

## CHAPTER 200

### SCALING EFFECTS ON BEACH RESPONSE PHYSICAL MODEL

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#### Abstract

A modified modeling law based on the comprehensive work by Wang, *et al.* (1990) has been derived with the assumptions that the suspended sediment transport is the dominant mode under storm wave condition inside the surf zone and the wave breaking index is preserved. This, in essence, adds to the constraint that wave height is properly modelled in accordance to the preservation of breaking index instead of simple vertical geometrical scale. Four different scaling laws including the one proposed presently were examined in the laboratory using 2-D wave tank and 3-D basin models. The model results were compared to prototype data from German large wave tank experiment. The model performances under different scaling laws were evaluated separately in the dune region and bar region of the entire model beach profile. Several statistics, including the errors of sand bar location, bar volume, bar profile, dune erosion volume, and dune profile, between model and prototype, were calculated and the result showed that the newly proposed scaling law presented the best overall model performance among the ones compared.

#### 1. Introduction

Beach and dune erosion as well as the shore profile changes that occur under storm waves and high water level are of basic interests in coastal engineering. And, by all means, physical model has been utilized frequently to improve our understanding of the process of beach and dune erosion and provide useful data for numerical models. Numerous papers have been written proposing various similitude relationships for beach response model study. At present there is no general solution which is also practical. Specific modeling laws are usually only applicable to certain restricted conditions. This paper is aimed at evaluating and improving the scaling law for distorted model guided by the modeling theory and through a series of laboratory

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experiments carried out at different physical scales with the main assumption that suspended sediment transport is the dominant mode in the surfzone.

## 2. Theoretical Approach

Many different scaling laws of beach response model have been proposed in the past. However, there is no clear indication of which is more appropriate than the others. The approach here is to follow the comprehensive work by Wang, *et al.* (1990) but with consideration of a more general wave breaking criterion to determine the proper wave height scale in model. The approach seems adequate since it shows that the modeling law can be established based on the concept accommodating the sediment transport. The theory applied is simple. Based on the equation balancing the spatial change of sediment transport rate and the temporal change of beach profile, the scaling of sediment transport rate can be shown to be

$$N_q = N_\delta N_\lambda / N_t,$$

where  $N$  indicates the ratio of prototype to model quantities such that  $N_q$  is the scale of volumetric sediment transport rate,  $N_\delta$  and  $N_\lambda$  are the vertical and horizontal length scales, respectively, and  $N_t$  is the morphological time scale. Then, by assuming the suspended load mode predominate the sediment transport, the scale of sediment transport rate can be also expressed in a depth average means as

$$N_q = N_\delta N_u N_c,$$

where  $u$  is the sediment transport velocity, and  $c$  is the sediment concentration. By further assuming that  $N_u$  can be determined in preserving the Froude number  $F = u/\sqrt{gh}$ , where  $h$  is the water depth, and  $N_c$  can be obtained from a perceptive model of sediment concentration under wave motion as

$$c \propto \frac{H f(\xi)}{T SW},$$

where  $H$  and  $T$  denote wave height and wave period, respectively,  $f(\xi)$  is a function of the surfzone parameter  $\xi = \tan \beta / \sqrt{H_o/L_o}$ ,  $\beta$  is the beach slope,  $H_o$  and  $L_o (= gT^2/2\pi)$  are the deepwater wave height and wave length,  $S$  is the submerged specific weight, and  $W$  is the sediment settling velocity, a pair of equations can be established:

$$N_T = N_\lambda N_H^{1/2} / N_\delta, \quad N_t = N_\lambda^2 N_W N_S / (N_\delta^{3/2} N_H^{1/2}),$$

under the condition that surfzone parameter is preserved, or  $N_\xi = 1$ , in model.

As a major difference to the concept utilized by Wang, *et al.* (1990), the wave height scale is now assumed here to be not equal to the vertical scale in general. Clearly, in modeling beach profile change, wave height inside the surf zone should be similar between model and prototype. Of course, by treating wave height as a vertical geometrical scale in essence implicitly assumes that wave height is proportional to the local water depth, i.e.,  $H = \gamma_b h$ , with the  $\gamma_b$  a constant value, which was originally proposed by McCowan (1894) and was widely used as breaking criterion. In this paper, however, a more general wave height scaling law is adopted here:

$$N_H = N_\gamma N_\delta,$$

Table 1: Classification of Beach Profile Modeling Laws.

