CHAPTER 199

A MODEL FOR BREACH GROWTH IN SAND-DIKES

by

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ABSTRACT

A mathematical model for breach growth in sand-dikes and dunes is presented. The model is based on the five-step breach erosion process as observed in several laboratory and field experiments. A simplified Galappatti (1983) pick up mechanism for sand from the bed is combined with Bagnold's (1963) modified (Visser, 1988) energetics-based sand transport conception to describe the breach erosion. The comparison of the model predictions with the data of the Zwin 89 field experiment shows good agreement.

1. INTRODUCTION

The Technical Advisory Committee on Water Defences (TAW) in The Netherlands is completing a probabilistic design method for dikes and dunes (hereafter both named dikes). This method will hold a procedure for the design and control of dikes based on a risk-norm (risk of inundation) instead of on a chance-norm (chance of exceeding a certain water level) as in the present deterministic methods, see Kraak et al. (1994). A risk-norm means that the inundation chance is combined with the consequences of flooding (deaths, loss of property and revenues, repair costs, etc.). To determine the consequences of an inundation, it is necessary to predict both the rate and speed of polder flooding, which are especially governed by the flow rate through the breach in the dike. This discharge rate largely depends on the process of breach growth.
The final aim of the investigation is a mathematical model, that describes the breach

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growth and the discharge rate through the breach in case of a dike-burst, as function of the parameters involved. These parameters are:

- cross-section of the dike (height, width, angles of the slopes);
- structure of the dike (dike material, revetments, foundation);
- hydraulic conditions (water level against the dike, wave load).

A first version of the model (Visser, 1988) was especially developed for the huge (about 75 m high) sand-dike of a proposed pumped-storage plant in The Netherlands. Visser et al. (1990) extended the model and confronted it with the data of the Zwin 89 field experiment, yielding reasonable agreement for the first stages of the erosion process. This model version was not yet applicable to the last two phases of the breach erosion process (see section 3). If applied, it would fairly overestimate the breach growth in this final phase.

This paper describes a new version of the model. Its improvements with respect to the previous version are:

- inclusion of a description of the breach erosion in phase IV);
- an improved description of the erosion mechanism in phases I, II and III; these improvements refer to both the physics and the mathematical treatment.

Phase V of the breach erosion process is not yet included in the model. This stage is important since it yields the ultimate breach dimensions. Further the model is still restricted to sand-dikes; effects of clay-layers and revetments have to be included in the near future.

2. ENTRAINMENT AND TRANSPORT OF SEDIMENT

Fig. 1 shows a typical cross-section of a sand-dike along the breach axis in the initial phase of the breach erosion process. A coordinate system (x,z) is adopted with coordinate x along the inner slope ($x = 0$ at the top of the dike) and coordinate z normal to the slope. H_w is the water level at sea, Z_T is the height of the top of the dike in the breach (both H_w and Z_T are measured above the base of the dike), and

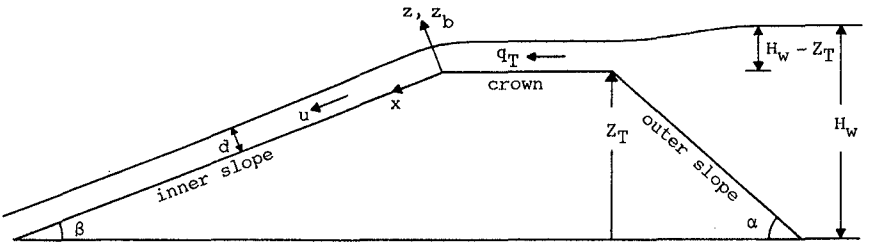

Fig. 1. Typical cross-section of a sand-dike along the breach axis.

