CHAPTER 153

WATERTABLE OVERHEIGHT DUE TO WAVE RUNUP ON A SANDY BEACH

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Abstract

Watertable overheight in beaches due to waves is investigated through laboratory experiments and field tests. Infiltration from wave runup onto the exposed beach creates a significant lifting of the coastal watertable. The infiltration velocity distribution is obtained from both the laboratory and field conditions. The inland overheight for the steady state (regular waves and no tide) is found to be $0.62\tan\beta \sqrt{H_o L_o}$ and that for the unsteady state (irregular waves) is estimated as $0.55\tan\beta \sqrt{H'_{\text{RMS}} L_o}$.

Introduction

The time averaged watertable in beaches stands considerably above the mean sea level due to waves and tidal effects. This is of practical importance for the stability of coastal structures, for the operation of coastal sewage disposal systems and for the accretion/erosion of the beach. The elevated watertable in coastal areas also influences hydrology, e.g., agricultural and soil conservation activities. The main factors for the overheight above the mean sea level are waves and tides as seen in Figure 1 (Nielsen et al. 1988; Aseervatham et al., 1993; Kang and Nielsen, 1994).

This figure shows the watertable variation in wells just landward of the high water mark on two beaches north of Sydney. Both beaches are subject to the same tidal influences, but only Palm beach is exposed to the ocean waves while Pittwater beach is protected from wave action. The data clearly demonstrates that the wave activity is a significant factor affecting the coastal overheight.
water table elevation. In this case, the waves raise the water table by up to 0.7m compared to the protected beach. The overheight inside the protected beach is due to the tide acting on a sloping beach face as explained by Nielsen (1990).

![Figure 1. Field data showing the significant lifting of the water table due to waves and tides.](image)

This paper describes the infiltration of water due to wave runup which creates a significant lifting of the coastal water table. The steady state asymptotic inland overheights are investigated under controlled wave conditions with different beach sands in a wave flume without tides. This overheight is compared with the field data.

**Infiltration from wave runup**

The effect of infiltration due to wave runup is clearly visible from the measured water table profiles shown in Figure 2. The water table profiles exhibit humps due to wave runup infiltration between the shoreline and the runup limit. These humps are particularly noticeable during the rising tide.

To model the water table in the area between the shoreline and the runup limit, a modified Boussinesq equation (1) which includes the runup infiltration effect must be used.
Figure 2. Watertable profiles measured on the rising tide at Kings Beach, Queensland, Australia.

\[ \frac{\partial \eta}{\partial t} = KD \frac{\partial^2 \eta}{\partial x^2} + \frac{U_f(x,t)}{n} \]  

where \( \eta \) denotes the watertable height averaged over a few surf beats, \( K \) is the hydraulic conductivity, \( D \) the aquifer depth, \( n \) the specific yield and \( U_f \) the infiltration velocity. For pure regular waves forcing without tidal effects, the situation may be considered quasi-stationary. Thus equation (1) becomes

\[ \frac{\partial^2 \eta}{\partial x^2} = -\frac{U_f(x)}{KD} \]  

This steady situation is achieved experimentally in a wave flume.

From these equations, \( U_f \) can be derived from measured watertable profiles. Two examples are shown in Figure 3. This Figure represents the smoothed relative infiltration velocity distributions against the relative shorenormal distance from the field and laboratory data. Infiltration velocity \( U_f \) is obtained by the Finite Difference Method. The normalised infiltration velocity has a maximum approximately 2/3 of distance from the shoreline to the runup.
limit for the field data and halfway between the shoreline and the runup limit for the laboratory data. Both the distributions show similar trends, but the magnitudes are different. This difference is possibly due to the assumed value of \( n=0.3 \) (for field) being too large, and it seems because field is unsteady with waves running onto dry sand or \( K \) is not correctly determined.

**Figure 3.** Relative infiltration velocity as a function of non-dimensional shorenormal distance. These infiltration velocity values were obtained through equation (1) for the field data (unsteady state) and equation (2) for the laboratory data (steady state).
The general nature of $U_1$ may be expressed by

$$U_1(x) = \begin{cases} C_1 \frac{K}{n} f(x) & \text{for } x_s \leq x \leq x_R \\ 0 & \text{for } x > x_R \end{cases}$$  \quad (3)$$

where $f(x)$ denotes the function of shorenormal distance $x$, $x_s$ and $x_R$ the horizontal coordinate of the shoreline and the runup limit respectively and $C_1$ a dimensionless infiltration coefficient.

Substituting this expression of $U_1$ into the equation (2), the watertable overheight $\eta$ will be independent of $K$ and thus independent of sand size. This fact is verified by the laboratory data in Figure 4.

**Steady state inland overheight due to waves**

Flume experiments were carried out to investigate the watertable response due to wave runup without tidal effects. Sands of two different sizes ($d_{50}=0.18\text{mm}$ and $0.78\text{mm}$) were used. The relative steady state inland overheight is plotted against the relative runup height in Figure 4.

![Figure 4. The relative inland overheight against the relative runup height in a steady state.](image-url)
From the regression analysis for both sand sizes, the steady state inland overheight above the still water level \((\eta_{in}-SWL)\) is obtained as

\[
\eta_{in}-SWL = 0.62(Z_{R}-SWL)
\]  

(4)

where \(Z_{R}-SWL\) is the runup height above the still water level. As mentioned earlier, the laboratory data in Figure 4 verify that the watertable overheight is independent of sand size and thus independent of the hydraulic conductivity \(K\). In analogy with Hunt's (1959) formula for the runup of regular waves, equation (4) can then be written as

\[
\eta_{in}-SWL = 0.62\tan\beta\sqrt{H_{o}L_{o}}
\]  

(5)

where \(\tan\beta\) is the beachface slope. The data in Figure 4 represent deep water wave heights ranging from 60mm and 180mm, periods ranging from 1.5sec to 2.9sec and aquifer depths ranging from 370mm to 440mm.

