CHAPTER 181

Numerical Modelling of Three-Dimensional Wave-Driven Currents in the Surf-Zone

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Abstract

The numerical modelling of time-averaged three-dimensional currents due to breaking waves is presented. The basic equations and the closures are described. The driving terms in the momentum equations are the radiation stresses derived from "organised" velocity of waves and the roller contribution. The model is validated on measurements collected in a flume. The application to the case of a rectilinear beach with obliquc incidence wave displays a helicoidal circulation. In the situation of a detached brcakwater with a normal wave, depth-integrated currents show two large symmetrical cells in the lee of the structure whereas streamlines near the bottom converge spirally toward the centres of the eddies.

Introduction

Longshore currents induced by breaking waves are often assumed to be responsible for changes in the shoreline plan-shape on extend of some kilometres (Horikawa 1988 , Southgate 1987 , Péchon, 1982). However the action of cross-shore currents have to be taken into account for the prediction of detailed bathymetry variations, for instance when appraising the impact nearby coastal structures (Hansen and Svendsen 1984 , Okayasu 1989).

A compound system is being developed to simulate wave, current and sediment transport patterns for the design of coastal projects. The module for time-averaged wave-driven 3D-currents is presented ; the physical concept and applications are described. In this article we focus on the wave-driven currents due to breaking phenomena exclusively. The wave is assumed to be regular and the wave pattern is supposed to be known here, resulting from the wave module of the system.

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The Three-Dimensional current model

The equations

For sake of clarity the Navier-Stokes equations are considered in the case of a two-dimensional motion in a flume :

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial w^2}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}$$
$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$$

where u, w are the horizontal and vertical components of the instantaneous velocity, p is the pressure. The viscous molecular terms are neglected in the highly turbulent environment.

As it is proposed in Svendsen and Lorentz (1989), the velocity is separated into three contributions : a mean current *(U, W)*, a purely periodic current (u_w, w_w) corresponding to wave motion, and turbulent fluctuations *(u', w')*. The time-averaged equations reads, the overbar indicating averaged quantities :

$$\frac{\partial U}{\partial t}+\frac{\partial U^2}{\partial x}+\frac{\partial UW}{\partial z}+\frac{\partial \overline{u_w^2}}{\partial x}+\frac{\partial \overline{u_w w_w}}{\partial z}=-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x}-\frac{\partial \overline{u'^2}}{\partial x}-\frac{\partial \overline{u'w'}}{\partial z}$$
$$\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}=0$$

and the vertical momentum equation leads to (Stive and Wind, 1982) :

$$\overline{p}=\rho g\,(\,\overline{\xi}-z\,)-\rho\overline{w'^2}-\rho\overline{w_w{}^2}$$

where ξ is the free surface level. By substitution in the momentum equation :

$$\frac{\partial U}{\partial t}+\frac{\partial U^2}{\partial x}+\frac{\partial UW}{\partial z}+\frac{\partial(\overline{u_w^2}-\overline{w_w^2})}{\partial x}+\frac{\partial \overline{u_w w_w}}{\partial z}=-\frac{1}{\rho}\frac{\partial \overline{\xi}}{\partial x}-\frac{\partial(\overline{u'^2}-\overline{w'^2})}{\partial x}-\frac{\partial \overline{u'w'}}{\partial z}$$

The derivative $\partial(\overline{u_w^2}-\overline{w_w^2})/\partial x$ is the wave radiation stress. The term $\partial\overline{u_w w_w}/\partial z$ created by waves is also considered whereas the term $\partial(\overline{u'^2}-\overline{w'^2})/\partial x$ is neglected except in the roller. Dropping the time derivative term, the 2 DV equations reduce to :

$$\frac{\partial U^2}{\partial x}+\frac{\partial UW}{\partial z}+\frac{\partial(\overline{u_w^2}-\overline{w_w^2})}{\partial x}+\frac{\partial \overline{u_w w_w}}{\partial z}=-g\frac{\partial \overline{\xi}}{\partial x}-\frac{\partial \overline{u'w'}}{\partial z}+t$$
$$\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}=0$$

where t is the contribution of the roller of the breaking waves, $t=-\partial\overline{u'^2}/\partial x$.