the angles of the outer and inner slope are α and β, respectively.
The entrainment of sand from the horizontal crown of the dike is very small compared with the pick up from the inner slope, see Steetzel and Visser (1992) and Visser (1994). The pick up of sand starts at the upstream end of the inner slope ($x = 0$). For large values of $u_\star/w_s = C_f^{1/2}u/w_s$ (is of order 10 in the present situation, so suspended load transport will dominate bed load transport), the entrainment and subsequent transport of suspended sediment along the inner slope can be approximated according to Galappatti (1983, see also Galappatti and Vreugdenhil, 1985) by:

$$s(x) \approx \frac{x}{l_a} s_s \qquad \text{for} \quad 0 \leq x \leq l_a \tag{1}$$

in which $s(x)$ is the sediment transport (volumes of particles) per unit width along the slope and l_a is the adaptation length of the suspended load transport:

$$l_a = \frac{ud}{w_s \cos\beta} = \frac{q_T}{w_s \cos\beta} \tag{2}$$

and s_s is the capacity of the suspended load transport:

$$s_s = \frac{0.01}{(w_s/u)(\cos\beta)^2} \frac{C_f u^3}{g\Delta} \qquad [\mathrm{m^2/s}] \tag{3}$$

where u is the depth-averaged flow velocity, d is the water depth (see Fig. 1), w_s is the fall velocity of sand in water, q_T is the discharge flow rate per unit width over the top of the dike, C_f is the friction coefficient for the bed ($C_f = g/C^2$, where C is the Chézy coefficient), $\Delta = (\rho_s - \rho)/\rho$, ρ_s is the mass density of sand, ρ is the water mass density and g is the acceleration of gravity, see Visser (1988). Equation (3) rests on a modified (Visser, 1988) Bagnold (1963) energetics-based sand transport conception for suspended sediment load. The efficiency factor 0.01 is according to Bagnold (1966).
Equation (3) emerges as the best formula out of 15 sand transport formulae in a test with the flume data of Steetzel and Visser (1992) and the data of the Zwin 89 experiment, with Van Rijn's (1984a, 1984b) formulation finishing second best. Most of the other formulae overestimate the measured sediment transport rates significantly, also those formulae which were developed for sand-water mixture flows at high shear stress (for instance Wilson, 1987) and for sediment transport on steep slopes (for instance Rickenmann, 1991). For the moment this conclusion (and the choice for (3)) holds for the first three phases of the breaching process when the flow is supercritical.

3. BREACH EROSION PROCESSES

3.1 Discharge rate

The water discharge rate q_T per unit breach width is described by a weir formula:

$$q_T = m(2/3)^{3/2} g^{1/2} (H_w - Z_T)^{3/2} \tag{4}$$

where m is the discharge coefficient (≈ 1.0). Equation (4) holds as long as the flow in the breach is not affected by the downstream water level, i.e. for phases I through IV (see paragraph 3.2).

3.2 Erosion of inner slope

The equation for the erosion of the inner slope is:

$$(1-p)\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = 0 \tag{5}$$

where p is the bed porosity, z_b is the position of the inner slope in z-direction (z is the coordinate normal to the inner slope, see Fig. 1). Substitution of (1), (2) and (3) into (5) yields:

$$\left|\frac{\partial z_b}{\partial t}\right| = \frac{0.01\, C_f}{(1-p) g \Delta} \left|\frac{\partial}{\partial x}\left(\frac{x u^4}{q_T \cos\beta}\right)\right| \qquad \text{for} \quad 0 \le x \le l_a \tag{6}$$

in which it has been assumed that the friction coefficient C_f is constant.

If q_T is constant (that is if $H_w - Z_T$ = constant, see equation (4)) and assuming $\cos\beta \approx 1$, it follows from (6) that:

$$\frac{\partial}{\partial x}\left|\frac{\partial z_b}{\partial t}\right| > 0 \qquad \text{for} \quad 0 < x < l_n \tag{7}$$

since the flow velocity u increases in positive x-direction for $0 < x < l_n$. So the erosion of the inner slope increases along the slope and the inner slope becomes steeper in x-direction and in time (see Fig. 2).

