CHAPTER 164

A QUASI-3D SURF ZONE MODEL

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ABSTRACT

This paper presents theoretical circulation patterns in both cross-shore and longshore directions on the plane beach slope. In the model an amended form of radiation stress which is consistent with the wave energy flux in the wave-current coexisting field is presented. Comparison of theoretical surf zone properties with laboratory experiments showed good agreements. Finally, a quasi-three dimensional model suitable for the entire nearshore zone is presented by linking the depth-integrated properties with vertical profiles.

1. INTRODUCTION

A prominent feature in the nearshore zone is the wave-induced current. It is commonly accepted that the primary driving force is the wave-induced radiation stress first introduced by Longuet-Higgins and Stewart (1961). Modeling this circulation has advanced considerably since the earlier development by Noda et al. (1974) and Ebersole and Dalrymple (1979). Both of these earlier models were driven by a wave refraction model with no current feedback. In recent years, coupled wave-induced circulation models have also been developed (Yoo and O'Connor, 1986; Yan, 1987; and Winer, 1988). All these models depthuniform circulation patterns and can, therefore, be classified as two-dimensional. They are not suitable for surf zone where current is vertically non-uniform. This feature is particularly prominent in the cross-shore direction because the onshore mass transport produced by the depth-varying momentum flux has to be compensated by the return flow due to the depth-uniform set-up force. This driving mechanism was first suggested by Dyhr-Nielsen and Sorensen (1970) and treated

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analytically by Dally (1980). Effort has since been made to produce cross-shore circulation patterns inside the surf zone such as the undertow model developed by Svendsen (1984), Buhr-Hansen and Svendsen (1984) and more recently, by Okayasu et al. (1988) and Yamashita and Tsuchiya (1990). It is only natural to attempt to develop circulation models that can address both horizontal and vertical variations in the nearshore zone. So far the effort is still few. De Vriend and Stive (1987) recently formulated a nearshore circulation model by employing a quasi-3D approach. At present, this approach is attractive from both theoritcal and computational point of view.

One of the handicaps of all the existing depth-varying models whether crossshore models or three-dimensional models is the pre-requisite on the large number of empirical coefficients that have to be assigned. This severely limits their application as one must be confident on the behavior of these coefficients under various natural conditions. Recently, Lee (1993) presented a formulation on current-wave interaction problems. In there, the radiation stress term in the momentum equation is amended, and two new conservation equations governing intrinic wave frequency and wave action are introduced. In this paper, this new formulation is utilized to develop a depth-varying circulation model. The difference between the new model and the existing models is quite significant. First of all, since radiation stress is the primary driving force for circulations an amended formula alters this force. Secondly, the new model has stronger theoretical basis and requires fewer empirical coefficients owing to the fact the model must satisfy the additional governing equations. An analytical model for a straight shoreline of uniform slope is introduced first to explore the nature of the model and to facilitate comparisons. A general version suitable for the entire nearshore zone is then developed by linking the depth-integrated properties with depth-varying models. The numerical technique is briefly addressed and examples are given.

2. HORIZONTAL CIRCULATION MODEL

The governing equations for the horizontal circulation model are obtained after depth integration and wave-averaging. In order to protect from losing the Eulerian mean quantities at the mean water level, the depth integration is taken prior to wave-averaging them. The strong presence of turbulence is a prominent feature in surf zone. Consequently, the fundamental equations governing the fluid motion should also include the turbulent effects. This is usually accomplished with the introduction of Reynolds stresses by time averaging over the turbulent fluctuations.

2.1 Depth-Integrated and Time-Averaged Equation of Mass

Integrating the continuity equation for incompressible fluid over depth and employing the kinematic boundary conditions on the free surface and on the bottom, we get

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\eta} u dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} v dz = 0$$
(1)

Now let the turbulence-averaged velocity vector, U(u, v, w), and the surface elevation, η , be decomposed into mean current and wave fluctuation, which will be distinguished by the subscript c and w, respectively; thus,

$$\mathbf{U} = \mathbf{U}_c + \mathbf{U}_w, \qquad \eta = \eta_c + \eta_w \tag{2}$$

where \mathbf{U}_{w} and η_{w} are the residual wave fluctuation which can be removed through the process of wave-averaging, and \mathbf{U}_{c} and η_{c} are the time-averaged value of velocity. The velocity at a particular water level with a mean position of (\mathbf{x}_{1}, z_{1}) is $\mathbf{U}(\mathbf{x}_{1}, z_{1} + \xi)$, where ξ is a location of the vertical trajectory being up and down with the residual wave fluctuation at \mathbf{x}_{1} .

