

## CHAPTER 170

### EXPERIMENTAL RESULTS OF WAVE TRANSFORMATION ACROSS A SLOPING BEACH

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#### Abstract

Experiments have been conducted to investigate the transformations of the wave characteristics induced by a uniformly decreasing sea bed. These transformations were recorded on both the frequency and the time domain representations of random wave trains. The coastal zone upwind of severe breaking was studied. Among the findings it is noted the tendency of the spectrum width parameter to decrease at a slowing rate as the waves propagate on shoaling waters. Also, the correlation coefficient between wave heights and periods seems to inversely follow the variations of a measure of the wave heights. A mathematical model has been developed based on existing results for shoaling, wave breaking and decay. This model applicable to a wide area of the surf zone predicts adequately the evolution of the joint probability between wave heights and periods provided by the experimental results.

#### Introduction

Wind wave modeling in the surf zone and the transitional waters is of central importance to coastal engineering. Albeit, shallow water waves in coastal areas are much more complex to model compared to waves in deep water. The transformations of random waves propagating into shallow water are governed by various processes, that are usually described by the source terms in an energy-balance equation. Apart from the wind action on the sea surface, the dissipation due to "white capping" and the nonlinear wave-wave interactions, factors that influence wave propagation in deep water, shallow water waves are additionally affected by processes such as shoaling, refraction, diffraction, reflection, bottom friction and wave breaking due to depth limitation.

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Depth-controlled shoaling and wave breaking play a dominant role in modifying the waves as they travel across the beach. The depth-induced wave breaking affects the whole frequency band in a way similar to the shoaling process. In fact, breaking can be regarded as the final stage in a combined shoaling/breaking transformation. This transformation can be tracked either on the frequency spectrum of the incoming waves or on the relevant statistics of wave heights and periods based on the representation of the wave train in the time domain.

The latter representation contains more information on the statistical structure of the sea state and seems to suit better the probabilistic approach to the design of coastal defences.

Existing Results

Various approaches have been used in the past to investigate and model the transformations to the probability density function (pdf) of the wave heights induced by wave shoaling/breaking due to depth limitation. The earlier models (Collins, 1970; Battjes, 1972; Kuo, 1974; Goda 1975) describe shoaling as only dependant on the local water depth. The common idea of all local-depth models is to cut off the portion of the wave height pdf beyond a breaker height controlled by water depth.

Collins and Battjes used a sharp cut-off of the Rayleigh pdf with all broken waves having heights equal to the breaker height  $H_b$ . Kuo and Kuo assumed the broken waves to have some height smaller than  $H_b$  after breaking and produced a sharply truncated Rayleigh pdf renormalized to unity. Goda's approach constitutes a refined version of the previous methods in that wave breaking occurs with linearly varying probability over a range of wave heights  $H_{b1}$  to  $H_{b2}$ , resulting in a distribution with a gradual cut-off around  $H_b$ . Figure 1 illustrates the above assumptions as regards the modification of the wave height pdf due to wave breaking.

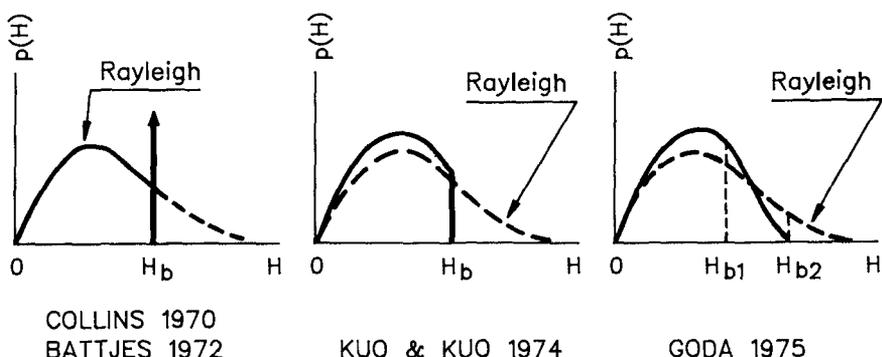


Fig.1 Modification of wave height pdf due to wave breaking

A second type of models (Battjes & Janssen, 1978; Thornton & Guza, 1983; Battjes and Beji, 1992) are based on integrating the energy-flux balance equation with wave height as it is transformed by the shoaling process along a path starting in deep water. Models applying the energy-flux balance calculate wave heights by employing the energy dissipation due to a bore, to model the shallow-water wave breaking. This involves a bore dissipation function adjusted through a variable parameter to each individual breaker type.