For $x = l_n$ the flow velocity u approaches the normal value for uniform flows:

$$u_n = \frac{(g d_n \sin\beta)^{1/2}}{C_f^{1/2}} = \frac{(g q_T \sin\beta)^{1/3}}{C_f^{1/3}} \tag{8}$$

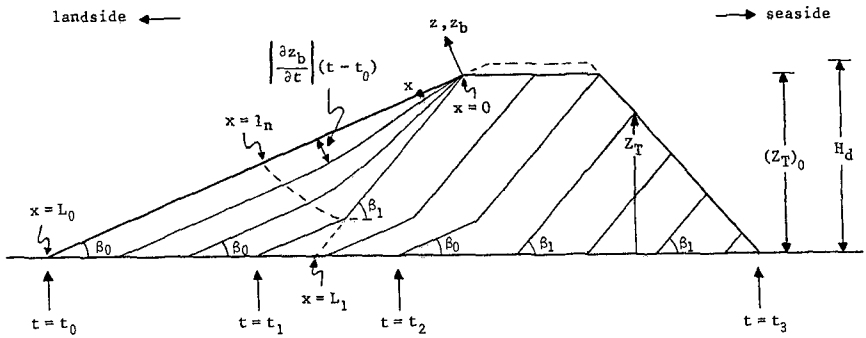

Fig. 2. Erosion of inner slope.

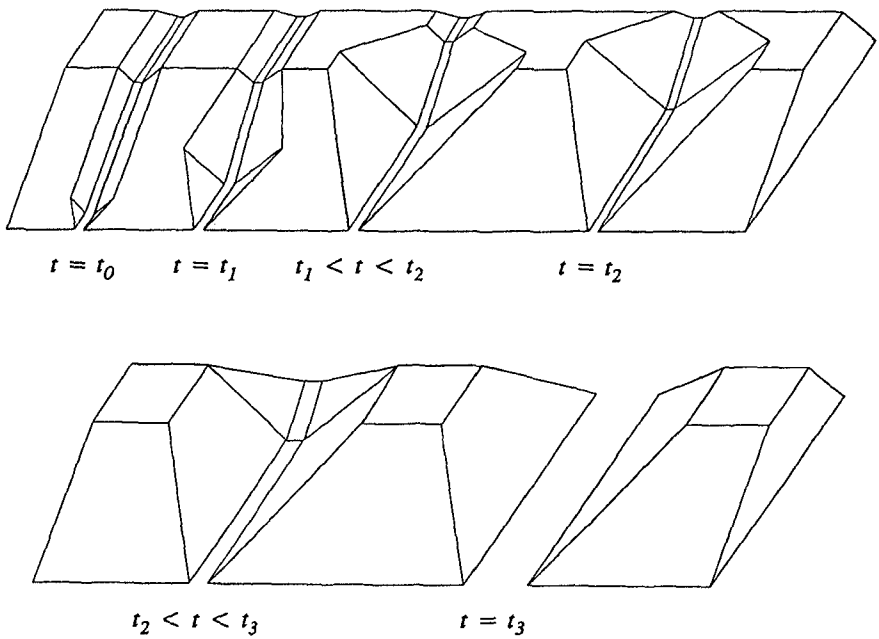

Fig. 3. Process of breach erosion.

Substitution of (1), (2), (3) and (8) into (5) gives:

$$\frac{\partial z_b}{\partial t} = -\frac{0.01}{(1-p)\Delta}\frac{\partial}{\partial x}\left(x u_n \tan\beta\right) \qquad \text{for} \quad l_n \le x \le l_a \tag{9}$$

If q_T and β are constant then u_n is constant and (9) becomes:

$$\frac{dz_b}{dt} = -\frac{0.01}{(1-p)\Delta} u_n \tan\beta \qquad \text{for} \quad l_n \le x \le l_a \tag{10}$$

This means that:

$$\frac{\partial}{\partial x}\left|\frac{dz_b}{dt}\right| = 0 \qquad \text{for} \quad l_n \le x \le l_a \tag{11}$$

i.e. the erosion of the inner slope is constant for these values of x, see Fig. 2.