Then, we obtain the wave-averaged value of velocity, U_c , as

$$\mathbf{U}_{\mathrm{c}}(\mathbf{x}_1,z_1)=rac{1}{T}\int_0^T\mathbf{U}(\mathbf{x}_1,z_1+\xi)dt$$

Substituting Eq. (2) and taking the wave-average after expanding in a Taylor series at $\eta = \eta_c$, Eq.(1) can be simplified as

$$\frac{\partial \eta_c}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} u_c dz + \frac{\partial}{\partial x} (\overline{\eta_w u_w})_{\eta_c} + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} v_c dz + \frac{\partial}{\partial y} (\overline{\eta_w v_w})_{\eta_c} = 0$$
(3)

The wave components are given by linear progressive wave theory as follow:

$$\frac{\partial \eta_c}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} u_c dz + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} v_c dz + \frac{1}{\rho} \left[\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} \right] = 0$$
(4)

where the x and y components of mass flux are defined as

$$M_x = \frac{E'k_x}{\sigma}, \qquad M_y = \frac{E'k_y}{\sigma}$$
 (5)

and E' is defined as $\rho g H^2/8$. The mass flux terms are considered the mass transport above the mean water level.

2.2 Depth-Integrated and Time-Averaged Equations of Momentum

Assuming that no horizontal viscous stress exist, the horizontal momentum equation in the x direction is integrated over depth to yield

$$\frac{\partial}{\partial t} \int_{-h}^{\eta} u dz + \frac{\partial}{\partial x} \int_{-h}^{\eta} u u dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} u v dz$$
$$= \frac{1}{\rho} \left[-\frac{\partial}{\partial x} \int_{-h}^{\eta} p dz + p|_{\eta} \frac{\partial \eta}{\partial x} + p|_{-h} \frac{\partial h}{\partial x} + \tau_{Wx} - \tau_{Bx} \right]$$
(6)

where $\tau_{Wx} = \tau_{zx}|_{\eta}$ is a wind stress in the x direction and $\tau_{Bx} = \tau_{zx}|_{-h}$ is the bottom friction. Substituting **U** and η defined in Eq. (2), the time-averaged

quantities is also obtained by expanding η in Taylor series at the mean water level, η_c ,

$$\frac{\partial}{\partial t} \int_{-h}^{\eta_c} u_c dz + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} u_c^2 dz + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} u_c v_c dz + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} \overline{u_w^2} dz + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} \overline{u_w v_w} dz + \frac{\partial}{\partial t} (\overline{\eta_w w_w})|_{\eta_c} + \frac{\partial}{\partial x} (2\overline{\eta_w u_c u_w})|_{\eta_c} + \frac{\partial}{\partial y} [\overline{\eta_w (u_c v_w + v_c u_w)}]_{\eta_c} = \frac{1}{\rho} \left[-\frac{\partial}{\partial x} \overline{\int_{-h}^{\eta_c} p dz} + \overline{p}|_{-h} \frac{\partial h}{\partial x} + \overline{\tau_{W_x}} - \overline{\tau_{B_x}} \right]$$
(7)

where the pressure at the free surface was assumed to be zero and then the total pressure is given as

$$p = \bar{p} + p_w = -\rho \overline{w_w^2(z)} + \rho g(\eta_c - z) + \rho g \eta_w K_p(z)$$
(8)

where K_p is the pressure response factor given by linear wave theory,

$$K_p = \frac{\cosh k(h+z)}{\cosh k(h+\eta_c)} \tag{9}$$

Substituting Eq (8), finally, the depth-integrated and time-averaged momentum equation in the x direction is obtained;

$$\frac{\partial}{\partial t} \int_{-h}^{\eta_c} u_c dz + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} u_c^2 dz + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} u_c v_c dz + \frac{1}{\rho} \frac{\partial S_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial S_{yx}}{\partial y} + g(h + \eta_c) \frac{\partial \eta_c}{\partial x} - \frac{\overline{\tau_{Wx}}}{\rho} + \frac{\overline{\tau_{Bx}}}{\rho} = 0$$
(10)

The momentum equation in the y direction can be similarly obtained,

$$\frac{\partial}{\partial t} \int_{-h}^{\eta_c} v_c dz + \frac{\partial}{\partial x} \int_{-h}^{\eta_c} u_c v_c dz + \frac{\partial}{\partial y} \int_{-h}^{\eta_c} v_c^2 dz + \frac{1}{\rho} \frac{\partial S_{xy}}{\partial x} + \frac{1}{\rho} \frac{\partial S_{yy}}{\partial y} + g(h + \eta_c) \frac{\partial \eta_c}{\partial y} - \frac{\overline{\tau}_{Wy}}{\rho} + \frac{\overline{\tau}_{By}}{\rho} = 0$$
(11)

For the case of linear progressive wave and mild slope, the radiation stress terms can be expressed in terms of wave characteristics as

$$S_{xx} = E'[n(\cos^2\theta + 1) - \frac{1}{2} + 2\cos\theta \frac{u_s}{C}]$$
(12)

$$S_{xy} = S_{yx} = E'[\sin\theta(n\cos\theta + \frac{u_s}{C}) + \cos\theta\frac{v_s}{C}]$$
(13)

$$S_{yy} = E'[n(\sin^2\theta + 1) - \frac{1}{2} + 2\sin\theta\frac{v_s}{C}]$$
(14)

where n = Cg/C and u_s and v_s are the time-averaged current velocity at mean water level. It is noted here that the definition of the radiation stress differs from that given by Longuet-Higgins and Stewart (1961) with the additional advective

terms. The amended form of radiation stress has been proven to be consistent with the wave energy flux in the wave-current coexisting field by Lee (1993).