To simplify the analysis both types of models assume that the waves are narrow-banded in frequency, so that all wave heights of the distribution are associated with the same average frequency. Therefore, starting in deep water, the wave heights are described by the single parameter Rayleigh pdf, which is modified accordingly as the waves propagate over the shoaling beach.

A further method to tackle the problem of random wave shoaling is the wave-by-wave analysis, that permits more realistic inclusion of the physics of the process. This approach selects randomly offshore waves from a known joint distribution between heights and periods, transforms individual waves and then reassembles the wave heights into probability distributions across the surf zone. In fact recent studies indicate that much of the behavior of random waves in the surf zone can be represented by the behavior of a set of individual regular waves (Dally and Dean, 1986; Ebersole 1987; Dally 1990; etc). Dally (1992) presented such a method by using as input the joint pdf of wave heights and periods proposed by Longuet-Higgins (1983). However, this density function is limited mainly to narrow banded spectra. It is noted that a simpler technique employing only the characteristic wave height and the wave period at peak has been suggested by Kamphuis (1994), avoiding the computations for many regular waves as originally proposed. However, this approach is suited rather for the spectral representation than for the joint pdf of the waves.

### Scope of Present Research

Motivation for the present investigation was provided by recent results regarding the joint pdf of wave heights and periods applicable to the more realistic broad banded power spectrum (Memos & Tzanis, 1994). The scope of this research is to study experimentally the transformations on the time and frequency domain incurred during the initial phases of shoaling in the transitional water depths prior to intense breaking. Also, a synthetic model on the joint pdf of shoaling/breaking waves in a wider zone of a uniformly sloping bed is presented along with some initial results.

The experiments have been conducted in the 50 m-long random wave flume of Imperial College, London and the data analysis was performed at the Technical University of Athens.

### Experimental Set-up

The experiments aimed at studying the behavior of random unidirectional waves propagating over a beach of decreasing depth and they were carried out in a 50m-long random wave flume. The set-up allowed data collection at probes placed over a flat beach sloping at 5.5%, while the water depth was kept around 1m over the horizontal bed of the flume (Fig. 2).

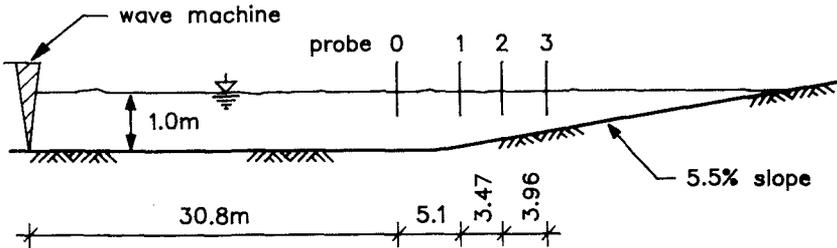


Fig.2 Experimental set- up

The random wave machine spanned the 2.78 m wide flume and was fed with signals of prescribed spectral characteristics. The data were collected from twin wire probes through an analog-to-digital converter. In all 27 runs were carried out, each one having a different input signal.

The transitional waters and outer surf zone was mainly investigated, where wave breaking and set-up occurred at low percentages, thus excluding the significant effect of wave reforming, which could otherwise "contaminate" the results of the present study of the shoaling process. Also, in this region the bed friction effect was rather reduced and could easily be ignored.

The investigation of the transformations referred to both the frequency and the time domain representations of the wave train.

### Main Findings and Discussion

The main results related to the transformation of the power spectrum across the outer surf zone are based on results of 4 representative experimental runs. These findings are the following:

- (a) In general the peak frequency of the input spectrum did not change significantly after the waves have travelled for 31m from the wave machine to the first probe. Also, the peak energy density and the frequency at peak tend to be stable as the waves propagate over the beach. This happened in all 27 runs. The shape of the spectrum around the peak was found to be

stable, too. It is noted that since we are studying the outer surf zone, with little wave breaking the spectrum tends to increase slightly in energy prior to major dissipation due to intense wave breaking.