Author	Geometric Distortion	Hydrodynamic Time Scale( $T$ )	Morphological Time Scale( $t$ )
Vellinga (1982)	$N_\delta = N_W^{0.44} N_\lambda^{0.78}$	$N_T = \sqrt{N_\delta}$	$N_t = N_T$
Hughes (1983)	$N_\delta = N_W^{2/3} N_\lambda^{2/3}$	$N_T = N_\lambda / \sqrt{N_\delta}$	$N_t = N_T$
Wang, <i>et al.</i> (1990)	$N_\delta = N_W^{2/5} N_\lambda^{4/5}$	$N_T = N_\lambda / \sqrt{N_\delta}$	$N_t = \sqrt{N_\delta}$
Wang, <i>et al.</i> (present)	$N_\delta = N_W^{2/5} N_\lambda^{4/5}$ $N_H = N_\delta^2 / N_\lambda$	$N_T = \sqrt{N_\lambda}$	$N_t = N_T$

$N$  =prototype to model scale ratio,  
 $\lambda$  =horizontal length scale,  $\delta$  =vertical length scale,  
 $W$ =sediment fall velocity,  $H$ =incident wave height.

where  $N_\gamma$  represents the scale ratio of the breaking index,  $\gamma_b$ .

A guideline on determining  $N_\gamma$  is proposed by examining the functional form of  $\gamma_b$  as developed by various investigators. The breaking index could be affected by both beach slope and deepwater wave steepness, with the latter likely to be minimal inside the surf zone. A general power law of functional form of  $N_\gamma$  is proposed as

$$N_\gamma = \left[ \frac{N_\delta}{N_\lambda} \right]^k,$$

where the value of  $k$  is likely in the range from 0 to 1. In the case of  $k=0$ , the proposed modeling law reduces to that of Wang's (1990). On the other extreme as  $k=1$ , which states that the scale of breaking index is linearly proportional to the scale of local beach slope, or

$$N_r = N_{\tan\beta} = N_\delta / N_\lambda,$$

assuming  $N_T = N_t$ , i.e., the number of incoming waves per unit time is preserved yields another modeling law as

$$N_T = N_t = N_\lambda^{1/2}, \quad N_H = N_\delta^2 / N_\lambda, \quad N_q = N_\delta N_\lambda^{1/2}, \quad N_\delta = (N_S N_W)^{2/5} N_\lambda^{4/5}.$$

Therefore, the modeling laws derived above and by Wang, *et al.* (1990) represent two limiting conditions of the breaking index scale ratio. Table 1 presents four different modeling laws covering the existing scope of the beach profile modeling law which will be tested against laboratory experiments in this study.

### 3. Laboratory Experiments

In order to verify the scaling laws shown in Table 1, a total of 38 laboratory experiments were conducted, with 25 experiments run in a 30 m long wave tank, 10 experiments completed in a 15 m long tilting flume and three experiments in the 12m wide, 30m long 3-D wave basin. The coarse sand with a median size of 0.2mm has been utilized in both wave tank and tilting flume for the distorted model ( $N_\delta \neq N_\lambda$ )

study. Three different horizontal length scales of  $N_\lambda=20, 30,$  and  $40,$  were selected for the experiments as compared to the prototype data from the German large wave tank experiments, which was also known as the GWK experiment (Dette and Uliczka, 1986). The prototype sand has a median size of  $0.33\text{mm}.$  Accordingly, the corresponding sediment settling velocity scale can be approximated by  $N_W=2,$  and the vertical length scales corresponding to  $N_\lambda=20, 30,$  and  $40,$  can be roughly determined to be equal to  $N_\delta=14.5, 20.0,$  and  $25.2,$  respectively, in the models.

As special cases, fine sand with median size equal to  $0.09\text{mm}$  was also replaced in the same  $30\text{m}$  long wave tank for the undistorted model experiments. For this sand,  $N_W = 6,$  as compared with the prototype sand. Three length scales of  $N_\lambda = N_\delta=20, 30,$  and  $40,$  were again selected for the experiments of undistorted model study. The scaling effects were then tested for all the models of different length scales by using different wave periods and wave heights in individual experiments. Three 3-D wave basin experiments were also conducted as to repeat some 2-D wave tank experiments under the same test conditions to examine the three dimensional effects. All the experimental cases tested in this study are summarized in Table 2.

#### 4. Results

The model performance from the experiments was monitored by surveys of model beach profile at different times. In the 2-D wave tank and wave flume experiments, only the center profile is surveyed. In the 3-D wave basin experiments, five profiles, which were evenly spaced across the basin, were surveyed. All the surveys were carried out at times of  $0\text{min}, 5\text{min}, 10\text{min}, 20\text{min}, 40\text{min},$  and  $80\text{min},$  in model. Since the data set is too voluminous to be included in this paper, only reduced information related to the specific study subject is presented. The test results were compared with the GWK prototype data as for evaluating the scaling laws. Of course, the modelled beach should behave closely to the prototype at the points when model wave height and wave period were scaled by a proper modeling law. Figure 1 presents a few typical model and prototype profiles, all shown in the prototype scale, at the final stage of the experiments. The examination of scaling effects will be carried out only for the cases of the distorted model in which the model behavior resembles more to the prototype. For the undistorted model study (Tests A22 to A26), the experiment results were not encouraging, which generally show small scattered bars and insignificant dune erosion as compared to the prototype. The reason is probably due to the fact that wave energy dissipates more quickly over the flatter bed of fine sand in the undistorted model. An example showing the undistorted model profile from Test A22 in the final model stage is presented in Figure 1.