The inner slope becomes steeper for $0 \le x < l_n$. However, the slope angle will not exceed a limit β_1, say $\beta_1 \approx \phi$ (ϕ is angle of repose). If this limit has been achieved on the entire stretch $0 \le x < l_n$ (on $t = t_1$), then the erosion rate becomes constant for $0 \le x < l_n$, as indicated by the lines for $t \ge t_1$ in Fig. 2.
So, if the breaching process starts at $t = t_0$ with the flow of water through a small initial channel in the crown and the inner (downstream) slope of the dike, then the following (subsequent) phases can be distinguished in this process (see Figures 2 and 3):

I. Steepening of the inclination angle (β) of (the channel in) the inner slope from an initial value β_0 up to a critical value β_1 at t_1 (see Fig. 1).
II. Continuation of the erosion of the inner slope resulting in a decrease of the length of the dike-top in the breach for $t_1 < t < t_2$; the inner slope angle remains (in this line of thoughts) at its critical value β_1.
III. Lowering of the top of the dike in the breach and a subsequent increase of the breach width for $t_2 \le t \le t_3$.
IV. After the complete wash-out of the dike in the breach, continuation of the breach growth in both vertical (scour hole) and horizontal direction for $t_3 < t \le t_4$. At t_4 the flow through the breach is critical, i.e. turns from supercritical ($Fr > 1$ for $t < t_4$) into subcritical ($Fr < 1$ for $t > t_4$).
V. Continuation of the increase of the breach width for $t_4 < t < t_5$. At t_5 the flow velocities in the breach become so small (incipient motion) that the breach erosion stops.

In phase I the width of the breach remains at its initial small value. At $t = t_1$ the breach width starts to increase at the downstream side of the crown (so in phase II the breach eats its way into the dike, see Fig. 3). On $t = t_2$ the width of the breach at the upstream side of the crown also starts to grow. As a first estimation this occurs directly proportional to the breach depth, see Visser (1988). The discharge

rate through the breach starts to increase at $t = t_2$. In the period $t_3 < t \leq t_4$ the scour hole gets its maximal depth. Stages IV and V are the most important stages, since in the period $t_3 < t < t_5$ most of the water is discharged through the breach and the ultimate dimensions of the breach are determined.

4. MODEL DESCRIPTION

4.1 Steepening of the inner slope (phase I)

The erosion of the inner slope is described by (5) with (1), (2), (3) and an equation for the flow velocity. The flow velocity u follows from (4) and an equation for the non-uniform flow depth along the stretch $0 < x < l_n$. Consequently a numerical method is necessary to describe the development of the entire inner slope for $t_0 < t < t_1$.

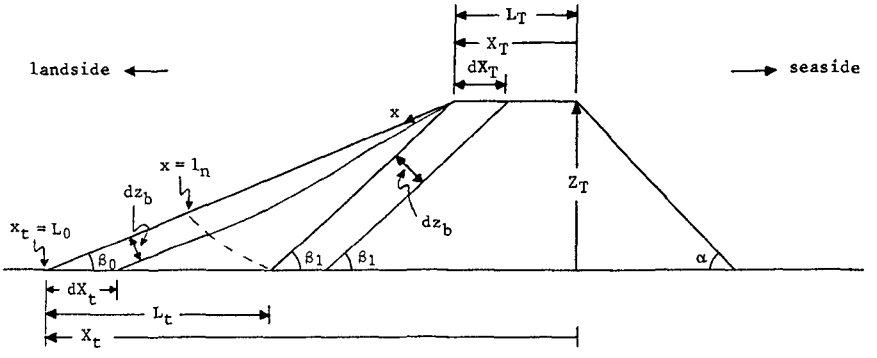

Fig. 4. Horizontal displacements dX_t and dX_T due to erosion dz_b of inner slope.