3. VERTICAL CIRCULATION MODEL FOR STRAIGHT SHORE-LINE

For the case of straight shoreline and parallel offshore contours the crossshore and longshore components are decoupled. This simplifies the mathematical manupulation considerably.

3.1 Theoretical Undertow Model

The vertical ciculation in the x-z plane is treated as quasi-steady. Since there is no y-direction (longshore) variation, the turbulence-averaged momentum equation in the x-direction (onshore) integrated from any level z to the mean water level can be written as

$$\frac{\partial}{\partial x} \int_{z}^{\eta} uudz - uw|_{z} = \frac{1}{\rho} \left[-\frac{\partial}{\partial x} \int_{z}^{\eta} pdz + p(\eta) \frac{\partial \eta}{\partial x} + \tau_{zx}|_{\eta} - \tau_{zx}(z) \right]$$
(15)

where the shear stress is assumed to be expressed in the form

$$\tau_{zx} = \rho \nu_t \frac{\partial u}{\partial z} \tag{16}$$

with ν_t defined as the total kinematic viscosity, which is composed of both eddy and molecular viscosities in the vertical direction. The shear stress at free surface, $\tau|_{\eta}$, is assumed only due to wind stress τ_W . Separating the velocity into the current and wave components and taking time-average, Eq. (15) becomes

$$\nu_t \frac{\partial u_c}{\partial z}|_z = -\frac{\partial}{\partial x} \int_z^{\eta_c} (\overline{u_w^2} - \overline{w_w^2}) dz - \frac{g}{2} \frac{\partial \overline{\eta_w^2}}{\partial x} + \frac{\partial}{\partial x} (2\overline{\eta_w u_c u_w})|_{\eta_c} - g(\eta_c - z) \frac{\partial \eta_c}{\partial x} + \frac{\overline{\tau_{Wz}}}{\rho}$$
(17)

The convective term of the mean current was assumed to be small enough compared to the rest. In shallow water, the first term on the right hand side becomes

$$\frac{\partial}{\partial x} \int_{z}^{\eta_{c}} (\overline{u_{w}^{2}} - \overline{w_{w}^{2}}) dz = g \frac{(\eta_{c} - z)}{(\eta_{c} + h)} \frac{\partial}{\partial x} \left[(\cos^{2} \theta + 1) \frac{H^{2}}{8} \right]$$
(18)

and the second term reduces to,

$$\frac{g}{2}\frac{\partial\overline{\eta_w^2}}{\partial x} = \frac{g}{16}\frac{\partial H^2}{\partial x}$$
(19)

The fourth term can be determined by the depth-integrated equation of momentum, Eq. (10), under the following assumptions; 1) the flow is in steady state, and 2) the effect of squared mean current are negligible. Then, Eq.(10) becomes

$$\frac{\partial \eta_c}{\partial x} = -\frac{1}{\rho g(h+\eta_c)} \left[\frac{\partial S_{xx}}{\partial x} - \overline{\tau_{Wx}} + \overline{\tau_{Bx}} \right]$$
(20)

Substituting Eqs. (18-20) into Eq. (17) and introducing non-dimensional variable, $z' = (\eta_c - z)/(\eta_c + h)$, results in,

$$-\frac{\nu_t}{\eta_c + h} \frac{\partial u_c}{\partial z'} \Big|_{z'} = z' \left\{ \frac{g}{16} \frac{\partial H^2}{\partial x} - 2g \frac{\partial}{\partial x} (\cos \theta \frac{u_s}{C} \frac{H^2}{8}) - \frac{\overline{\tau}_{Wx}}{\rho} + \frac{\overline{\tau}_{Bx}}{\rho} \right\} - \frac{g}{16} \frac{\partial H^2}{\partial x} + 2g \frac{\partial}{\partial x} (\cos \theta \frac{u_s}{C} \frac{H^2}{8}) + \frac{\overline{\tau}_{Wx}}{\rho}$$
(21)

According to the above equation, we can estimate the shear stress at the mean water level,

$$-\frac{\nu_t}{\eta_c + h} \frac{\partial u_c}{\partial z'}|_{\eta_c} = -\frac{g}{16} \frac{\partial H^2}{\partial x} + 2g \frac{\partial}{\partial x} (\cos\theta \frac{u_s}{C} \frac{H^2}{8}) + \frac{\overline{\tau_{W_x}}}{\rho}$$
(22)

and the shear stress at the bottom,

$$-\frac{\nu_t}{\eta_c + h} \frac{\partial u_c}{\partial z'}|_{-h} = \frac{\overline{\tau_{Bx}}}{\rho}$$
(23)