##### 4.1 Evaluation of Scaling Effects

The evaluation of scaling law is performed by investigating the model beach profile changes in separate dune and bar regions instead of the entire profile. The dune region is defined as from the landward end of the profile to the seaward end of the first slope in initial profile and the bar region is defined as from the beginning of the second slope to the seaward end of profile. Figure 2 shows a schematic of the defined dune and bar regions. The scaling effects were evaluated in the dune region

Table 2: Summary of Experimental Cases.

Test Facility	Test ID#	Wave period (sec)	Wave height (cm)	Water depth (cm)	Grain size (mm)	Horizontal scale, $N_\lambda$	Vertical scale, $N_\delta$
Wave Tank (2-D)	A1	1.00	11.50	52.0	0.20	20.0	14.46
	A2	1.14	10.50	34.6			
	A3	1.20	11.25	52.0			
	A4	1.20	12.75	52.0			
	A5	1.33	10.00	52.0			
	A6	1.33	11.00	35.3			
	A7	1.33	11.25	35.3			
	A8	1.33	12.00	52.0			
	A9	1.33	12.75	52.0			
	A10	1.33	13.00	52.0			
	A11	1.33	17.50	52.0			
	A12	1.45	10.50	52.0			
	A13	1.45	13.50	52.0			
	A14	1.45	18.00	52.0			
	A16	1.33	10.00	40.0	0.20	30.0	20.0
	A17	1.15	9.50				
	A18	1.00	9.50				
	A19	0.80	5.50	40.0	0.20	40.0	25.2
	A20	1.00	9.00				
	A21	1.15	9.50				
	A22	1.34	7.50	40.0	0.09	20.0	20.0
	A23	1.34	7.50			30.0	30.0
	A24	1.10	5.00			30.0	30.0
	A25	1.10	5.00			40.0	40.0
	A26	1.05	3.75			40.0	40.0
Wave Flume (2-D)	T1	1.30	11.50	35.3	0.21	20.0	14.46
	T2	1.30	11.50				
	T3	1.30	12.00				
	T4	1.30	12.50				
	T5	1.30	12.50				
	T6	1.30	13.00				
	T7	1.30	13.00				
	T8	1.30	15.00				
	T9	1.30	16.50				
	T10	1.65	16.50				
Wave Basin (3-D)	B1	1.14	10.50	35.3	0.20	20.0	17.1
	B2	1.14	10.50				14.6
	B3	1.33	12.50				14.6

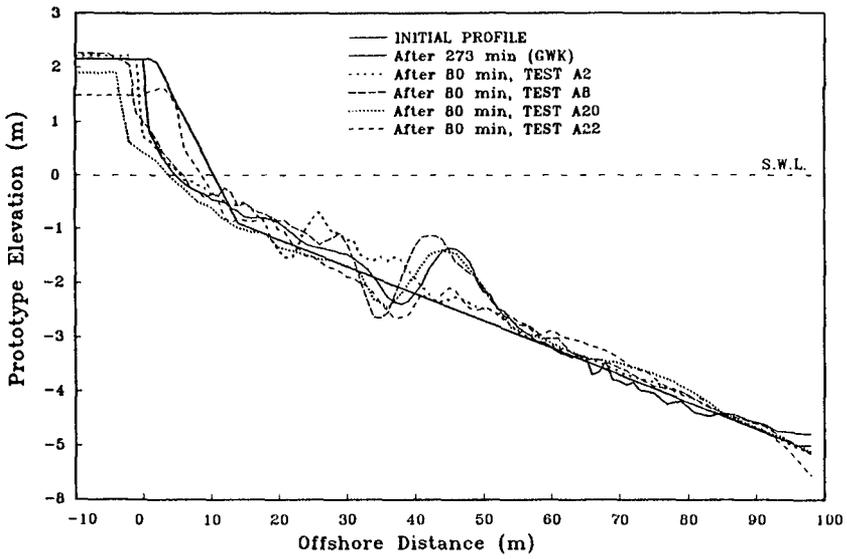


Fig.1: Comparison of model and prototype profiles in equilibrium state.