**Runup distributions for irregular waves**

It is natural to expect that the \(U_{1}\)-distribution is closely related to the runup distribution. In general the runup distribution has been found to be of the Rayleigh type. That is

\[
P(Z_{R}>Z) = \exp\left(-\frac{(Z-Z_{100})^{2}}{L_{R}^{2}}\right)
\]  

(6)

where \(Z_{100}\) is the highest level transgressed by 100% of the waves during the recording interval, \(L_{R}\) is the vertical runup scale of the distribution, see e.g. Nielsen and Hanslow (1991). Furthermore Saville (1962) and Battjes (1971) suggested that individual runup percentiles \(R_{n}\) correspond to the same wave height percentile through 'the principle of equivalency' which is based on Hunt's (1959) formula for the regular wave runup

\[
R_{n} = Z_{n}-SWL = \tan\beta\sqrt{H_{o}L_{o}}
\]  

(7)

where \(Z_{n}\) is the level transgressed by \(n\) percent of waves and \(H_{o}\) the height exceeded by \(n\) percent of the deep water waves. The vertical runup scale \(L_{R}\) for irregular waves, which is RMS runup according to the Rayleigh distribution, can then be estimated in terms of \(H_{oRMS}\) value by

\[
L_{R} = R_{RMS} = \tan\beta\sqrt{H_{oRMS}L_{o}}
\]  

(8)
where $H_{o RMS}$ is the deep water root mean square wave height.

Relative runup height for regular waves and relative vertical runup scale for irregular waves were plotted together against beachface slope $\tan \beta$ in Figure 5. This comparison is due to the concept that the runup length scale $L_R$ for irregular waves plays a somewhat similar role to that of $Z_R$ for regular waves. Both the field and laboratory runup data show the increasing function of the beachface slope.

Overheight for irregular waves

The $U_T$-distribution due to irregular waves will be different from that due to regular waves. The reason is that while all of the regular waves run up to the same level, the runup limit, the individual waves in a train of irregular waves all reach different levels.

Field data of the watertable overheight due to waves are plotted in Figure 6. It is clearly seen that the overheight is an increasing function of the parameter $\tan \beta \sqrt{H_{o RMS} L_0}$ which is the runup height of the RMS wave according to Hunt's formula (1959). This correlation corresponds to the overheight due to regular waves being proportional to $\sqrt{H_{o RMS} L_0} \tan \beta$, see Figure 4. The lower group of data in the Figure 6 corresponds to a period when the exposed beach (Palm beach)
watertable does not increase much even though the RMS wave height is increasing. This is mainly due to refraction.

Figure 6. Watertable overheights due to waves: The differences in watertable levels between the exposed Palm Beach and the protected Pittwater Beach.

Figure 7. Watertable overheights due to waves: The differences in watertable levels between the exposed Palm Beach and the protected Pittwater Beach. The same data are used as shown in Figure 6, but refraction is accounted for by using $H'_{oRMS}$ instead of $H_{oRMS}$. 
After including the effect of refraction, the same data as shown in Figure 6 are plotted in Figure 7. That is, the equivalent perpendicular deep water RMS wave height $H'_{oRMS}$ is used

$$H'_{oRMS} = K H_{oRMS} = \sqrt{\cos \alpha_o} H_{oRMS}$$

(9)

where $K_r$ is the refraction coefficient and $\alpha_o$ wave angle between the deep water wave crest and the shoreline. It is seen that the inclusion of refraction improves the correlation.

Figure 7 shows that the watertable overheight due to waves $\eta_w$ is well correlated with $\tan \beta \sqrt{H'_{oRMS} L_o}$. Linear regression gives

$$\eta_w = 0.55 \sqrt{H'_{oRMS} L_o} \tan \beta$$

(10)

and with the vertical scale $L_R$ of the runup distribution given by

$$L_R = 0.80 \tan \beta \sqrt{H_{oRMS} L_o}$$

(11)

based on the data from the present study and Nielsen & Hanslow (1991), this gives ($\alpha_o$ was not known for all data sets)

$$\eta_w = 0.69 L_R$$

(12)

which corresponds qualitatively to the regular wave result, see Figure 5.

**Conclusions**

1. There is a significant lifting of the coastal watertable due to the infiltration from wave runup.

2. The infiltration velocity from the field and laboratory data has a maximum value roughly midway between the shoreline and the runup limit.

3. For regular waves and no tide, steady state inland watertable overheight ($\eta_w - SWL$) was found to be $0.62 \tan \beta \sqrt{H_{oRMS} L_o}$ for both coarse and fine sand sizes and thus independent of the hydraulic conductivity $K$. 
4. For irregular waves with tides, the inland overheight due to waves $\eta_w$ is estimated by $0.55 \tan \beta \sqrt{H'_{\text{rms}}} L_o$ and the relation $\eta_w = 0.69 L_R$ was found.

5. Use of the equivalent perpendicular deep water RMS wave height $H'_{\text{rms}}$ gives better estimation of watertable overheight.

References


