CHAPTER 141

A NONLINEAR SURF BEAT MODEL

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Abstract

A nonlinear, short-wave-averaged (surf beat) model is presented. The model is based on that of Roelvink (1993), but the numerical techniques used in the solution are based on the so-called weighted-averaged flux (WAF) method (eg Watson et al, 1992), with time-operator splitting used for the treatment of some of the source terms. This method allows a small number of computational points to be used, and is particularly efficient in modelling breaking long waves. The short-wave (or primary-wave) energy equation is solved using a more traditional Lax-Wendroff technique. Results of validation indicate that the model performs satisfactorily in most respects.

Introduction

There are two classical characteristic time-scales in a system of waves: that of the individual waves and that of the wave groups. On the time scale of the wave group the short-wave averaged momentum flux ("radiation stress") and mass flux ("wave-induced mass flux") vary slowly. This variation in time and space of radiation stress and mass flux generates long waves with periods and wavelength similar to the group periods and lengths. These long waves may travel with the wave groups or they may be released as free waves if the wave groups forcing them change rapidly, e.g. due to breaking in the surf zone. The free waves either escape to deep water ("leaky modes") or are trapped at the shoreline by refraction as "edge waves". These long waves are often collectively called infragravity waves or "surf beat".

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In the recent decades numerous observations have shown that the energy at surf beat frequencies can be substantial, and in some cases even exceed that of the short waves (Wright et al, 1982). Likewise, the shoreline amplitudes arising from infragravity waves can be comparable to the run-up height of short waves (Guza and Thornton, 1982; 1985). The importance of low-frequency motion for the nearshore dynamics can be inferred from the fact that natural sedimentary coasts often exhibit morphological features (bars and cusps) with length scales considerably in excess of those of the wind waves. This is found to be particularly pronounced in very shallow water. The effect of surf beat on morphology was found to increase offshore transport and to move bars in a seaward direction while reducing their amplitude (Roelvink, 1993). Recent work done at HR Wallingford (Bowers, 1992) highlighted the need for accurate numerical modelling of surf beat: physical model experiments using realistic short crested waves demonstrated the importance of surf beat as well as incident set-down in causing long waves inside harbours. This surf beat is produced by groups of waves breaking on the shoreline surrounding the harbour. Due to the large length of shoreline needed for realistic modelling of the generation of surf beat in a physical model, an efficient mathematical model of surf beat generation would be far more cost effective than extensive additional coastline moulding in a physical model together with additional lengths of wave-maker. The work in this report has been undertaken as a first step in meeting this need in that only cross-shore wave modes are considered.

In the early sixties Longuet-Higgins and Stewart developed the concept of "radiation stress", by which they explained how groups of high waves are accompanied by a depression of the mean water surface. In other words, groups of short waves force a long wave, which is known as the set-down or the bound long wave. This concept successfully explained many of the early observations of surf beat, and forms the basis of many subsequent surf beat models.

In one of the earliest such models, Symonds et al. (1982) assume that within the inner surf zone, the short waves are "saturated", meaning that the variations on wave group scale have vanished and the radiation stress gradients are constant in time. Outside the surf zone, they assume that the horizontal variation of radiation stress is negligible (and thereby do not include the effect of bound waves). In the transition region, the breaking-point moves back and forth; in this region there is a radiation stress gradient varying in time. This gradient acts as a local forcing, comparable to a wave maker which generates waves both in the onshore direction and (with opposite sign) in the offshore direction. The onshore directed wave is subsequently reflected off the beach and interferes with the offshore directed wave. Depending on the dimensionless width of the surf zone, the relative phase of the two free outgoing wave components change, resulting in an enhancing or damping of the total free wave radiated from the surf zone. The fact that such an amplitude variation with the dimensionless surf zone width exists was confirmed in laboratory experiments by Kostense(1984); however, the

quantitative agreement between the model and experiments was not entirely convincing.

Schäffer and Svendsen (1988) improved this model concept by including the forcing outside the surf zone responsible for the bound long waves. In their model this forcing is reduced in the surf zone but does not vanish completely since they relax the rigid assumption of a saturated inner surf zone. Schäffer and Jonsson (1990) compared this model with Kostense's (1984) data and found considerably better agreement; remaining discrepancies can be ascribed to the lack of bottom friction and the use of linearized equations in their model.