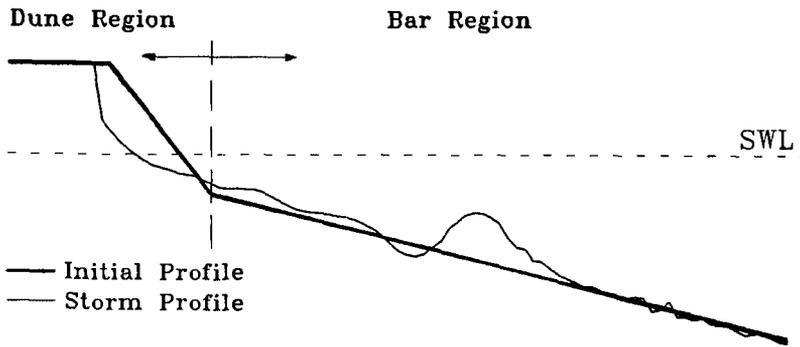


Fig.2: A schematic of dune and bar profile regions.

by calculating the error of dune erosion volume and the RMS error of dune profile as compared to the prototype. In the model bar region, the errors of bar crest location and bar volume, and RMS error of bar profile were calculated as compared to the prototype. The RMS error of the modelled beach profile is defined as

$$\epsilon = \left[ \frac{1}{n} \sum_{i=1}^n (h_{M,i} - h_{P,i})^2 \right]^{1/2},$$

where  $h_M$  and  $h_P$  are the corresponding profile elevations obtained from model and prototype, respectively. Other errors, including the errors of dune erosion volume, bar location and bar volume, are defined in the dimensionless form as

$$\frac{A_M - A_P}{A_P},$$

where  $A_M$  and  $A_P$ , respectively, are the model and prototype quantities used in the calculation of errors. Figures 3 to 7 show these errors computed and compared against model wave height for different model wave periods from all tested cases with  $N_\lambda = 20$  and  $N_\delta = 14.5$ . These errors were computed based on the survey of model profile at the model time of 40min and the final survey of prototype profile at the prototype time of 273min. This model profile from the survey at the model time of 40min is deemed to correspond to the final prototype profile upon the basis of correct morphological time scale. In Figure 3, a somewhat positive linear relationship between model wave height and error of dune erosion volume is observed. This indicates that greater dune erosion in model is likely to occur under larger waves. However, the effect of wave period upon dune erosion in model is less clear revealing a rather weak relationship between the two. The same patterns are also seen in Figures 5 and 6 for the errors of bar volume and bar location. That is, larger bar volume and further seaward bar location in model are likely to occur in the case of higher waves. In Figures 4 and 7, the RMS errors of modelled profile from the prototype in the dune and bar regions, respectively, were compared against model wave height for different wave periods. In these figures, it is seen that the effect of wave height to the RMS errors is less significant. However, the effect of wave period becomes important since a proper model wave period can result a consistent small RMS error which indicates a better fit of modelled profile to the prototype.

Based on the result of errors compared in Figures 3 to 7, it is clear that a proper modeling law should yield smallest absolute value of errors. That is, the zero crossings appeared in Figures 3, 5, and 6, and the least RMS errors in Figures 4 and 7 should correspond to the best modeling law. In order to examine the performance of the four modeling laws presented in Table 1, the errors corresponding to these modeling laws were also identified and marked in these Figures. Clearly, the modeling law derived in the paper presents the best model result in both dune and bar regions as compared to the prototype.

#### 4.2 Evaluation of Morphological Time Scale

The morphological time scale of a modeling law was evaluated by comparing dune and bar profile properties in model and prototype at different time intervals.

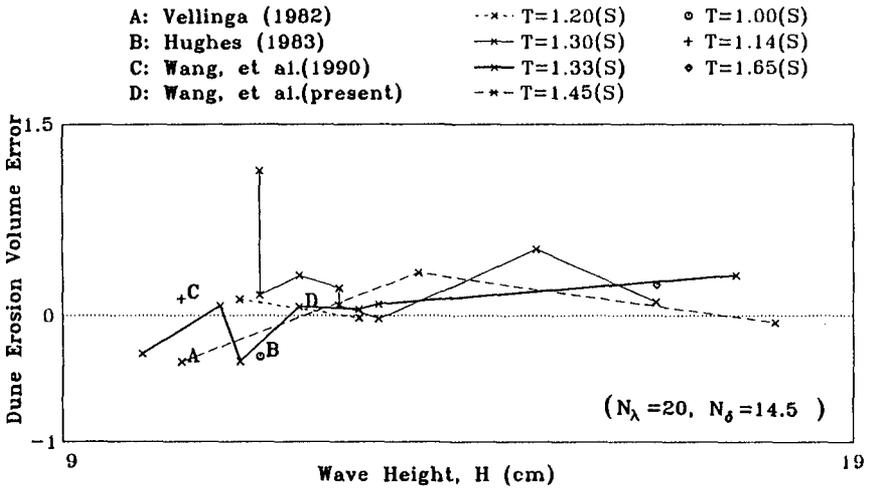


Fig.3: Comparison of error of dune erosion volume.