The closures

The terms which depends on the periodic current of breaking waves are expressed in function of the input wave characteristics and the energy dissipation (Longuet-Higgins 1970, Dingemans et al 1986, Deigaard and Fredsoe 1989) :

$$\frac{\partial\,(\overline{u_w^2}-\overline{w_w^2})}{\partial x} = \frac{1}{\rho h} D$$

$$\frac{\partial \overline{u_w w_w}}{\partial z} = \frac{1}{2\rho h} D$$

with D : energy dissipation
C : wave celerity
h : mean water depth

It is noted that the contribution of non-breaking waves in the expression of the radiation stress is not included here. Apart from the roller, Reynolds stresses are given using the eddy viscosity concept :

$$\overline{u'w'} = -\nu_t \frac{\partial U}{\partial z}$$

A uniform turbulence viscosity distribution is adopted, following Svendsen et al (1987) :

$\nu_t = Mh\,(\frac{D}{\rho})^{1/3}$ The constant M is taken to be 0.03

The contribution of the roller in the term $-\partial\overline{u'^2}/\partial x$ can be expressed by approximating the horizontal velocity profile as suggested by Svendsen (1984) :

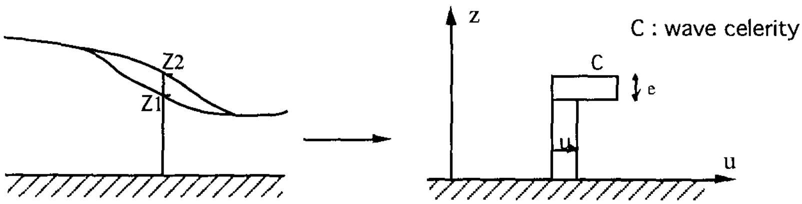

Fig.1 Vertical profile of velocity in the surfzone

He proposed : $\overline{e} = 0.9\,H^2 / L$ with H : wave height
L : wave length

So we obtain $\displaystyle\overline{\int_{Z_1}^{Z_2} u'^2\,dz} = C^2\,\overline{e} = 0.9\,\frac{C^2}{L} H^2$

The time-averaged entrainment force t due to the roller is specified uniform from the wave trough to the mean water level, on a layer thickness of $H/2$. It follows :

$$t = -\frac{2}{H}\frac{\partial}{\partial x}\left(0.9\,\frac{C^2 H^2}{L}\right) = -\frac{1.8}{H\,T}\frac{\partial\,C H^2}{\partial x}$$

Introducing the flux of energy E_f, in shallow water :

$$E_f = \frac{1}{8} \rho g H^2 C_g \approx \frac{1}{8} \rho g H^2 C$$

$$t = \frac{14}{\rho g} \frac{D}{HT} \quad \text{where } D = -\frac{\partial E_f}{\partial x} \text{ energy dissipation}$$

T : wave period

The exchanges of momentum and mass through the surface

In quasi-3D models (Wind and Stive 1987, Sanchez-Arcilla et al. 1992) the equations are generally solved from the bottom to the trough level and a shear stress is specified at the boundaries. Moreover the mass flux above trough level is given as an input.

Since the present model is fully three-dimensional, the time-averaged equations have to be solved from the bottom to the mean water level. The total flux of momentum due to breaking waves which is introduced in the computational domain is specified through the radiation stresses distributed over the water column and the force t on a thickness $H/2$. It follows that it is not relevant to impose any additional shear stress at the surface in this model ; so the shear stress is set to zero.