The above stability of the shape of the spectrum and of the frequency at peak have been reported also by other researchers (Cai et al., 1992; Resio, 1987; etc).

- (b) A systematic modification of the rear face of the spectrum was taking place as the waves propagated into shallower waters. This is due to conservative non-linear effects in accordance with Kitaigorodskii's modification of the tail of the spectrum in shallow water behaving there like  $f^{-3}$  rather than  $f^{-5}$ , where  $f$  the frequency (Bouwes and Komen, 1983).
- (c) A transfer of energy to the forward face of the spectrum was also noticed. This, has again been verified by other investigators (Mase & Kirby, 1992; Resio, 1987).
- (d) A second peak is usually being developed at twice the peak frequency of the spectrum, presumably due to bound waves (Bendykowska and Werner, 1989; Memos, 1990; etc.).

All points mentioned previously can be observed in Fig. 3.

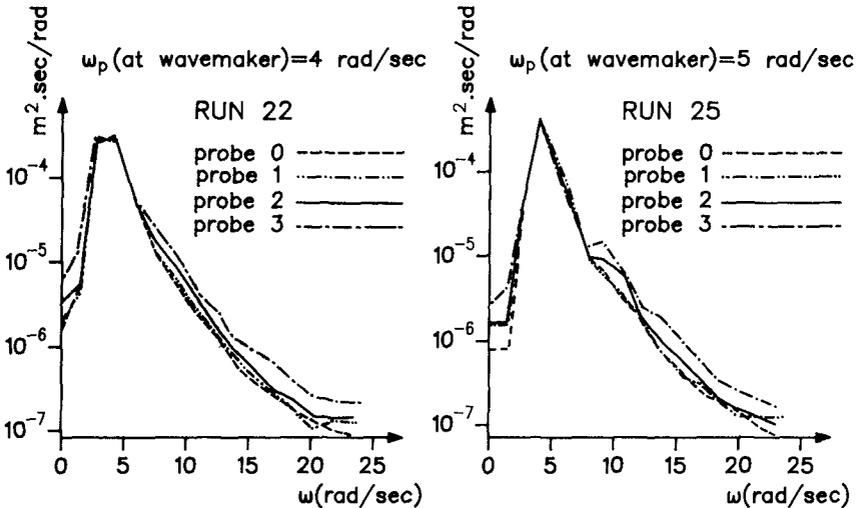


Fig. 3 Modification of wave energy spectrum

- (e) A systematic narrowing of the spectrum established by the decrease of spectral width  $\epsilon$  as the waves propagate from deep water toward the surf zone was detected (Fig. 4). It appears that a lower limit of  $\epsilon$  around 0.4 is reached at the most innershore probe, regardless of the initial width of the input spectrum (with  $\epsilon > 0.4$ ). This seems to be a new result.

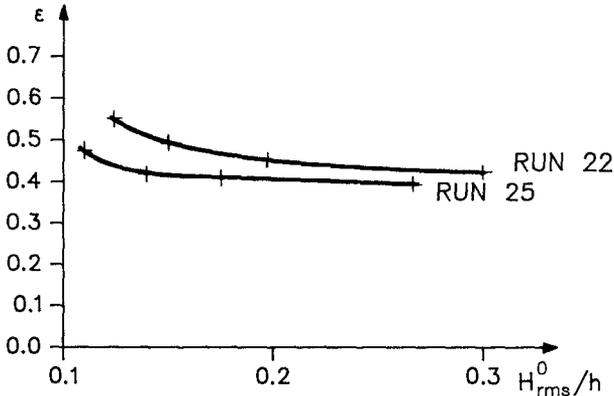


Fig. 4 Variation of bandwidth parameter  $\epsilon$  across the beach

Referring to the variation of wave heights and periods across the slope, the following points can be made for the time domain representation of the waves:

- (f) No major deviation of the wave heights pdf from the Rayleigh distribution occurs within the considered zone. Only a small redistribution of energy seems to take place around the peak of the pdf of wave heights (Fig. 5). This has been reported, also, by Thornton & Guza (1983).
- (g) A mild increase of the mean wave period with wave propagation was also detected (Fig.6).
- (h) The joint pdf of wave heights  $H$  and periods  $T$  showed an overall stability in line with the stability displayed by the energy spectrum. This can be seen in Fig. 7, where the corresponding joint pdfs for the 4 locations have been plotted for one run. However, a trend of more  $H$ - $T$  pairs accumulating close to the modal values was uncovered by estimating the double integral  $\int \int p dHdT$  for, say,  $p > 1.0$ . The results for 2 runs shown in Fig. 8 exhibit this tendency. This accumulation can also be verified in the marginal distribution of wave heights along the mean period, as shown in Fig. 9.