Roelvink (1993) developed a nonlinear surf beat model based on short-wave averaged mass and momentum conservation equations (Phillips 1977). To get the radiation stress, a mean short wave energy transportation equation was solved simultaneously, which included the effect of variation of mean water level on the energy evolution. Numerical results for incident bichromatic waves agreed with the Kostense (1984) data to different degrees depending on the bottom friction coefficient used. For incident irregular waves, a narrow band assumption was used to overcome the difficulty of determining the group velocity of irregular waves, which is needed in the calculation of radiation stress. Comparisons of numerical results with Van Leeuwen's (1992) (random) experimental data showed good agreement. This model was then applied to the study of the effect of surf beat on cross-shore beach morphology.

The numerical models mentioned above are all based on the 'wave-averaged' approach, i.e. averaging the mass and momentum conservation equations over a short wave period and using the concept of radiation stress to express the short-wave momentum flux. The disadvantage of this approach is that questionable assumptions are made about the validity of linear theory for the propagation of breaking waves within the surf zone. An alternative to this is to use short-wave-resolving models to study the full wave motion, including the generation processes of low frequency wave. This is usually done using either the nonlinear shallow water wave equations (in the inner surf zone) or the Boussinesq equations (or both). Whilst being more satisfactory from a theoretical point of view, this approach obviously involves considerably more computational time and expense than the wave-averaged approach. In this work we therefore chose to follow the wave-averaged route.

Equations of motion

The shallow-water wave equations for the conservation of mass and momentum (in 1-D) can be written (in so-called conservation form, and in the absence of bottom friction) as

$$d_t + (du)_x = 0 \tag{1}$$

$$(du)_t + (u^2 d + \frac{1}{2} g d^2)_x = g d h_x \tag{2}$$

where u is the depth-averaged fluid velocity, d is the total water depth, g the gravitational acceleration constant, and h the still water depth (x is the offshore coordinate, and $\zeta = d\text{-}h$ is the free surface elevation). The term on the right, arising from the bottom slope w.r.t. x, is referred to as a source term. These equations resolve the primary wave motion. In order to exclude this type of motion, the original equations must be time as well as depth averaged. This results in a similar continuity equation (see below), but additional terms appear in the momentum equation. To treat these properly we start from the momentum equation of Phillips (1977) describing time and depth-averaged 1-D motion,

$$\frac{\partial}{\partial t}\overline{\int_{-h}^{\zeta} \rho u \, dz} + \frac{\partial}{\partial x}\overline{\int_{-h}^{\zeta} (\rho u^2 + p) \, dz} - p_b \overline{\frac{d}{dx} h} + \overline{\tau_b} = 0 \tag{3}$$

where ρ is water density, and p, p_b are pressure and pressure at the bottom respectively, and an overbar denotes an average over a short wave period. The following decomposition is made:

$$u = U + u' \tag{4}$$

where U is the long wave velocity and u' is the fluctuating component (due to the primary waves). We define

$$d = h + \zeta \tag{5}$$

so that d now denotes a mean total depth. Following Roelvink (1993) we define fluxes

$$Q_t = \overline{\int_{-h}^{\zeta} u \, dz} \tag{6}$$

and

$$Q_w = \overline{\int_{-h}^{\zeta} u' \, dz} \tag{7}$$

We introduce the velocity V such that

$$V = \frac{Q_t}{d} = U + \frac{Q_w}{d} \tag{8}$$

and after some manipulations, and using the shallow water assumption, the final continuity and momentum equations are arrived at:

$$\frac{\partial}{\partial t}d + \frac{\partial}{\partial x}(Vd) = 0 \tag{9}$$

$$\frac{\partial}{\partial t}(Vd) + \frac{\partial}{\partial x}\left[dV^2 + \frac{1}{2}gd^2 \right] = gd\frac{\partial h}{\partial x} - \frac{\partial}{\partial x}\left[\frac{S_{xx}}{\rho}\right] + \frac{\partial}{\partial x}\left[\frac{Q_w}{d}\right]^2 - \frac{\overline{\tau_b}}{\rho} \tag{10}$$

It can be seen that these equations have a similar form on the left hand side to (1) and (2). We use this correspondence to apply similar flux-conservative techniques to the solution of the wave-averaged equations (see below). The final equation to consider is the equation describing the transformation of short wave energy. It is the variation in this energy that will drive the incoming bound waves. Assuming (Roelvink, 1993) that the characteristic short wave frequency is constant in space and time we get an energy equation of the form