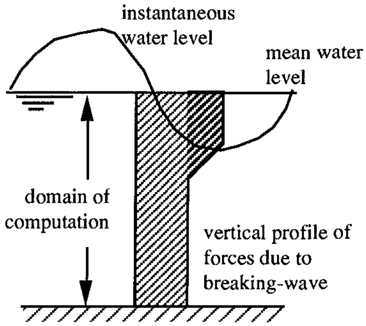

Fig. 2 Vertical distribution of forces

In nature the wave-induced mass-flux above the mean water level is not nil in and out the surf-zone and the horizontal variation of this quantity induce also mass flux through the time-average surface. Walstra et al (1994) take this effect into account in a 2DV model by adding a term in the continuity equation and they allow vertical velocities through the surface boundary. Since this type of circulation occurs for breaking as well as non-breaking waves, we chosen here to solve it separately with a Boussinesq type model which reproduces it automatically. This part is not presented in this paper.

Applications

Cross-shore currents in a flume

The numerical model has been applied with standard parameters to the case of a flume which had been investigated on a physical model (Buhr-Hansen and Svendsen, 1986).

The bottom profile is parabolic (fig. 3). The generated wave is regular with a period of 2.0 s and at the breaking point the wave height is 0.17 m and the still water depth is 0.20 m. The wave field specified in the numerical model is the one measured on the experimental set-up.

The computed undertow added to the mean currents due to wave motion has been compared to measurements collected in the surf-zone. In this case the currents due to wave motion (non-breaking contribution) is estimated with a non-linear wave theory (analytical solution of Serre equations) depending on the wave height. The numerical results were in good agreement with measured velocities.

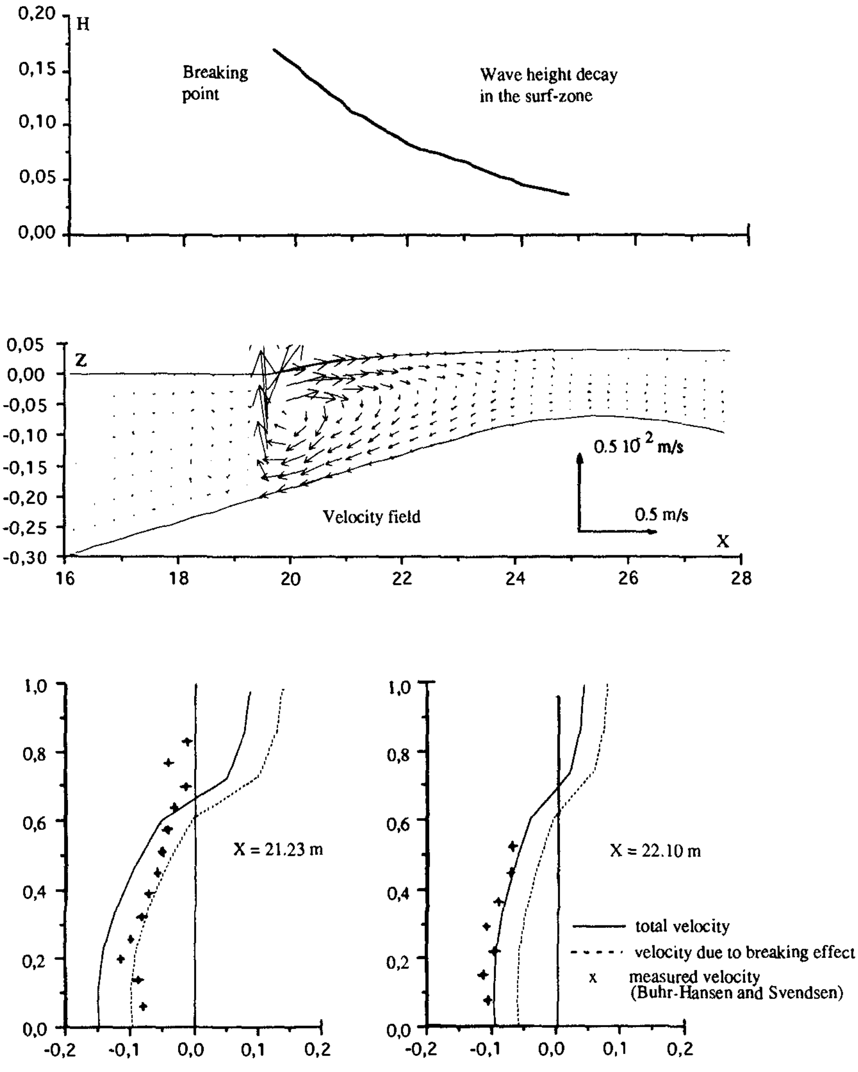

Fig. 3 Wave-driven currents in a flume
Comparison with measurements

Three-dimensional circulation along a rectilinear beach.