An analytical solution for stage I is possible along the stretch $l_n \leq x \leq l_a$, where equation (10) describes the erosion rate dz_b/dt. The horizontal displacement dX_t of the toe of the inner slope can simply be expressed in dz_b (see Fig. 4):

$$dX_t = \frac{1}{\sin\beta_0} dz_b \tag{12}$$

Substitution of (4), (8), (10), $\beta = \beta_0$ and $Z_T = (Z_T)_0$ into (12) yields for $t_0 \leq t \leq t_1$:

$$\frac{dX_t}{dt} = -k_0\sqrt{H_w - (Z_T)_0} \qquad \text{for} \quad l_n \leq x_t \leq l_a \tag{13}$$

where x_t is the x-coordinate of the toe of the inner slope (see Fig. 4) and:

$$k_0 = \frac{0.0082}{(1-p)\Delta} (m/C_f)^{1/3} g^{1/2} \frac{(\sin\beta_0)^{1/3}}{\cos\beta_0} \tag{14}$$

If $l_n = L_1$ at $t = t_1$ then:

$$\int_{t_0}^{t_1} dX_t = -L_t \tag{15}$$

Substitution of (13) into (15) finally yields:

$$t_1 - t_0 = \frac{L_t}{k_0 \sqrt{H_w - (Z_c)_0}} \tag{16}$$

If $L_1 > l_n(t_1)$ then ξL_t as defined in Fig. 5 should replace L_t in equation (16).

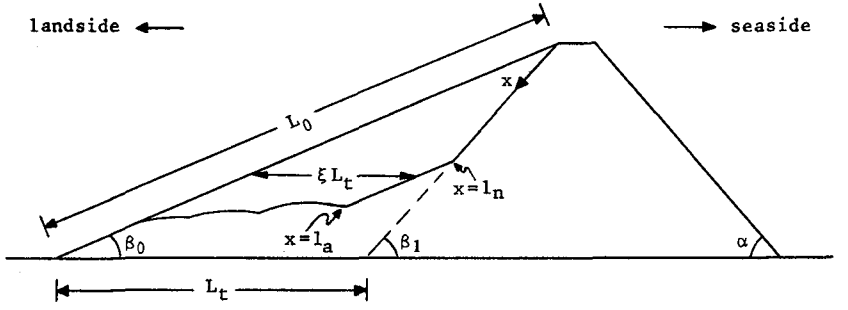

Fig. 5. Erosion of inner slope for relatively high dikes.

4.2 Decrease of crown length (phase II)

At $t = t_1$ the steepness of the inner slope is at its critical value (angle β_1). From now on the rate of erosion is constant along the entire stretch $0 \le x \le l_a$, see Fig. 2. So the erosion of the inner slope for $t_1 < t < t_2$ is entirely determined by the erosion at the toe of the slope ($x = L_1$) as long as $L_1 < l_a$:

$$-L_1(1-p)\frac{dz_b}{dt} = s(L_1) = \frac{L_1}{l_a}s_s(L_1) \tag{17}$$

Generally $L_1 > l_n$ (see Visser, 1994), so $u(L_1) = u_n$. Substituting (2), (3) and (8) with $\beta = \beta_1$ into (17) yields, in agreement with (10):

$$\frac{dz_b}{dt} = -\frac{0.01}{(1-p)\Delta}u_n \tan\beta_1 \qquad \text{for} \quad 0 \le x \le L_1 \tag{18}$$

Fig. 4 shows that in the breach the length of the dike-top (initial value L_T) decreases for $t_1 < t < t_2$ due to the erosion of the inner slope. The relation between the decrease of the length of the dike-top (dX_T) and the erosion of the inner slope (dz_b) is:

$$dX_T = \frac{1}{\sin\beta_1}dz_b \tag{19}$$

Substitution of (4) with $Z_T = (Z_T)_0$, (8) with $\beta = \beta_1$ and (18) into (19) gives:

$$\frac{dX_T}{dt} = -k_1\sqrt{H_w - (Z_T)_0} \qquad \text{for} \quad t_1 < t < t_2 \tag{20}$$

with:

$$k_1 = \frac{0.0082}{(1-p)\Delta}(m/C_f)^{1/3}g^{1/2}\frac{(\sin\beta_1)^{1/3}}{\cos\beta_1} \tag{21}$$