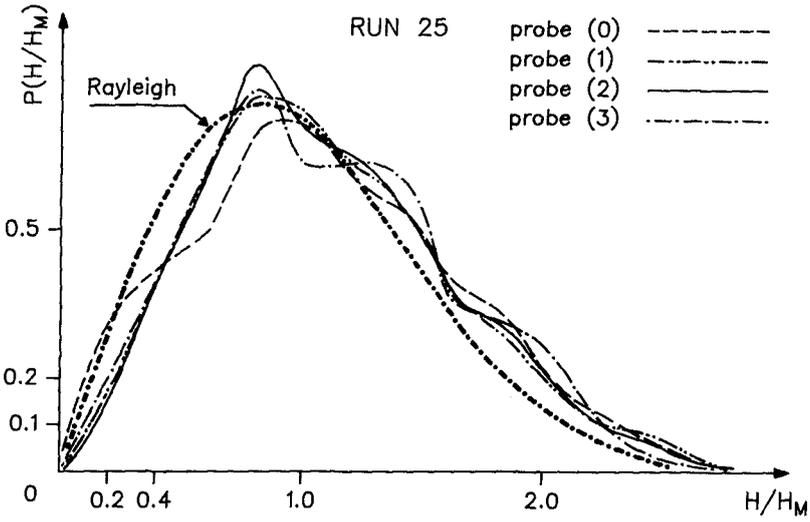


Fig. 5 Evolution of wave height distribution

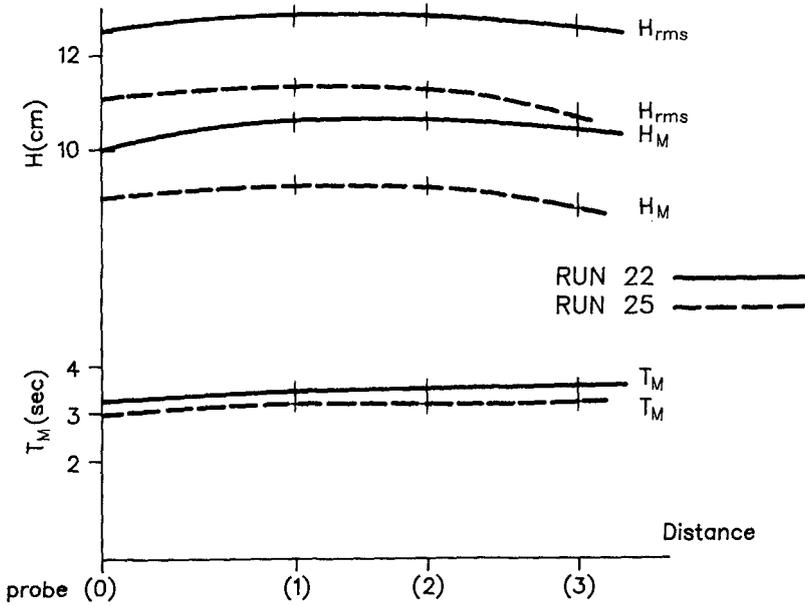


Fig.6 Variation of wave height and period across the beach

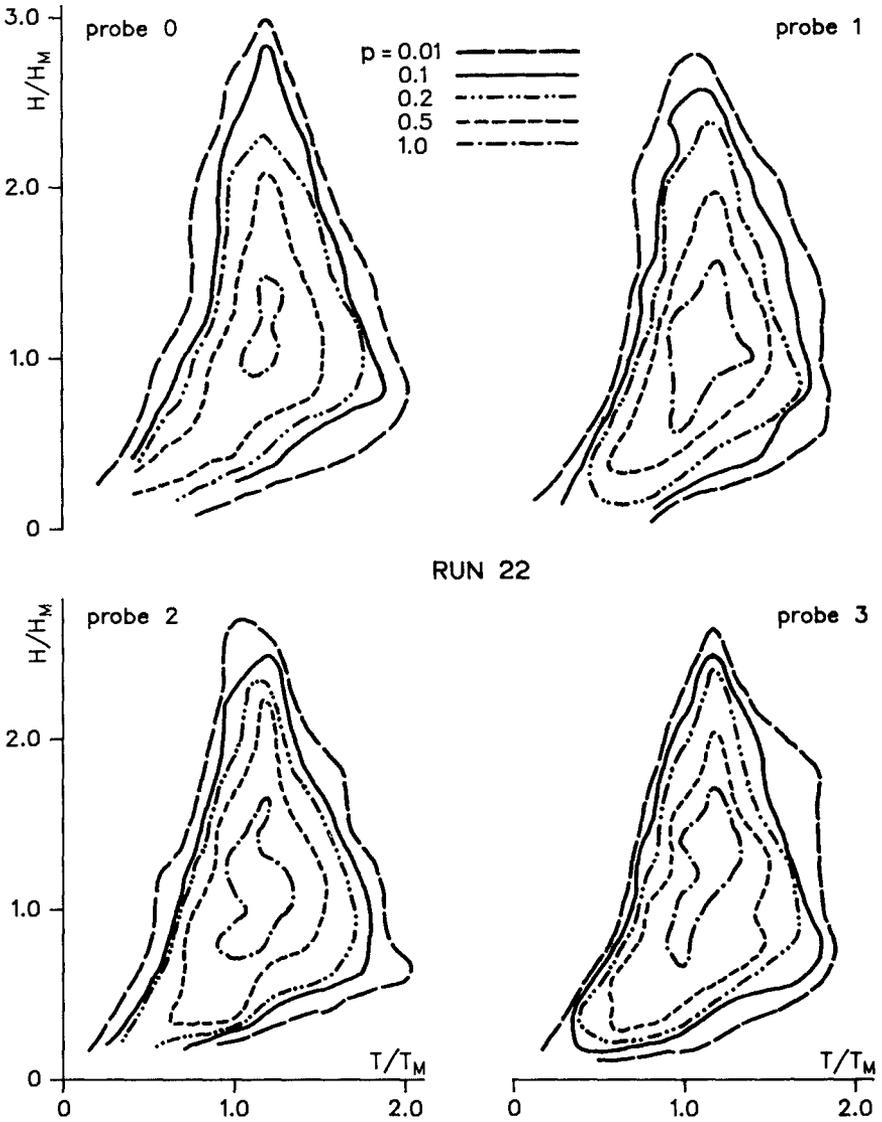


Fig. 7 Joint pdf of H-T at 4 gauges

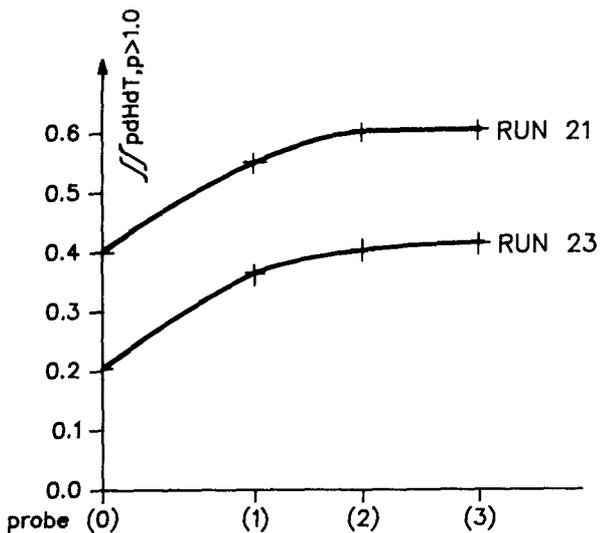


Fig. 8 Higher probabilities for higher waves

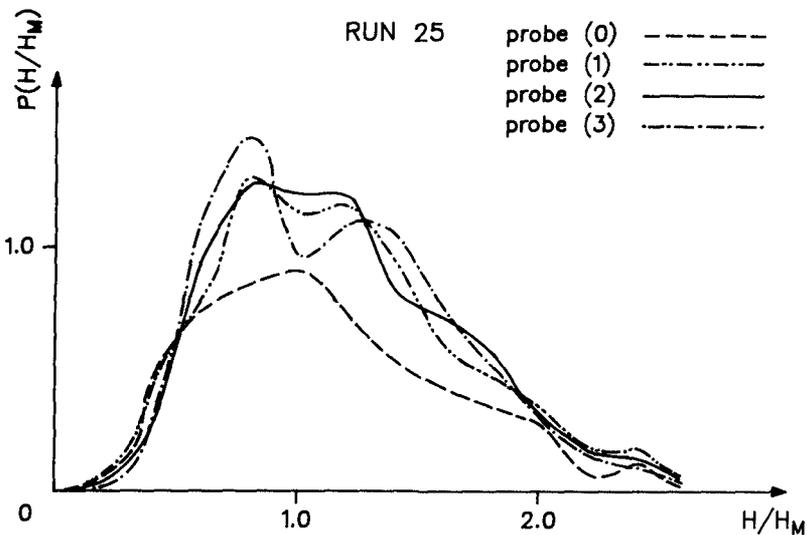


Fig. 9 Marginal distribution of H along the mean period

- (i) The variation of the correlation coefficient  $r$  between wave heights and periods can be seen in Fig. 10. A similar trend with that of the

spectrum bandwidth parameter  $\varepsilon$  can be observed, the difference lying in that  $r$  reaches a minimum value within the outer surf zone under examination. This seems to correspond to the variation of the wave heights over the slope. For decreasing  $H$  we obtain increasing  $r$  and vice versa (cf Figs 6 and 10).