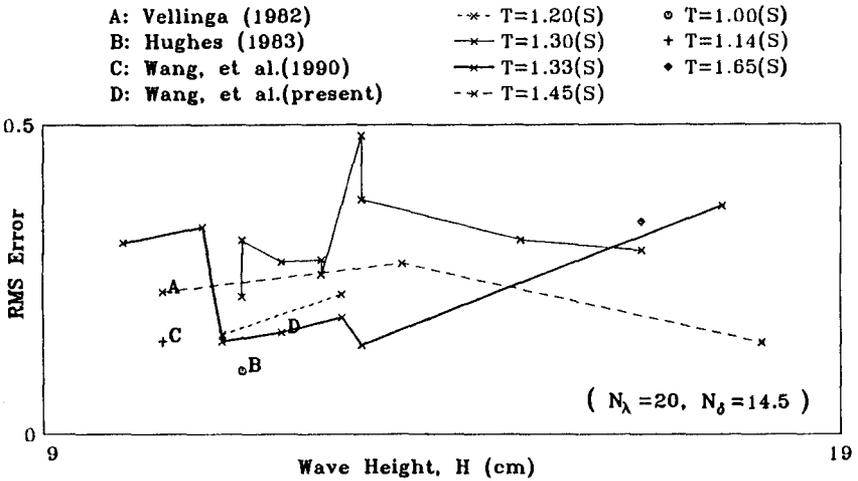


Fig.4: Comparison of RMS error of dune profile.

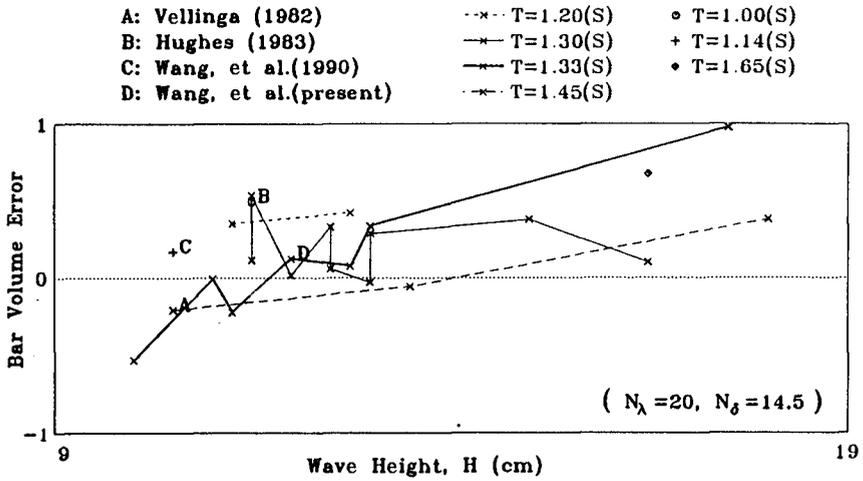


Fig.5: Comparison of error of bar volume.

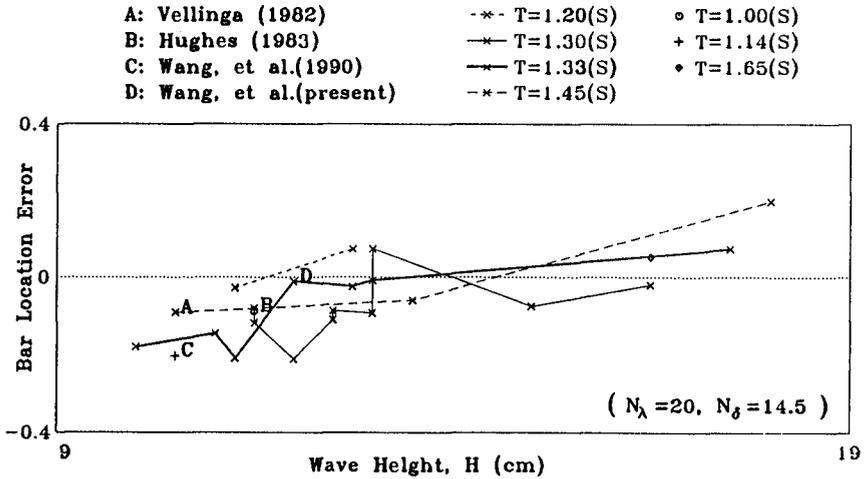


Fig.6: Comparison of error of bar crest location.

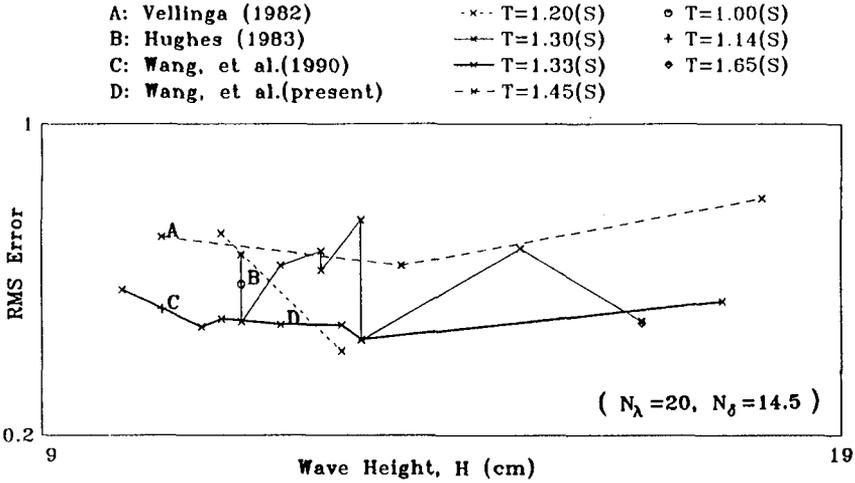


Fig.7: Comparison of RMS error of bar profile.