Now we assume that the turbulent motions originating from the surface wave breaking is governed by an constant eddy viscosity, and that the boundary layer remains thin. Then, the above equation can be solved explicitly to give the following solution,

$$u_c(z') = u_s + C_{x1} z'^2 + C_{x2} z'$$
(24)

where

$$C_{x1} = -\frac{\eta_c + h}{2\varepsilon_z} \left\{ \frac{g}{16} \frac{\partial H^2}{\partial x} - 2g \frac{\partial}{\partial x} (\cos \theta \frac{u_s}{C} \frac{H^2}{8}) - \frac{\overline{\tau_{Wx}}}{\rho} + \frac{\overline{\tau_{Bx,tb}}}{\rho} \right\}$$
(25)

$$C_{x2} = -\frac{\eta_c + h}{\varepsilon_z} \left\{ -\frac{g}{16} \frac{\partial H^2}{\partial x} + 2g \frac{\partial}{\partial x} (\cos \theta \frac{u_s}{C} \frac{H^2}{8}) + \frac{\overline{\tau_{Wx}}}{\rho} \right\}$$
(26)

where ε_z implies the constant eddy viscosity which will be estimated in Section 3.3. According to this equation, the mean flow pattern inside the surf zone in the main region is essentially parabolic. The analytical solution for a plane beach is presented to simplify mathematical operation and to facilitate comparisions with data.

The profile given by Eq. (24) contains 3 physical parameters, the surface current, the eddy viscosity and the bottom friction. One of them can be eliminated by constraint that, in the cross-shore direction, the net flow has to be equal to zero for a steady case, or

$$\int_{-h}^{\eta_c} u_c dz + \frac{M_x}{\rho} = 0 \tag{27}$$

where M_x is wave-induced mass flux. Eliminating τ_b from Eqs.(24-26) and (27), the coefficients can be rewritten as

$$C_{x1} = 3(\overline{u} - u_s) - \frac{3}{2} \frac{\eta_c + h}{\varepsilon_z} P, \qquad C_{x2} = \frac{\eta_c + h}{\varepsilon_z} P$$
(28)

where

$$P = \frac{g}{16} \frac{\partial H^2}{\partial x} - 2g \frac{\partial}{\partial x} (\cos \theta \frac{u_s}{C} \frac{H^2}{8}), \qquad \overline{u} = -\frac{M_x}{\rho(\eta_c + h)} = -\frac{g}{8} \frac{\cos \theta H^2}{C(\eta_c + h)}$$
(29)

3.2 Theoretical Longshore Current Model

The longshore current profile can be obtained in a similar manner. For a straight shoreline, the surface gradient as well as the radiation stress gradient in the y-direction can be neglected. Again, neglecting the convective term of the mean current, the momentum equation in the y direction yields,

$$\frac{\partial}{\partial x} \int_{z}^{\eta_{c}} \overline{v_{w} u_{w}} dz + \frac{\partial}{\partial x} (\overline{\eta_{w} v_{c} u_{w}})|_{\eta_{c}} + \frac{\partial}{\partial x} (\overline{\eta_{w} u_{c} v_{w}})|_{\eta_{c}} = \frac{1}{\rho} \left[\overline{\tau_{Wy}} - \rho \nu_{t} \frac{\partial v_{c}}{\partial z}|_{z} \right]$$
(30)

Since the second and third terms become zero (Lee, 1983), Eq. (36) is reduced to

$$\frac{\partial}{\partial x} \int_{z}^{\eta_{c}} \overline{v_{w} u_{w}} dz = \frac{\overline{\tau_{Wy}}}{\rho} - \nu_{t} \frac{\partial v_{c}}{\partial z}|_{z}$$
(31)

In shallow water, the LHS becomes

$$\frac{\partial}{\partial x} \int_{z}^{\eta_{c}} \overline{v_{w} u_{w}} dz = \frac{(\eta_{c} - z)}{(\eta_{c} + h)} \frac{\partial}{\partial x} [g \cos \theta \sin \theta \frac{H^{2}}{8}]$$
(32)

by linear wave theory. Substituting Eq. (32) into Eq. (31) gives the following:

$$-\frac{\nu_t}{\eta_c + h} \frac{\partial v_c}{\partial z'}|_{z'} = -z' \frac{\partial}{\partial x} [g \cos \theta \sin \theta \frac{H^2}{8}] + \frac{\overline{\tau_{Wy}}}{\rho}$$
(33)

Integrating Eq.(33) with respect to z' with introduction of depth-independent ε_z we obtain,

$$\overline{v}(z') = v_s + C_{y1} z'^2 + C_{y2} z' \tag{34}$$

where

$$C_{y1} = \frac{\eta_c + h}{2\varepsilon_z} \frac{\partial}{\partial x} [g\cos\theta\sin\theta \frac{H^2}{8}], \qquad C_{y2} = -\frac{\eta_c + h}{\varepsilon_z} \frac{\overline{\tau_{Wy}}}{\rho}$$
(35)