The studied domain had a constant bottom slope equal to 2/100 , the off-shore water depth was 9.0 meters. The wave height at the deeper limit was 2.0 m, the period 8.0 s and the incidence 25° (fig. 4). All boundaries were closed in this test, with a slipping condition.

Wave propagation was computed with a simple refraction model and the decay in the surf-zone was deduced from the energy equation where the energy dissipation in the bore is assumed to be similar to an hydraulic jump one.

In this situation an helicoidal wave-driven flow occurs in the domain (fig. 5) and the velocity near the bottom is oriented offshore. This qualitative result is confirmed by field observations (Ingle 1966) but measurements are not available for validation. Moreover it is pointed out that the "non-breaking contribution" of wave motion is not included in this result.

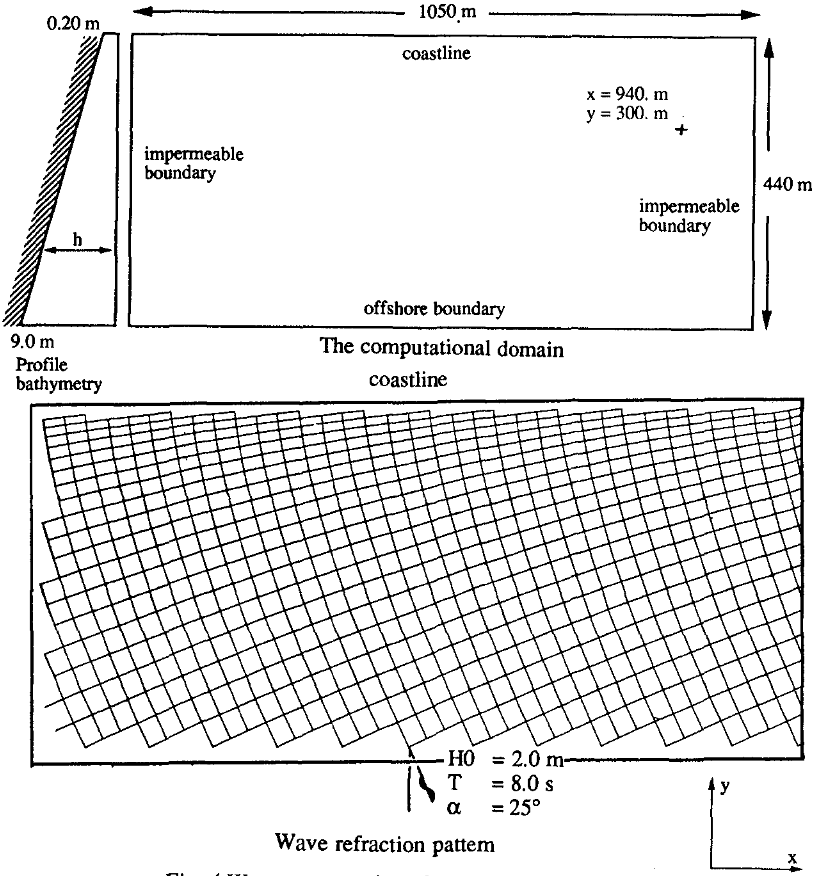

Fig. 4 Wave propagation along a rectilinear coast

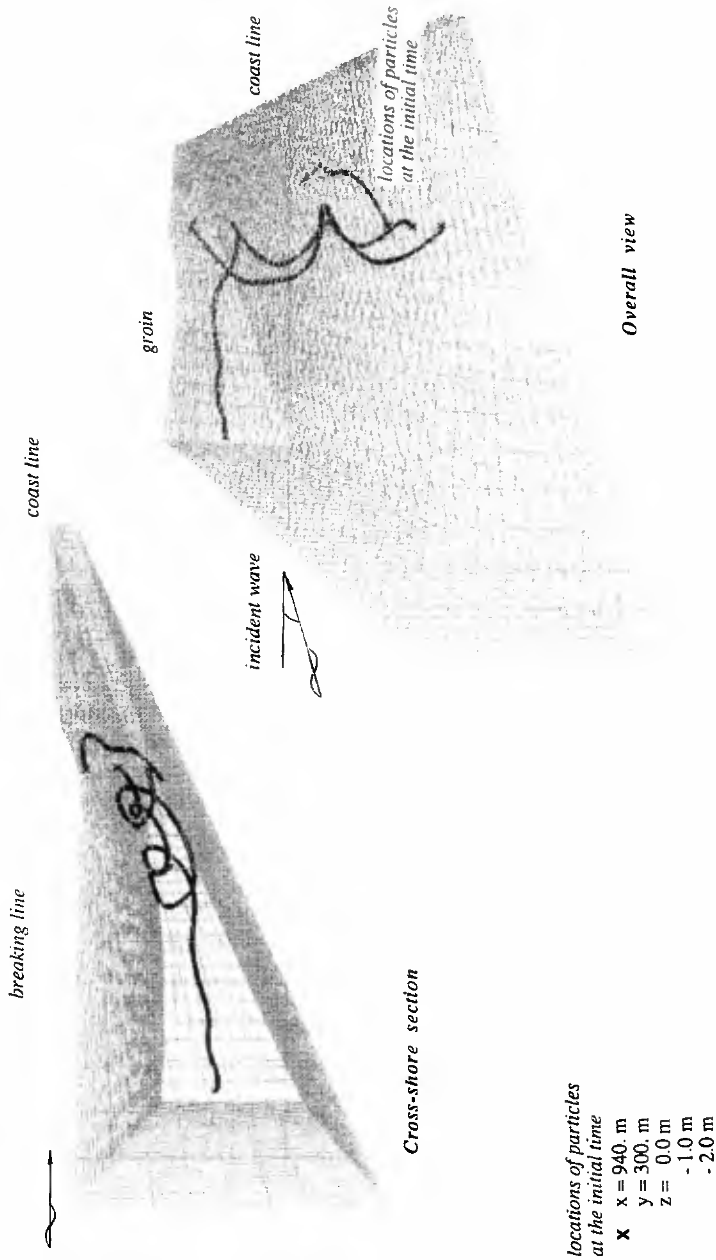

Fig. 5 Currents induced by breaking waves - Particles tracks

Currents near a the detached breakwater

The bottom slope is constant and equal to 3/100. The still water depth is 5.0 meters at the off-shore frontier and it is 0.5 meter at the shoreline. The off-shore boundary is open while the three others are closed and combined with slipping conditions. The detached breakwater is 100 meters long and it is located at 100 meters from the shoreline. The still water depth at the toe of the structure is 3.5 meters.

The wave at the open boundary is 2.5 meters high, its period is 8.0 seconds and its incidence is normal to the coast. For this preliminary simulation the wave field is schematised, the wave-driven terms are nil in the lee of the structure and the wave height decay elsewhere in the surf-zone is given by a formula. Of course this wave field is very crude but the aim of this test is to display vertical heterogeneity of the velocity pattern. A more accurate computation of the wave field is planned in the future.

The depth-averaged flow pattern shows two large and symmetrical eddies behind the breakwater (fig.6). The maximum velocity reaches 1.5 m/s. In order to see three-dimensional effects, particles tracks computed with the bidimensional velocity fields near the bed, near the surface and with depth-averaged one are also visualised on figure 6. The streamlines near the bottom are oriented toward the centre of the eddy while the others describe large cells. This difference between the depth-averaged and the near-bed flow patterns is crucial for sediment transport modelling.

Conclusion

The computation of time-averaged wave-driven currents in the surf-zone for schematical 3D cases gives satisfying qualitative results. The model have already been compared successfully with measurements collected in experimental flumes but additional validation tests are needed in three-dimensional situations.