The modelled beach profile results from three experiments, each presented the best fit model profiles to the final prototype data from three different horizontal scale groups, were selected for this evaluation. The three experiments are Test B3 ( $N_\lambda = 20$ ) from the 3-D wave basin experiment, and Tests A18 ( $N_\lambda = 30$ ) and A20 ( $N_\lambda = 40$ ) from the 2-D wave tank experiment. The 3-D model profiles used in the evaluation are those averaged from the survey of five profiles across the basin to eliminate the minor 3-D effect in model. Based on these experimental data and prototype data, three different morphological time scaling laws as of Hughes (1983), Wang, *et al.* (1990) and the present one shown in Table 1 were compared in terms of the temporal changes of dune erosion volume, RMS elevation difference to initial dune profile, bar volume and bar location, and RMS elevation difference to initial bar profile. The RMS elevation difference to initial profile is defined as

$$e = \left[ \frac{1}{n} \sum_{i=1}^n (h_{S,i} - h_{I,i})^2 \right]^{1/2},$$

where  $h_I$  and  $h_S$ , respectively, are the profile elevations from the initial survey and one following survey at a later time. The results of comparison of these errors computed for the three morphological time scaling laws are shown in Figures 8 to 12.

Figure 8 presents the result of time changes of dune erosion volume for all three scaling laws compared. The comparison shows good model result from the 3-D wave basin experiment for  $N_\lambda=20$  but not the 2-D experiment for  $N_\lambda = 30$  and 40. The reason is probably due to poor compaction and dry condition of sand in the dune region in the 2-D experiment. It appears that none of the model results as well as the prototype have reached equilibrium at the end of the run. For the RMS elevation difference to initial dune profile, all three modeling laws appeared to perform reasonably well in Figure 9 as the scaled model values clustered in a narrow range around the prototype result. For the temporal changes of bar crest location, all

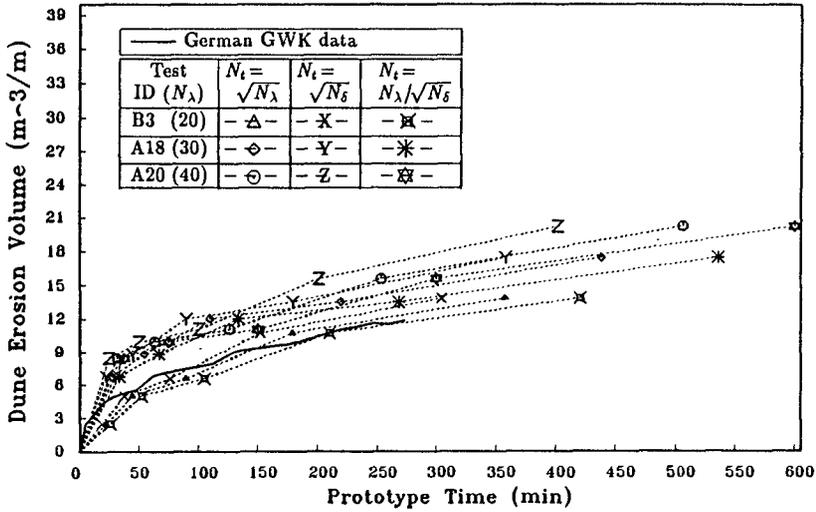


Fig.8: Comparison of temporal changes of dune erosion volume.

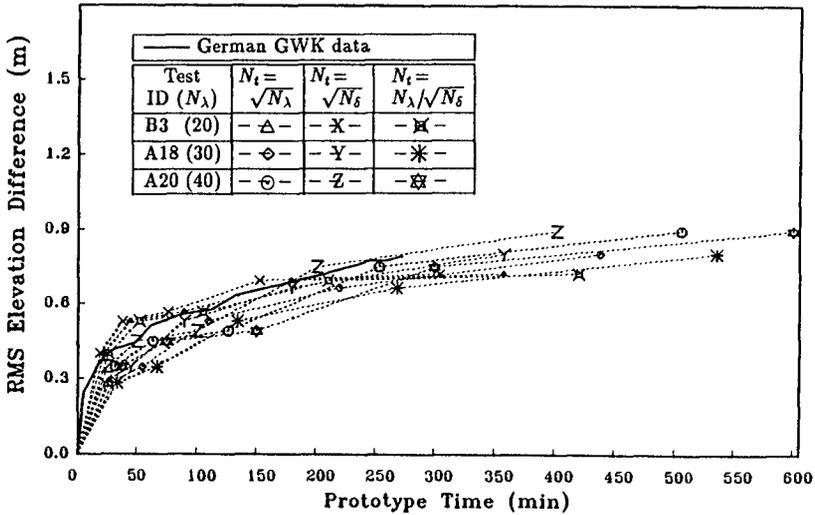


Fig.9: Comparison of RMS elevation difference of dune profile.