Integration of (20) gives:

$$t_2 - t_1 = \frac{L_T}{k_1\sqrt{H_w - (Z_T)_0}} \tag{22}$$

Visser (1988) argues that due to the increase of the breach width an extra amount of sand falls into the flow, slowing down the breach erosion in vertical direction (with a factor f compared with a 2-D situation; to a lesser extent this applies also to phase I). The factor f will vary from phase to phase. Assuming f to be constant in each phase, equation (22) becomes for the 3-D situation:

$$t_2 - t_1 = \frac{L_T}{f_1 k_1\sqrt{H_w - (Z_T)_0}} \tag{23}$$

4.3 Decrease of crown level (phase III)

At $t = t_2$ the top of the dike in the breach starts to drop. The relation between the fall dZ_T and the rate of erosion dz_b of the inner slope follows from a simple geometrical consideration (see Fig. 6):

$$dZ_T = \frac{\sin\alpha}{\sin(\alpha + \beta_1)} dz_b \tag{24}$$

Substitution of (4), (8) with $\beta = \beta_1$ and (18) into (24) yields:

$$\frac{dZ_T}{dt} = -k_2\sqrt{H_w - Z_T} \quad \text{for} \quad t_2 \le t \le t_3 \tag{25}$$

where:

$$k_2 = \frac{\sin\alpha \sin\beta_1}{\sin(\alpha + \beta_1)} k_1 \tag{26}$$

At $t = t_2$ the width of the breach at the upstream end of the dike-top starts to increase. Visser (1988) argues that the breach width (so also the depth-averaged breach width B) increases linearly with the growth of the breach depth $H_d - Z_T$:

$$\frac{dB}{dt} = r\frac{d(H_d - Z_T)}{dt} \quad \text{for} \quad t_2 \le t \le t_3 \tag{27}$$

where r is a coefficient with a theoretical value (for sand-dikes) of about 2.2 for the depth-averaged breach width and about 3.8 for the breach width at the top of the dike.
Due to the increase of the breach width an extra amount of sand falls into the flow, slowing down the breach erosion in vertical direction (with a factor f_2 in this stage, see paragraph 4.2). Hence equation (25) becomes:

$$\frac{dZ_T}{dt} = -f_2 k_2\sqrt{H_w - Z_T} \quad \text{for} \quad t_2 \le t \le t_3 \tag{28}$$

in which the factor f_2 is (see Visser, 1988):

$$f_2 = \frac{B + 2q_T/u}{2B} \tag{29}$$

Integration of (28) gives with $Z_T = 0$ at $t = t_3$:

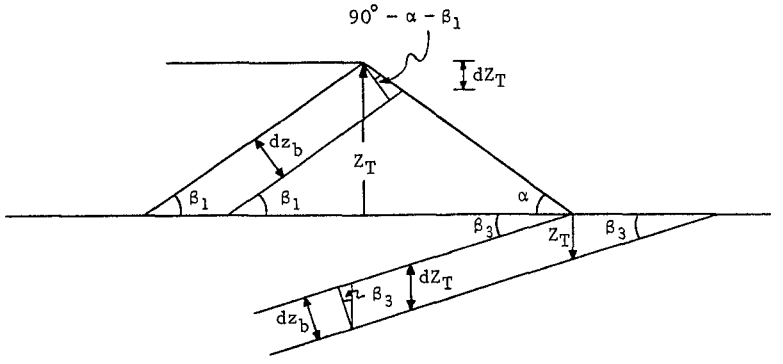

Fig. 6. Relation between dZ_T and dz_b in phases III and IV.

$$Z_T(t) = H_w - \left[\frac{f_2 k_2}{2}(t - t_3) + \sqrt{H_w}\right]^2 \quad \text{for} \quad t_2 \le t \le t_3 \tag{30}$$

Substitution of $Z = (Z_c)_0$ at $t = t_2$ into (30) gives:

$$t_3 - t_2 = \frac{2}{f_2 k_2}\left[\sqrt{H_w} - \sqrt{H_w - (Z_T)_0}\right] \tag{31}$$