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}(c_g E) = -D \tag{11}$$

In the model, the radiation stress, S_{xx}, and the primary-wave-induced flux, Q_w, are represented by the expressions of Longuet-Higgins & Stewart (1964); bottom shear stress is represented in the usual quadratic form. D, representing the short wave decay due to breaking, is formulated using both the method of Roelvink (1993) and of Battjes & Janssen (1978). In the runs performed so far there has been little difference between the two methods. In view of this we chose Roelvink's method,

$$D\ (\ E\ ,\ d\ ) = \{\ 1\ -\ \exp\left[\ -\ \left(\ \frac{H}{\gamma d}\ \right)^n\ \right]\ \}\ 2\ \alpha f_p E \tag{12}$$

it being rather simpler. Here, following Roelvink we take α =1.0 and n = 10. The parameter γ will vary depending on the type of wave being studied. Its value is given in each case studied. f_p is the characteristic measure of the (constant) frequency. For waves other than monochromatic waves we take it to be an average of the constituent frequencies. Using linear theory it can be shown that

$$H = \sqrt{8E/(\rho g)} \tag{13}$$

so the system is now closed.

Numerical methods

In the WAF method applied to the shallow-water wave equations the equations are solved in so-called flux-conservative form (eg (1) and (2)). Both the original shallow water wave equations and the surf beat model equations can be written in the same flux-conservative vector formulation

$$\underline{U}_t + \underline{F}_x = \underline{S} \tag{14}$$

where $\underline{U}$ represents the conserved quantities, and $\underline{F}$ represents the flux of these quantities. The term $\underline{S}$ is the so-called source term, and it is this term that differs depending on whether the equations are just depth averaged or both depth and time averaged. This vector equation is solved in finite-difference form, using fluxes at intermediate locations:

$$\underline{U}_i^{n+1} = \underline{U}_i^n + \frac{\Delta t}{\Delta x}\left(\underline{F}_{i-1/2} - \underline{F}_{i+1/2} \right) + \underline{S}_i \, \Delta t \tag{15}$$

where $t = n\Delta t$ is the initial time, and $t = (n+1)\Delta t$ is the time at which we want to find a solution. The flux, which is found by solution of the local Riemann problem at each cell, is then averaged over the cell width at the half time level, and a total-variation-diminishing adjustment is made by means of upwinding, in order to reduce spurious oscillations. The slope term can cause problems with accuracy, but these are circumvented by making an appropriate transformation, thus allowing it to be incorporated into the local Riemann problem solution (see Watson et al. (1992) for details). The boundary condition at the seaward end allows waves to propagate out of the solution domain without reflection, and the shoreline is defined as the position at which the depth decreases below a specified amount.

For the surf beat problem, we can decompose the source term into the slope term and an additional term

$$S^* = -\frac{\partial}{\partial x}\left(\frac{Sxx}{\rho} \right) + \frac{\partial}{\partial x}\left(\frac{Qw^2}{d} \right) - \frac{\overline{\tau}_b}{\rho} \tag{16}$$

The problem caused by this additional term can be solved by applying the time-operator splitting (TOS) method. Firstly we solve the following equations to get a intermediate solution d* and V*:

$$\frac{\partial}{\partial t} d^* + \frac{\partial}{\partial x}\left(V^* d^* \right) = o \tag{17}$$

$$\frac{\partial}{\partial t}\left(V^* d^* \right) + \frac{\partial}{\partial x}\left[V^{*2} d^* + \tfrac{1}{2} g d^{*2} \right] = g d^* hx \tag{18}$$

These equations have the same form as (1) and (2), so they can be solved using the same method for solving those equations, including the treatment of seaward and shoreline boundary conditions. Secondly we solve the equations:

$$d_t = 0 \tag{19}$$

$$V_t = S^* \tag{20}$$

starting with the intermediate solutions. This is the solution to equations (9) and (10) given by the TOS method. The accuracy of this solution is first order (Watson et al 1992), which is lower than that of equations (1) and (2). This decrease of accuracy due to TOS method was remedied by using a transformation similar to that of Watson et al (1992), but differing by the quantity S^*. Thus, at each time step and for the difference calculation at each grid point we transform the problem into a reference frame that is accelerating at the rate

$$a^* = -gh_x + S^* \tag{21}$$

In this frame the new variables are

$$\xi = x - \frac{a^*}{2}t \tag{22}$$

$$\tau = t \tag{23}$$

$$W = V - a^*t \tag{24}$$

$$D = d \tag{25}$$

Substitution of (22)-(25) into (9) and (10) gives

$$D_\xi + (WD)_\xi = 0 \tag{26}$$

$$W_\xi + WW_\xi + gD_\xi = 0 \tag{27}$$

Once the solution of the (26) and (27) has been found (using the same method as for (1) and (2)), the solution of (9) and (10) can be effected by using relations (22)-(25). The solutions so obtained,

$$d(x,t) = D\left(x - \frac{a^*}{2}\Delta t^2 , t \right) \tag{28}$$

$$V(x,t) = W\left(x - \frac{a^*}{2}\Delta t^2, t \right) + a^*\Delta t \tag{29}$$

provide the basis for the modification of the TOS method by inclusion of the second order terms. Thus, noting that