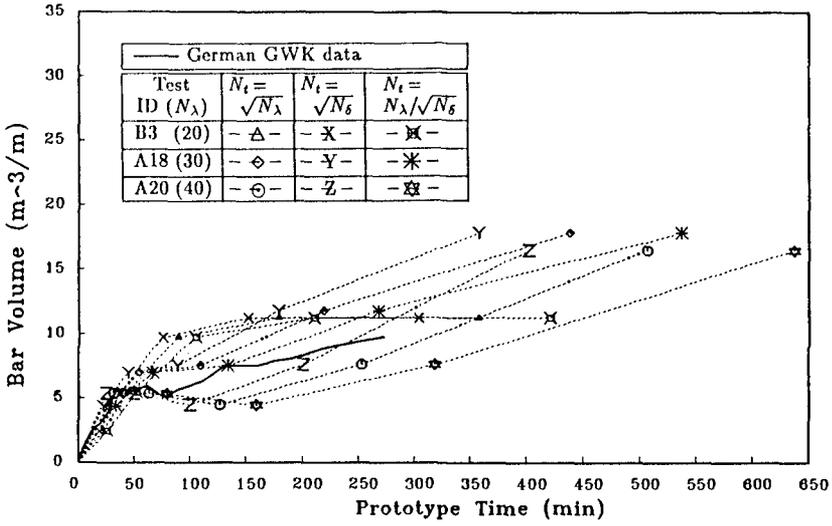


Fig.10: Comparison of temporal changes of bar volume.

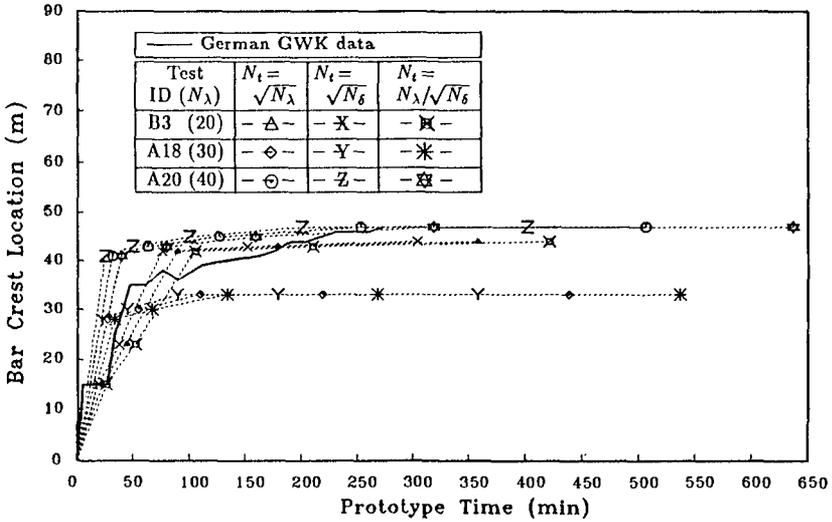


Fig.11: Comparison of temporal changes of bar crest location.

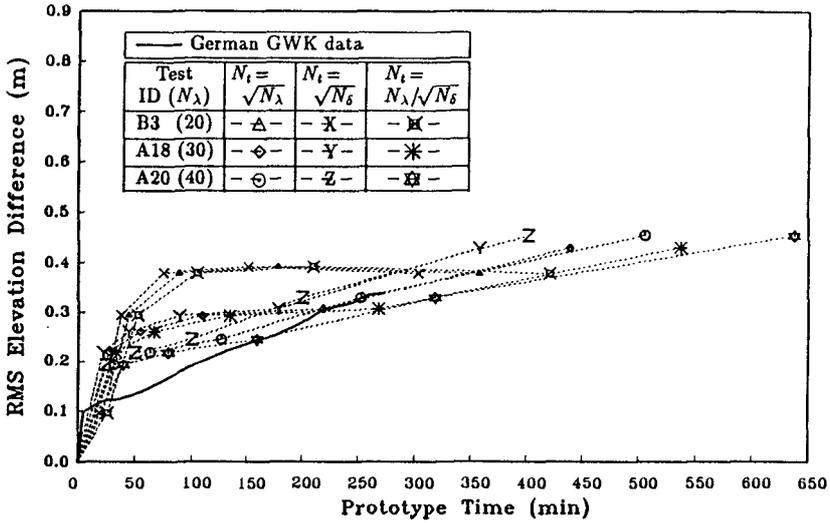


Fig.12: Comparison of RMS elevation difference of bar profile.

three modeling laws appeared to give reasonable results for the data compared to the prototype with the exception of those from Test A18. For the bar volume and RMS elevation difference of bar profile, the modeling law derived in the paper apparently showed the best fit to the prototype data than the other two modeling laws compared. Table 3 presents a summary of the model length and time scales computed based on the four modeling laws and those estimated from the best experimental results for the case of  $N_\lambda = 20$  and  $N_W = 2$ . Certainly, the newly derived modeling law shows overall better performance than the others compared.

