.

Following conclusions can be achieved after presentation and discussion of the results:

- The periodic wave theories of the respective order show good agreement with measured regular wave kinematics as well as with regular wave surface elevations.
- The introduced LAGRANGEian approach gives good results concerning surface elevations and orbital velocities and needs a lower order for calculation than other periodic wave theories.
- Using LAGRANGEian approach on the basis of STOKES wave theories, the application range of the STOKES theory can be extended over a wider range of wave parameter.
- Stretching techniques and engineering methods on the basis of linear wave theory seem to be not the right physical tool for the calculation of wave kinematics in irregular waves. The main reason for the failure of these methods is the linear treatment of the super harmonics of the surface elevation.
- Using a superposition method with the LAGRANGEian approach, nonlinearities are automatically appearing. Especially in deep water a 1st order LAGRANGEian approach allows to calculate higher order irregular wave kinematics as well as higher order irregular surface elevations in a mathematical and physical conclusive way.

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