Or, alternatively, C_{y1} can be expressed in terms of radiation stress as

$$C_{y1} = \frac{\overline{\eta} + h}{2\varepsilon_z} \frac{1}{\rho} \frac{\partial S_{xx}}{\partial x}$$
(36)

For steady longshore current, the depth-integrated radiation stress is balanced by the bottom friction under no wind. The depth-averaged mean longshore current is given by,

$$\overline{v} = v_s + \frac{C_{y1}}{3} + \frac{C_{y2}}{2}$$
(37)

3.3 Estimation of Surface Velocity and Eddy Viscosity

The solutions for cross-shore and longshore current profiles will be complete if the surface velocity and eddy viscosity are determined. Lee (1993) has shown that the surface current can be determined semi-analytically by virtue of wave action and intrinsic frequency conservation equations; both of them are surface conditions. The solutions briefly summarized here.

The surf zone is assumed to be coherent in that the essential wave-like periodic motion is retained and is quasi-steady when time-averaged over wave period. In this case, the wave action and wave frequency conservation equations are, respectively,

$$\nabla_h \cdot (\mathbf{U}_s \frac{H^2}{\sigma_s}) = 0, \qquad \nabla_h \cdot (\mathbf{U}_s \sigma_s) = 0 \tag{38}$$

Here σ_s denotes σ inside the surf zone. The wave energy equation is modified to reflect dissipation,

$$\nabla_h \cdot \left[(\mathbf{Cg} + \mathbf{Cg}_D + \mathbf{U}_s) \frac{H^2}{\sigma_s} \right] = 0$$
(39)

with Cg_D representing a disspation velocity. Eliminating a U_s term of Eq. (39) based on the first equation of Eqs.(38), the cross shore component of Eq. (39) provides

$$H = \frac{\beta_H}{(Cg_{Dx} - Cg_x)} \tag{40}$$

and also the surface currents are obtained,

$$u_s = \beta_C (Cg_{Dx} - Cg_x), \qquad v_s = \beta_L (Cg_{Dy} - Cg_y) \tag{41}$$

with β 's the constants of proportionality. They further assume on two-dimensional beaches of uniform slope that the dissipation inside the surf zone is dominated by the influence of the initial condition at the breaking point and \mathbf{Cg}_D to be equal to $-\beta \mathbf{Cg}_b$. The above equations can then be written as

$$H = \frac{\beta_H}{\cos\theta(\beta Cg_b - Cg)}$$

$$u_s = \beta_C \cos \theta (\beta C g_b - C_g), \qquad v_s = \beta_L \sin \theta (\beta C g_b - C_g)$$
(42)

Thus, the wave height and surface currents are determined explicitly. Figure 1 shows the comparison of wave height variation in the surf zone between the present theory and the laboratory data by Horikawa and Kuo (1966). Figure 2 compares the theory with the laboratory longshore current data measured by Visser (1991).

The vertical eddy viscosity for both cross-shore and longshore components is treated as the same and is estimated from the longshore mean current strength. From Section 3.2 we can obtain,

$$\frac{\overline{\tau_{By,tb}}}{\rho} = \frac{6\varepsilon_z}{\overline{\eta} + h} (\overline{v} - v_s)$$
(43)

In a flow field where the turbulent-induced stress dominanates the bottom friction, the bottom stress can be approximated by,

$$\frac{\overline{\tau_{By,tb}}}{\rho} \approx \frac{\sin\theta}{C_a} D \tag{44}$$

where $C_a = \omega/k$ and D is the local rate of energy dissipation. Based on Eq.(39), D can be expressed as

$$D = -\nabla \cdot \left[\beta C g_b \frac{\mathbf{K}}{k} \frac{\omega}{\sigma_s} E'\right]$$

Eliminating the bottom stress term from Eqs.(43) and (44), an expression for the eddy viscosity is obtained,

$$\frac{\varepsilon_z}{\overline{\eta} + h} = \frac{\sin\theta D}{6C_a\rho(\overline{v} - v_s)} \tag{45}$$

which relates the eddy viscosity to the the mean longshore current strength. In the following computation we simply assume that mean longshore current strength is proportional to the surface current strength, that is,

$$\overline{v} = \gamma v_s \tag{46}$$

with γ the ratio of mean current to the surface current. Therefore, the eddy viscosity can be simplified as the following explicit expression,

$$\frac{\varepsilon_z}{\eta_c + h} = -\frac{1}{24(1 - \gamma)\beta_L} \frac{|u_{orb}|}{(1 - Cg/(\beta Cg_b))} \frac{\partial}{\partial x} (\cos\theta H)$$
(47)

3.4 Data Comparisons

In this subsection, each theoretical solution is compared with data measured on the plane beach of uniform slope. Wind stress effect is omitted in the solutions.