5. Summary and Conclusions

A new modeling law for distorted beach model was derived by modifying the one proposed by Wang, *et al.* (1990). The new modeling law preserves not only the sediment transport velocity parameter and surf zone parameter but also the wave breaking index in model. This, in essence, adds to the constraint that wave height may need to be modelled differently from the simple vertical geometrical scale.

In order to investigate the scaling effects, a total of 38 beach model experiments were conducted in the 2-D wave tank and wave flume, and 3-D wave basin. The scaling effects were studied by varying the wave height and wave period in individual experiments and the model results were compared to the prototype data from German GWK experiment. Three different horizontal length scales,  $N_\lambda = 20, 30,$  and  $40,$  were selected in the model experiments. Two different sands, with median sizes equal to  $0.09\text{mm}$  and  $0.2\text{mm},$  respectively, were used in model for undistorted and distorted model studies. The prototype sand has a median size of  $0.33\text{mm}.$

Besides the investigation of scaling effects, four different scaling laws including the new one derived in the paper were also tested against the laboratory model results. The evaluation of scaling laws was carried out by examining the changes of modelled

Table 3. Comparison of Model Performance ( $N_\lambda = 20$ ,  $N_W = 2$ ).

Author	$N_\delta$	$N_H$	$N_T$	$N_t$
Vellinga (1982)	14.5	14.5	3.8	3.8
Hughes (1983)	14.5	14.5	5.3	5.3
Wang, <i>et al.</i> (1990)	14.5	14.5	5.3	3.8
Wang, <i>et al.</i> (present)	14.5	10.5	4.5	4.5

Experimental data	$N_\delta$	$N_H$	$N_T$	$N_t$
Bar location	14.5	11.7	4.5	4.5
Bar volume change	14.5	12.7	4.5	4.5
Dune volume change	14.5	12.4	4.5	4.5

beach profile properties in the separate dune and bar profile regions. Several test parameters, including the errors of dune erosion volume and RMS error of dune profile, bar volume and bar location, and RMS error of bar profile, were utilized to evaluate the model behavior as compared to the prototype. The major conclusions are listed below:

1. Based on the comparison of errors of the model dune erosion volume near the final model stage, greater dune erosion in model is found to occur under larger waves. And, this result is generally not influenced by varying the wave period in model. As comparing the errors of bar volume and bar location, more bar volume and further seaward bar location are found to occur also in the case of larger waves. The smaller absolute value of these errors corresponds to the better cases with the model behavior similar to the prototype. In terms of the RMS errors of modelled beach profile computed in the dune and bar regions, it is seen that the effect of wave height to the RMS errors is less significant. However, the factor of wave period becomes important since a proper model wave period can result a consistent small RMS error which indicates a better fit of modelled profile to the prototype. Accordingly, by applying these results to evaluate the four modeling laws presented in Table 1, the one derived in the present paper appears to have the overall better performance than the other modeling laws.

2. The evaluation of three different morphological time scaling laws, as of Hughes (1983), Wang, *et al.* (1990) and the present one shown in Table 1, by comparing dune and bar profile properties in model and prototype at different times shows that there is less clear which one performs better than the others. In general, all three time scaling laws yield good result to the temporal changes of dune erosion volume and dune profile in model. In terms of bar volume and bar location, the beach model responses seem to develop the bar a little too fast and too large in the initial model stage. However, the results show good agreement to the prototype near the final model stage. It appears that none of the model and prototype profile have reached the equilibrium state at the end of the run.

3. Overall evaluation of the four modeling laws compared in this study shows that the one derived in the present paper performs better than the other three modeling laws. The new modeling law assumes that the fluid motion time scale is the same as the morphological time scale and is equal to square root of the horizontal length scale. The consequences are interesting that both the number of incoming waves per unit time and the deepwater wave length are preserved in model. On the other hand, the new modeling law requires to model the wave height differently from the simple vertical geometrical scale. According to the new modeling law, wave height higher than that from the vertical length scale ratio is needed as to preserve the wave breaking index in the distorted model.

4. The experiments of undistorted model using fine sand of median size of 0.09mm was not successful. The expected bar and dune profile could not be obtained in model. The reason is probably due to the fact that wave energy dissipates more quickly over the flatter bed of fine sand in the undistorted model.

5. The experiments supported the validity of applying distorted model for the prediction of beach and dune erosion in the nearshore zone. However, the present study focuses only on the modeling effects of storm profile. It is necessary to also test the modeling law upon the beach accretion.

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