If the depth of the initial breach ($H_d - (Z_T)_0$, see Fig. 2) is small compared with the dike-height H_d (so also small compared with H_w), then (31) reduces to:

$$t_3 - t_2 = \frac{2}{f_2 k_2}\sqrt{H_w} \tag{32}$$

4.4 Continuation of breach growth (phase IV)

Locally the dike has been completely washed out ($Z_T = 0$) at $t = t_3$ and the breach continues to grow in both vertical (scour hole: $Z_T < 0$) and horizontal direction for $t > t_3$. The equation for the discharge rate q_T per unit of breach width is for $t_3 \le t \le t_4$:

$$q_T = m(2/3)^{3/2} g^{1/2} (H_w)^{3/2} \tag{33}$$

The scour hole has an upstream slope (β_3, bed elevation decreasing in flow direction: β_3 is not equal to β_1) and a downstream slope (bed elevation increasing in flow direction). It is assumed that the breach growth in phase IV is determined

by the erosion of the upstream slope of the scour hole. Then equation (10) describes also the erosion in vertical direction in phase IV:

$$\frac{dz_b}{dt} = -\frac{0.01}{(1-p)\Delta} u_n \tan\beta_3 \qquad \text{for} \quad l_n \le x \le l_a \tag{34}$$

The relation between the increase of the depth of the scour hole dZ_T and the rate of erosion dz_b of the upstream slope of the scour hole follows from a simple geometrical consideration (see Fig. 6):

$$dZ_T = \frac{1}{\cos\beta_3} dz_b \tag{35}$$

Substitution of (8) with $\beta = \beta_3$, (33) and (34) into (35) and including a factor f yields:

$$\frac{dZ_T}{dt} = -f_3 k_3 \sqrt{H_w} \qquad \text{for} \quad t_3 \le t \le t_4 \tag{36}$$

with:

$$k_3 = \frac{0.0082}{(1-p)\Delta} (m/C_f)^{1/3} g^{1/2} \frac{(\sin\beta_3)^{4/3}}{(\cos\beta_3)^2} \tag{37}$$

Integration of (36) gives with $Z_T = 0$ at $t = t_3$:

$$Z_T(t) = -f_3 k_3 \sqrt{H_w} (t - t_3) \qquad \text{for} \quad t_3 \le t \le t_4 \tag{38}$$

It is assumed that (27) holds also in phase IV; then substitution of (38) into (27) gives the increase of the breach width for $t_3 \le t \le t_4$. For the initial stage of phase IV this assumption seems reasonable. It is, however, rather obvious that the validity of (38), and consequently also (27), has its limits, otherwise large breach depths are necessary to explain the existence of relatively wide breaches.
One of the aims of a large scale experiment performed October 1994 (Zwin 94 field experiment) has been to solve this uncertainty about the growth of the scour hole and its relation with the increase of the breach width in phase IV. The experimental procedure and results of this field experiment will be presented in a forthcoming publication.

5. COMPARISON WITH FIELD DATA

The present breach erosion model is tested to the data of the Zwin 89 experiment. This large scale experiment was performed in the Zwin Channel (an tidal inlet in the south-western part of The Netherlands) in December 1989, see Visser et al. (1990). The dimensions of the sand-dike in the Zwin 89 experiment have been: $H_d = 2.2$ m, $L_T \approx 7.5$ m, $\beta_0 = 18.4°$ and $\alpha = 39°$. The 50 m long sand-dike was constructed, exclusively for the experiment, with local sand $D_{50} \approx 0.22$ mm. A small pilot channel (initial breach), about 9 m long, about 1 m wide and with a depth $H_d - (Z_c)_0 \approx 0.35$ m was made in the dike-top to ensure breaching near the middle of the Zwin Channel.

The breaching process was both video-taped and photographed. Levelling-staffs in the crown of the dike provided the proper length-scale for the readings from the video-tape and the photographs. The main result of these readings, i.e. the development of the 'depth-averaged' breach width $B(t)$, at the downstream end of the crown of the dike, is shown in Fig. 7. These data differ slightly from those in Visser et al. (1990), where breach width $B(t)$ was given as averaged (both along the breach length and in depth) value of observed breach width.