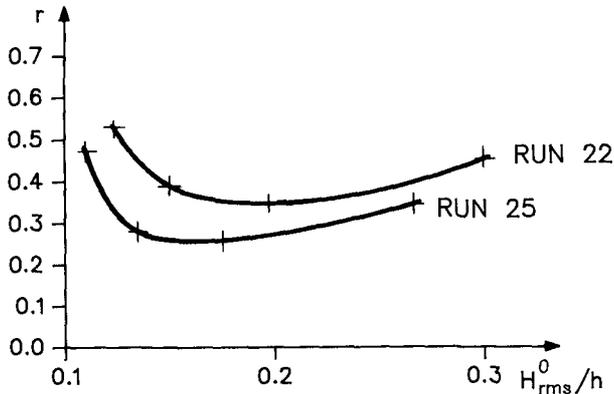


Fig. 10 Variation of  $r(H,T)$  across the beach

This relation can be said that is generally valid in wind waves: for large enough wave heights the inherent correlation between heights and periods becomes looser and the extreme wind waves are in general associated with a narrow band of periods. The same phenomenon is also demonstrated in the scatter diagrams of short term wave heights and periods, which are nearly symmetrical for large wave heights with respect to a vertical axis, implying little correlation between  $H$  and  $T$ .

Based on the experimental data the relationship between the variation of  $r$  and wave height is depicted in Fig. 11, which clearly shows that indeed there exists a kind of law between the two variables.

The horizontal axis denotes the relative increase of the wave height in terms of  $\Delta\bar{H}/\bar{H}$ ,  $\bar{H}$  mean value, and the vertical gives the variation  $\Delta r$ . A main feature of the graph is that for an increase in  $\bar{H}$  we have a corresponding decrease in  $r$ , and vice versa, as already noted previously.

#### A Simple Wave Shoaling/Breaking Model

A mathematical model based on existing results for depth-induced shoaling, wave breaking and decay has been developed to check the experimental data. The wave-by-wave approach, referred to previously, has

been applied. It is noted that this technique is better suited for the surf zone rather than the transitional waters where our experiments were confined. However, its performance in such depths was quite acceptable, as it will be shown later.