<u>Undertow Model</u>

The velocity profile is calculated by Eq. (24). Four parameters are to be designated; they are, γ : the ratio of mean current to surface current; β : the dissipation coefficient; β_C : the cross-shore current coefficient; and β_L : the longshore current coefficient.

Figure 3 shows the comparisions of the computed vertical profiles of the crossshore current with those measured by Buhr-Hansen and Svendsen (1984). The test conditions were: slope =1:34.25; $H_o = 0.12m$ and T=1 sec. The parameters used in the computations are: $\beta = 1.17$; $\beta_C=0.07$; $\beta_L=5.0$ and $\gamma=0.982$. The limiting wave height at breaking point is determined by the Miche's criterion with $\kappa=0.78$. Figure 4 plots the profile changes across the untire surf zone using the same parameters as given above. In order to examine the effect of the advection term, the results when the term is neglected are also represented as dotted lines in Figure 5. The effect seems to show the significant deviation from the measurements as it is close to the shoreline under the same input condition. However, the difference also seems to show the overall agreement with the experiments by small reduction of the γ value as shown in Figure 5.

Longshore Current Model

Figure 6 shows the comparasions between computed profiles and the laboratory data measured by Visser (1991). The test conditions were: slope=1:10, $H_o = 9.6cm$, T = 1sec and $\theta_o = 16.4^\circ$. The values of parameters are as follows; $\beta=1.2$, $\beta_L=5.0$, and $\gamma=0.96$. It is seen that γ plays an important role. A maximum value 1 results in a uniform longshore current profile whereas a value 2/3 results in a no-slip bottom velocity. From the comparisons with experimental data, a value near 0.95 is suggested. Figure 7 plots the longshore current profile variations across the surf zone.

Figure 8 illustrates the three-dimensional current profiles inside the surf zone using the same conditions as Figure 5 with the exception that the input wave is oblique at 10° in deep water clockwise to the shoreline normal. The threedimensional current forms a clockwise spiral from top to bottom.

3.5 Model Adoption for General 3-D Tophography

The theoretical models so far developed are for parallel contours. For irregular bathymetries, getting the surface velocity as the surface boundary condition might be ineffective for modelling, so the bottom shear stress in terms of depthaveraged current is considered as the boundary condition instead of the surface velocity. For the prediction of a longshore current this alternative way gives the exactly same result. For that of the undertow, however, this will give the different result. The bottom shear stress suggested by Longuet-Higgins (1970) is now modified for both cross-shore and longshore directions by

$$\overline{\tau_{B,tb}} = \rho F_w |u_{orb}| \gamma \bar{\mathbf{U}} \tag{48}$$

where F_w can be estimated in terms of wave characteristics as

$$F_w = \frac{\beta}{4\gamma\beta_L} \frac{\nabla \cdot (\mathbf{K}H/k)}{(\beta - Cg/Cg_b)} \tag{49}$$

When the bottom shear stress given in Eq. (49) is applied as a boundary condition instead of the surface velocity, three coefficients of the undertow model, C_{x1} , C_{x2}

and u_s , are written by

$$C_{x1} = -\frac{\eta_c + h}{2\varepsilon_z} (P + \overline{\tau_{Bx,tb}})$$
(50)

$$C_{x2} = \frac{\eta_c + h}{\varepsilon_z} P \tag{51}$$

$$u_{s} = \bar{u} - \frac{C_{x1}}{3} - \frac{C_{x2}}{2} \tag{52}$$

The result is shown in Figures 9-10 for the same experimental conditions as used by Buhr-Hansen and Svendsen (1984) given in Section 3.4. The 'S.B.C.' indicates the full theory obtained by the surface boundary condition, and the 'B.B.C.' indicates the approximate theory obtained by the bottom shear stress with neglecting the advection term. The full theory was obtained by $\gamma=0.982$, the approximate theory by $\gamma=0.978$. The comparison with experiments is still in agreement. Therefore, instead of the boundary condition given by surface currents, the bottom shear stress is used in the practical model for the complicated bathymetry, and the advection terms are omitted.

4. QUASI THREE-DIMENSIONAL MODEL

The depth-integrated horizontal model is now combined with the vertical theoretical model to a quasi-3D model. This quasi-3D model looks promising since it provides three-dimensional information at almost the same cost of a two-dimensional horizontal model although it produces the relatively simple variation of vertical profile.