The comparison of the model prediction with the data of the Zwin 89 experiment

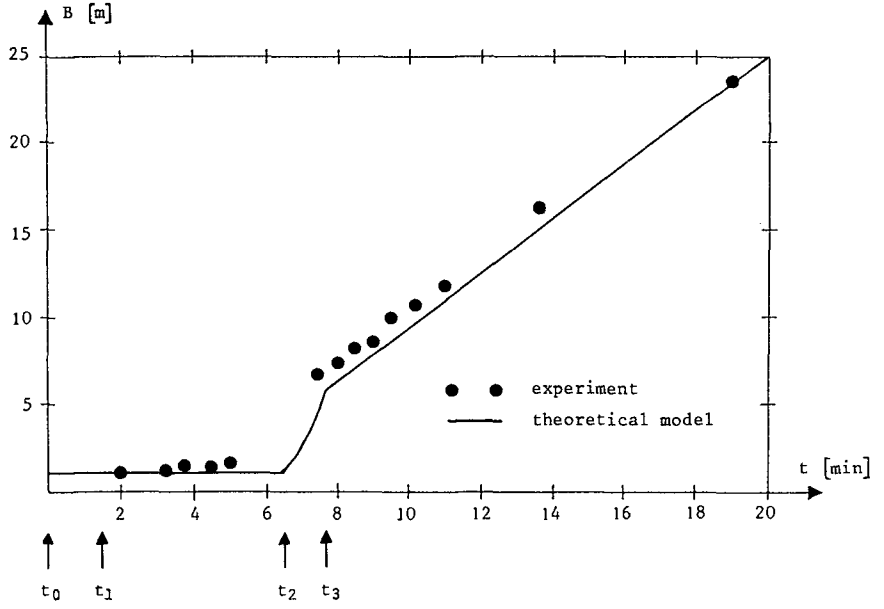

Fig. 7. Comparison of measured (Zwin 89 experiment) and computed breach width $B(t)$ at the upstream end of the dike-top.

has been done with the following values for the different parameters: $p = 0.4$, $\Delta = 1.65$, $m = 1.0$, $f_1 = f_2 = f_3 = 0.6$ (estimated with equation (29)), $C_f = 0.015$, $\beta_1 = 32°$ (see Visser et al., 1990) and $r = 2.2$ (for depth-averaged breach width). For β_3 the value found by Delft Hydraulics (1972) for scour holes has been adopted: $\beta_3 \approx 12°$ ($\tan\beta_3 \approx 0.2$). This is a very crude assumption since there exists no universal value for this angle.
Setting $t_0 = 0$, substitution of these values into (16) with (14) yields $t_1 = 1.5$ min (in this phase: $H_w - (Z_T)_0 \approx 0.13$ m), substitution into (22) with (21) gives $t_2 - t_1 = 5.0$ min (in phase II: $H_w - (Z_T)_0 \approx 0.17$ m), so $t_2 = 6.5$ min, and into (31) with (26) gives $t_3 - t_2 = 1.2$ min (in phase III: $H_w \approx 2.1$ m), so $t_3 = 7.7$ min. The increase of the breach width $B(t)$ is given by (27) with (28) in phase III and by (27) with (36) in phase IV. The result of the model prediction for $B(t)$ for the Zwin 89 experiment are shown in Fig. 7. The kink for $t = t_3$ is due to keeping β at β_1 for $t_2 \leq t \leq t_3$ while in reality β will decrease from β_1 to β_3 in this phase.
The experimental data (flow velocities and water levels measured upstream and downstream form the breach) indicate that $t_4 \approx 20$ min, see Visser et al. (1990). Hence Fig. 7 shows the development of the breach width $B(t)$ in phases I through IV.

6. DISCUSSION

The agreement between the present breach erosion model and the data of the Zwin 89 experiment is good. It must be emphasized, however, that there still some uncertainties that has to be solved. These uncertainties relate to:

- the development of the scour hole (depth, slope angle β_3) in phase IV and its effect on the increase of the breach width;
- the magnitude of the factors f_1, f_2 and f_2.

As yet the present version of the model does not describe the breach width in phase V, in which at the end the breach erosion stops and the breach gets its final dimensions.
It is expected that the data of the recent Zwin 94 field experiment (see paragraph 4.4) will significantly contribute to the understanding of the breach erosion process in phases IV and V and to the completion of the breach erosion model in 1995.

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