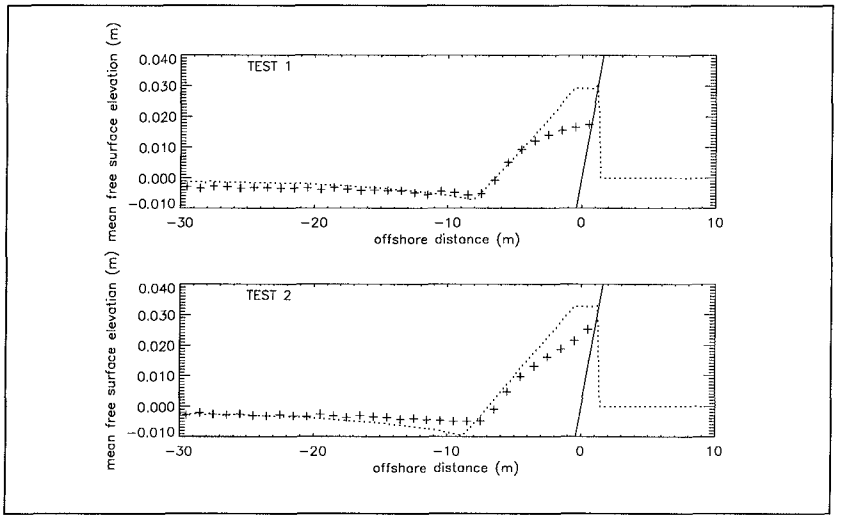

Figure 1. Mean surface elevation from model and experiments (Stive, 1983). Crosses from experiments. (a) Test 1 (b) Test 2.

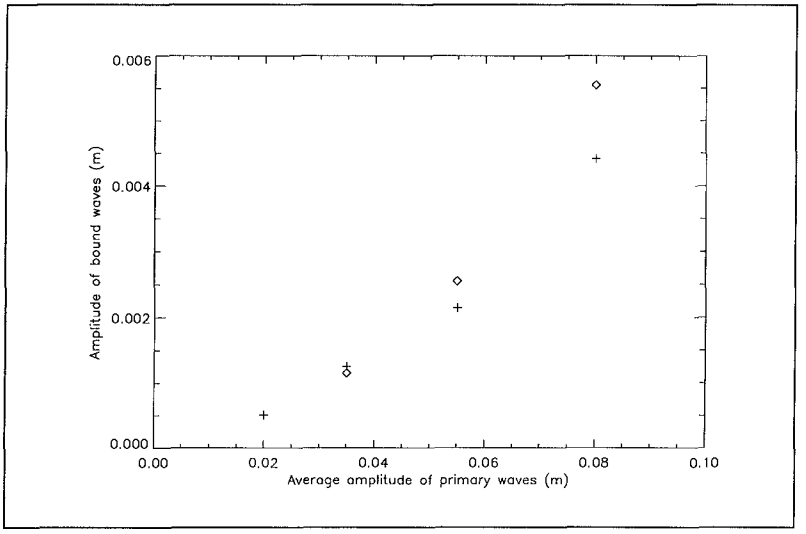

Figure 2. Amplitude of bound waves vs primary wave amplitude. Diamonds from experiments (Kostense, 1984). Crosses from model.

$$d^*(x,t) = D\left(x + \frac{1}{2}gh_x\Delta t^2,\ t\right) \tag{30}$$

$$V^*(x,t) = W\left(x + \frac{1}{2}gh_x\Delta t^2,\ t\right) - gh_x\Delta t^2 \tag{31}$$

the final solutions of the surf beat model are

$$d\ (x,\ (n+1)\ \Delta t) = d^*(x,\ (n+1)\ \Delta t) - \frac{\partial d^*}{\partial x}\ (\tfrac{1}{2}\ S^*\ \Delta t^2) \tag{32}$$

$$V\ (x,\ (n+1)\ \Delta t) = V^*\ (x,\ (n+1)\ \Delta t) - \frac{\partial V^*}{\partial x}\ (\tfrac{1}{2}\ S^*\Delta t^2) + S^*\Delta t \tag{33}$$

During the numerical calculation discussed above, the short wave energy equation (11) should be solved simultaneously to give the value of energy E for the calculation of radiation stress S_{xx} and short wave mean flux Q_w. A one-step second-order Lax-Wendroff difference scheme was used to solve the equation.