Even for the general tophography, velocity variation with respect to depth may be approximated as the function of parabola of 2nd order.

$$u_c = C_{x1} z'^2 + C_{x2} z' + C_{x3}$$
(53)

$$v_c = C_{y1} z'^2 + C_{y2} z' + C_{y3}$$
(54)

where C_1 , C_2 and C_3 are expressed in terms of wave characteristics and current quantities such as H, $h + \eta_c$, Q(depth integration of velocity vector), and $\overline{\tau_W}$:

$$C_{x1} = -\frac{\eta_c + h}{2\varepsilon_z} \left\{ \frac{g}{16} \frac{\partial H^2}{\partial x} - \frac{\overline{\tau_{Wx}}}{\rho} + \frac{\overline{\tau_{Bx,tb}}}{\rho} \right\}$$
(55)

$$C_{x2} = -\frac{\eta_c + h}{\varepsilon_z} \left\{ -\frac{g}{16} \frac{\partial H^2}{\partial x} + \frac{\overline{\tau_{Wx}}}{\rho} \right\}$$
(56)

$$C_{x3} = \frac{Q_x}{\eta_c + h} - \frac{C_{x1}}{3} - \frac{C_{x2}}{2}$$
(57)

$$C_{y1} = -\frac{\eta_c + h}{2\varepsilon_z} \left\{ \frac{g}{16} \frac{\partial H^2}{\partial y} - \frac{\overline{\tau_{Wy}}}{\rho} + \frac{\overline{\tau_{By,tb}}}{\rho} \right\}$$
(58)

$$C_{y2} = -\frac{\eta_c + h}{\varepsilon_z} \left\{ -\frac{g}{16} \frac{\partial H^2}{\partial y} + \frac{\overline{\tau_{Wy}}}{\rho} \right\}$$
(59)

$$C_{y3} = \frac{Q_y}{\eta_c + h} - \frac{C_{y1}}{3} - \frac{C_{y2}}{2}$$
(60)

Substituting Eqs. (53) and (54) into the convective acceleration terms yields

$$\int_{-h}^{\eta_c} u_c^2 dz = \left[\frac{Q_x^2}{(h+\eta_c)^2} + T_{xx} \right] (h+\eta_c)$$
(61)

$$\int_{-h}^{\eta_c} u_c v_c dz = \left[\frac{Q_x Q_y}{(h+\eta_c)^2} + T_{xy} \right] (h+\eta_c)$$
(62)

$$\int_{-h}^{\eta_c} v_c^2 dz = \left[\frac{Q_y^2}{(h+\eta_c)^2} + T_{yy} \right] (h+\eta_c)$$
(63)

where

$$T_{xx} = \left[\frac{4C_{x1}^2}{45} + \frac{C_{x2}^2}{12} + \frac{C_{x1}C_{x2}}{6}\right]$$
(64)

$$T_{xy} = T_{yx} = \left[\frac{4C_{x1}C_{y1}}{45} + \frac{C_{x2}C_{y2}}{12} + \frac{C_{x1}C_{y2}}{12} + \frac{C_{x2}C_{y1}}{12}\right]$$
(65)

$$T_{yy} = \left[\frac{4C_{y1}^2}{45} + \frac{C_{y2}^2}{12} + \frac{C_{y1}C_{y2}}{6}\right]$$
(66)

Substituting into Eq. (10) leads to the following x-directional modified momentum equation:

$$\frac{\partial Q_x}{\partial t} + \frac{\partial}{\partial x} \left[\frac{Q_x^2}{h + \eta_c} + (h + \eta_c) T_{xx} \right] + \frac{\partial}{\partial y} \left[\frac{Q_x Q_y}{h + \eta_c} + (h + \eta_c) T_{xy} \right] + \frac{1}{\rho} \frac{\partial S_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial S_{yx}}{\partial y} + g(h + \eta_c) \frac{\partial \eta_c}{\partial x} - \frac{\overline{\tau}_{Wx}}{\rho} + \frac{\overline{\tau}_{Bx}}{\rho} = 0$$
(67)

The modified momentum equation of y direction from Eq. (11)

$$\frac{\partial Q_y}{\partial t} + \frac{\partial}{\partial x} \left[\frac{Q_x Q_y}{h + \eta_c} + (h + \eta_c) T_{yx} \right] + \frac{\partial}{\partial y} \left[\frac{Q_y^2}{h + \eta_c} + (h + \eta_c) T_{yy} \right] + \frac{1}{\rho} \frac{\partial S_{xy}}{\partial x} + \frac{1}{\rho} \frac{\partial S_{yy}}{\partial y} + g(h + \eta_c) \frac{\partial \eta_c}{\partial y} - \frac{\overline{\tau}_{Wy}}{\rho} + \frac{\overline{\tau}_{By}}{\rho} = 0$$
(68)

As noted below, the bottom friction consists of turbulent shear stress and bottom frictions due to viscous and streaming flows, which can be expressed as

$$\overline{\tau_B} = F_w |u_{orb}| \overline{\mathbf{U}} + F_c |u_{orb}| (\mathbf{U}_{B,tb} + \mathbf{U}_{strm})$$
(69)

where $\mathbf{U}_{B,tb}$ represents the bottom velocity induced by turbulent flow and \mathbf{U}_{strm} is the streaming velocity in the oscillatory boundary.