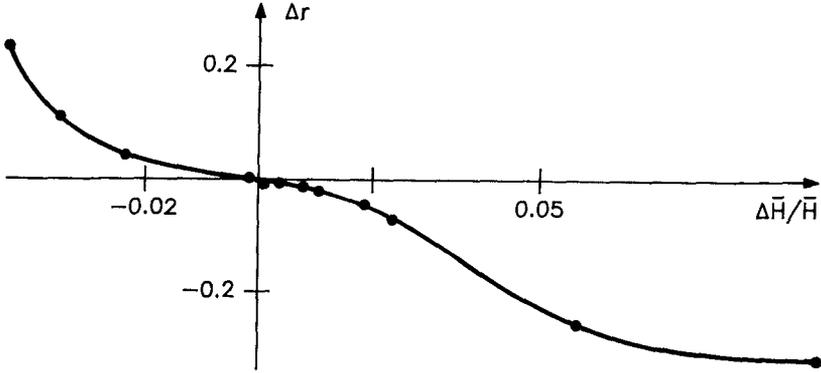


Fig.11 Relation between  $\Delta\bar{H}$  and  $\Delta r$ .

Cnoidal wave theory has been assumed to apply for the shoaling process and as a good first approximation the record at probe (0) was assumed to represent deep water conditions. Following Shuto (1974) as modified by Goda (1975), the shoaling transformations given below were taken into account.

$$H = H_0 K(h), \quad h \geq h_1 \quad (1)$$

$$H = H_0 K(h_1)(h_1/h)^{2/7}, \quad h_2 \leq h \leq h_1 \quad (2)$$

where  $H_0, L_0$  deep-water wave height and wavelength

$H, L$  wave height and wavelength at depth  $h$

$$K(x) = 1/\{ [1+(4\pi x)/L \sinh(4\pi x/L)] \tanh(2\pi x/L) \}^{1/2} \quad (3)$$

$$h_1^2 = 0.209 H_0 L_0 K(h_1) \quad (4)$$

$$h_2 = 0.8 h_1 \quad (5)$$

When  $h < h_2$ , then solution of the following equation provides the shoaling coefficient  $K_1 = H/H_0$

$$K_1(\sqrt{K_1 - B}) - C = 0 \quad (6)$$

where  $B = 1.382h/h_0 (H_0/L_0)^{1/2}$  (7)

$C = C_2 (L_0/h)^{3/2} / (2\pi H_0/L_0)^{1/2}$  (8)

$C_2 = 3.693 K(h_1) (h_2/L_0)^{3/2} [(0.558 H_0/L_0)^{1/2} - h_2/L_0]$  (9)

The breaking criterion proposed by Weggel (1972) has been assumed. This reads as follows

$$\frac{H_b}{L} = (1/7) \tanh \left[ \frac{7h_b}{L} \frac{b}{1+\alpha(h_b/gT^2)} \right] \tag{10}$$

where  $H_b$  wave height at breaking

$T$  wave period

$\alpha = 43.75 [1-\exp(-19m)]$  (11)

$b = 1.56 / [1+\exp(-19.5 m)]$  (12)

$m$  bed slope

$h_b$  still water depth at incipient breaking.

Finally, wave decay after breaking has been included in the model after Dally et al. (1985) as follows.

$$\frac{H}{H_b} = \left[ (1+c)(h/h_b)^{(G/m)-0.5} - c(h/h_b)^2 \right]^{1/2} \tag{13}$$

where  $H$  the height of the decaying wave

$$c = \frac{\Gamma^2}{m[(2.5)-(G/m)]} (h/H)_b^2 \tag{14}$$

$G$  decay coefficient ( ~0.2 or lower)

$\Gamma \sim 0.35 \div 0.40$

Applying the above synthetic model to the experimental conditions mentioned earlier and taking as deep water conditions those recorded at probe (0), joint pdfs at locations (1), (2) and (3) can be produced for direct comparison with the experimental data. Figure 12 contains the model results for run 22 which are comparable with the experimental results shown in Fig. 7.