The wave pattern in practical studies can be complex and its prediction requires a sophisticated wave model, especially if hydrodynamic results are used to determine sediment transport. It can be provided by bidimensional infragravity wave models (based on Boussinesq or Serre equations) which also simulate wave height decay in the surf-zone (Hamm et al. 1993, Karambas et al. 1992, Schäffer et al. 1992). The total velocity in the surf zone is the combination of the three-dimensional currents induced by breaking waves and the instantaneous wave velocity. However some care have to be taken in the summation of the two velocity fields because the current resulting from the wave model also partly includes the breaking effect. So the component which comes from breaking in the wave model must be subtract before adding the two velocity fields.

Acknowledgements

This work was carried out as part of the G8 Coastal Morphodynamics research programme. It was funded partly by the Service Central Technique du Secrétariat d'Etat à la Mer and the Commission of the European Communities, Directorate General for Science, Research and Development, under contract n° MAS2-CT92-0027.

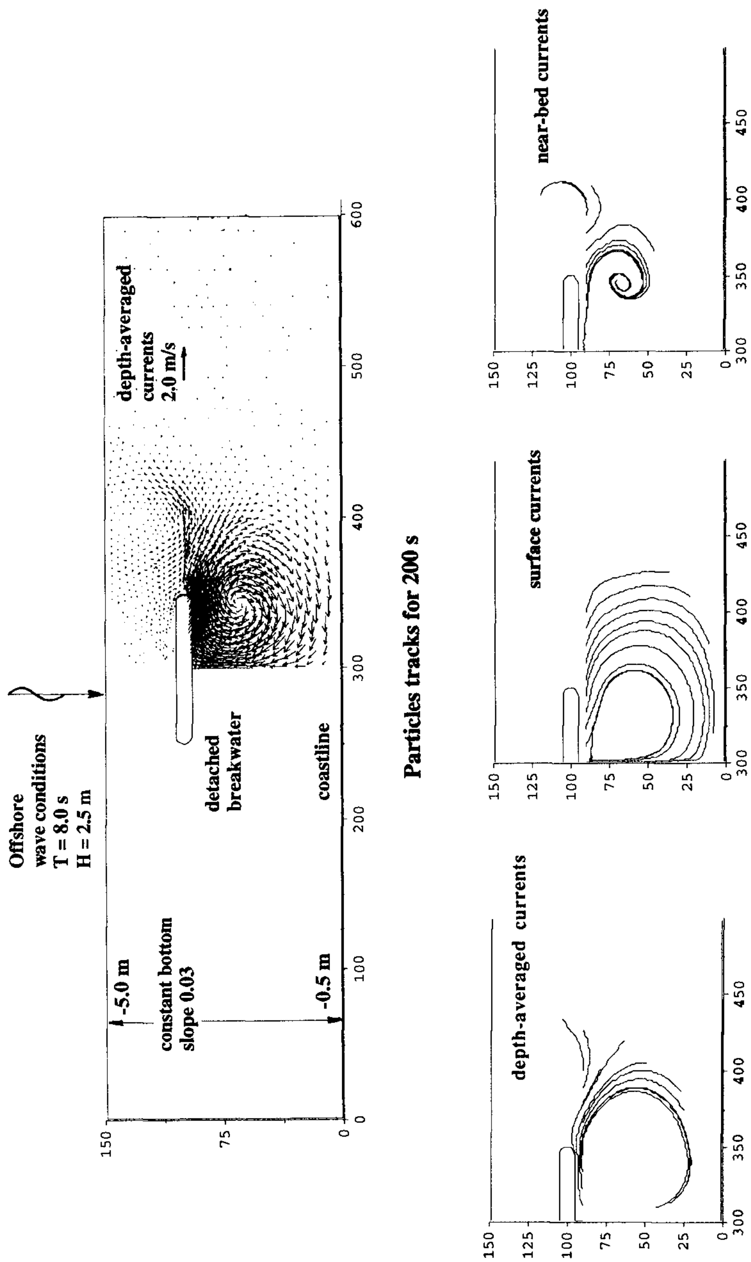

Fig. 6 Circulations near a detached breakwater

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