The continuity equation results in the same equation as before.

$$\frac{\partial \eta_c}{\partial t} + \frac{\partial}{\partial x}(Q_x + M_x) + \frac{\partial}{\partial y}(Q_y + M_y) = 0$$
(70)

5. CONCLUSION

The surface advective terms were added to the conventional radiation stress by taking Taylor series expansion at the mean water level. The resulting radiation stress was proven by Lee (1993) to be consistent with the wave energy flux in the wave-current coexisting field.

The surface properties obtained from the surf zone model enabled us to develop the theory for the vertical circulation model which had suffered obscurity of boundary conditions. In addition, the friction coefficient and eddy viscosity applicable to the turbulent flow in a surf zone have been estimated in terms of energy dissipation. The developed model yielded the theoretical results comparable with laboratory experiments.

Based on the examination of the theoretical model for vertical profiles of currents in steady state, a quasi-three dimensional circulation model suitable for the entire nearshore zone is developed by linking the depth-integrated properties with the vertical profiles.





Fig. 3. Vertical profiles of cross-shore current.



Slope 1/10

Fig. 6. Vertical profiles of longshore current.



Fig. 4. Comparison with experiments presented by Hansen and Svendsen (1984).







0.5

0.5



and Svendsen (1984).



3-D FLOW SPIRAL



Fig. 9. Vertical profiles of cross-shore current by using bottom shear stress.

REFERENCES

- Buhr-Hansen, J. and Svendsen, I.A. 1984. "A theoretical and experimental study of undertow," Proc. 19th ICCE, ASCE.
- Dally, W.R. 1980. "A numerical model for beach profile evolution," Master's Thesis, Civil Eng., Univ. of Delaware, Dept., Newark.
- De Vriend, H.J., and Stive, M.J.F. 1987. "Quasi-3D modelling of nearshore currents," Coastal Eng., 11: 565-601.
- Dong, P., and Anastasiou, K. 1991. "A numerical model of the vertical distribution of longshore currents on a plane beach," Coastal Eng., 15: 279-298.
- Ebersole, B.A., and Dalrymple, R.A. 1979. "A numerical model for nearshore circulation including convective accelerations and lateral mixing," Ocean Engineering Report No. 21, Dept. of Civil Eng., Univ. of Delaware, Newark, Delaware.
- Horikawa, K., and Kuo, C.T. 1966. "A study of wave transformation inside the surf zone," Proc. 10th ICCE, ASCE, Tokyo, 217-233.
- Lee, J.L. 1993. "Wave-current interaction and quasi-3D modelling in nearshore zone," Ph.D. Dissertation, Coastal and Oceanographic Engineering Department, University of Florida, Gainesville.
- Longuet-Higgins, M.S. 1970. "Longshore currents generated by obliquely incident sea waves, I," J. Geophys. Res., 75(33): 6778-6789.
- Longuet-Higgins, M.S. and Stewart, R.W. 1961. "The changes in amplitude of short gravity waves on steady non- uniform currents, J. Fluid Mech., 10:529-549.
- Nielson, D.M., and Sorensen, T. 1970. "Sand transport phenomena on coasts with bars," Proc. 12th ICCE, ASCE, pp.855-866.
- Noda, E., Sonu, C.J., Rupert, V.C., and Collins, J.I. 1974. "Nearshore circulation under sea breeze conditions and wave-current interactions in the surf zone," Tetra Tech Report TC-149-4.
- Okayasu, A., Shibayama, T., and Horikawa, K. 1988. "Vertical variation of undertow in the surf zone," Proc. 21st ICCE, ASCE, pp.478-491.
- Svendsen, I.A. 1984. "Wave heights and set-up in a surf zone," Coastal Eng., 8: 303-329.
- Svendsen, I.A., and Lorenz, R.S. 1989. "Velocities in combined undertow and longshore currents," Coastal Eng., 13: 55-79.
- Svendsen, I.A., and Putrevu, U. 1990. "Nearshore circulation with 3-D profiles," Proc. 22nd ICCE, ASCE, pp.241-254.
- Visser, P.J. 1991. "Labaratory measurements of uniform longshore currents," Coastal Eng., 15: 563-593.
- Winer, H.S. 1988. "Numerical modeling of wave-induced currents using a parabolic wave equation," Ph.D dissertation, Coastal and Oceanographic Engineering Department, Univ. of Florida, Gainesville.
- Yamashita, T, Tsuchiya, and Suriamihardja, D.A. 1990. "Vertically 2-D nearshore circulation model," Proc. 22nd ICCE, ASCE, pp.150-163.
- Yan, Y. 1987. "Numerical modeling of current and wave interaction on an inlet-beach system," Technical Report No. 73, Coastal and Oceanographic Engineering Department, Univ. of Florida, Gainesville.
- Yoo, D., and O'Connor, B.A. 1986. "Mathematical modeling of wave-induced nearshore circulations," Proc. 20th ICCE, ASCE, pp.1667-1681.