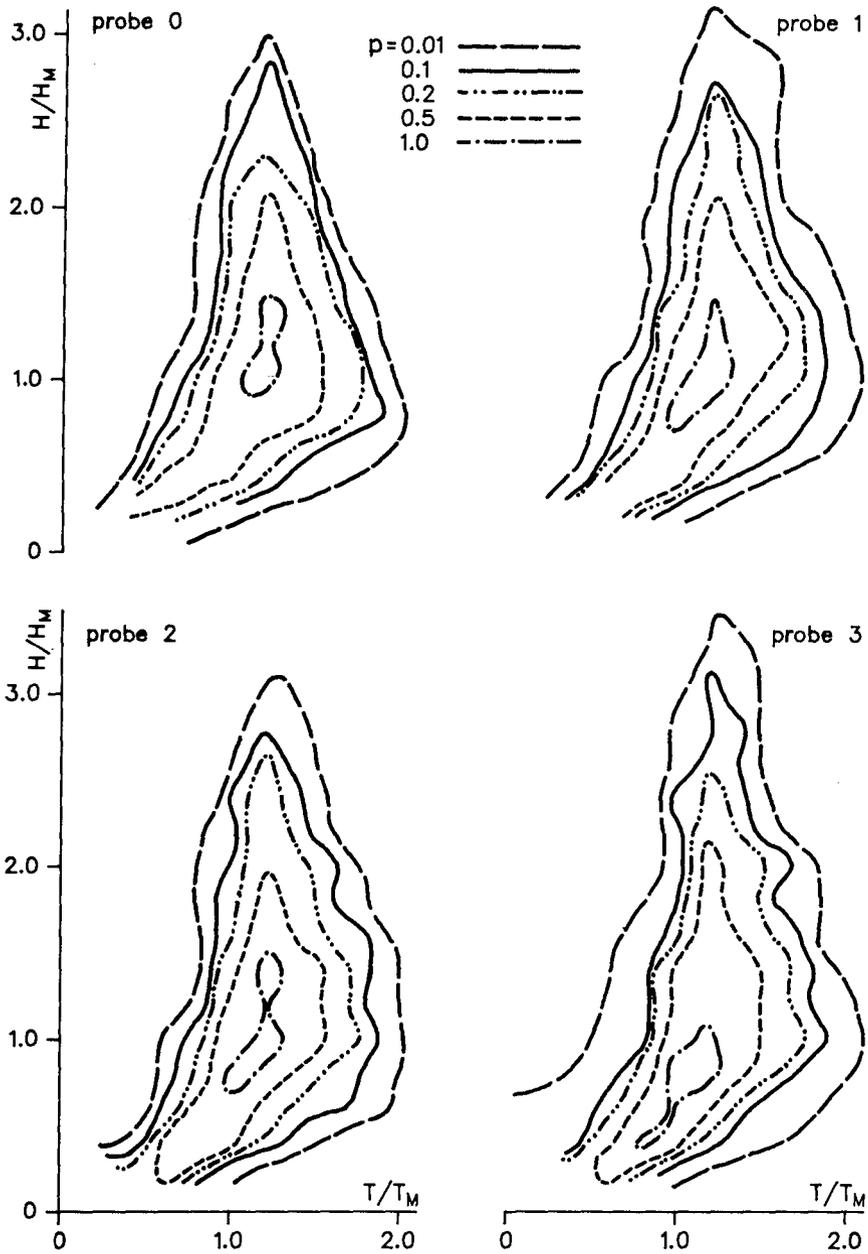


Fig. 12 Model results for run 22

The model results verifies the observed only slight variability of the joint probability structure, as the waves propagate over the sloping bed.

Agreement between the two sets of results is quite acceptable, despite the fact that the wave-by-wave technique is better suited for the surf zone inshore of probe (3), as noted above. Results for other runs indicate the same trend and agree similarly well with experimental data.

### Conclusions

Focusing on the new findings of the present research, we can conclude that the variations of both the bandwidth parameter  $\epsilon$  and the correlation coefficient  $r$  between wave heights and periods across the outer surf zone, follow a trend resulting from a general law governing the correlation between wave heights and periods for large heights. The correlation coefficient  $r(H,T)$  is closely related to the variations of a measure of the wave height across the beach, while the decrease of  $\epsilon$  seems to flatten out within the surf zone.

Also, that a synthetic model developed on existing results can adequately predict the variation of the joint probability density function between waveheights and periods in the outer surf zone of a sloping beach. Improvement of the model is currently underway by abolishing the assumption that the deep water conditions occur at probe (0) and, also, by assuming Stokian waves of the 3rd order rather than cnoidal waves for the shoaling part of the process. This description of the waves may suit better a transitional zone of deeper waters as those present in the experiments.

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