Results

The surf beat model has so far been validated only by comparison with monochromatic (Stive, 1983) and bichromatic wave tests (Kostense, 1984). Both of these tests were performed on plane beaches.

Two tests were examined in Stive's data, corresponding to spilling and plunging breaker types: Test 1 (spilling; wave height = 0.145m, period = 1.79s), and Test 2 (plunging; wave height = 0.145m, period = 3.00s). The flume consisted of a flat bed section of depth 0.70m (continuing for a length of 10m) and then a non-erodible constant slope of 1:40. In Figure 1(a) and 1(b) the mean free surface elevation in the two cases is compared with the model result. It can be seen in Fig. 1(a) that in the first case modelling is generally good everywhere other than at the shoreline, where the mean shoreline is clearly overpredicted (this is equally true of Test 2). To obtain accurate modelling of the region of the breaker point

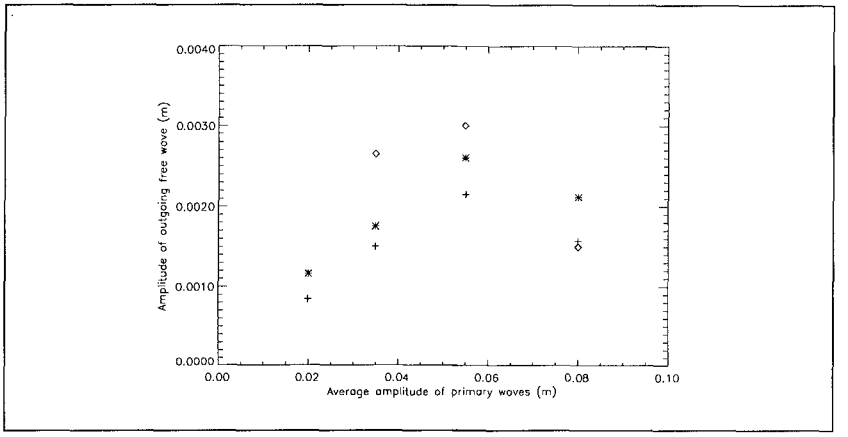

Figure 3. Amplitude of free waves vs primary wave amplitude. Diamonds = experiments (Kostense, 1984). Asterisks = model (f_w=0.01). Crosses = model (f_w=0.05).

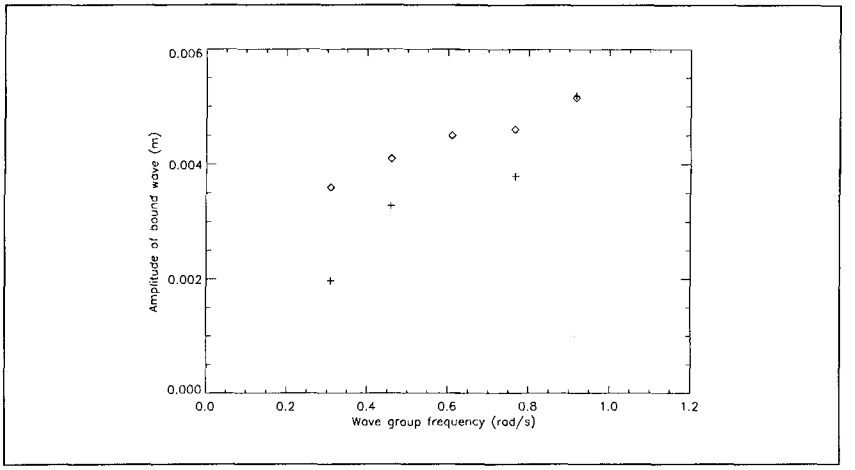

Figure 4. Amplitude of bound waves vs group frequency. Diamonds = experiment (Kostense, 1984). Crosses = model.

in Test 1, a value of $\gamma = 0.88$ was used. The same value gave the best fit for Test 2, but in this case modelling clearly is not as good. It is not clear why modelling in this region should be inferior for Test 2. The model of Thornton & Guza (1983) of wave height decay, on which the model of Roelvink is based, works well for plunging as well as spilling breakers.

In the tests of Kostense (1984), the primary waves were made up of two frequencies generated in a water depth of 0.50m which broke on a plane cemented beach of a 1:20 slope after travelling over a horizontal stretch. Figures 2 and 3 are, respectively, the variation of the bound wave amplitude and the reflected free wave (out-going) amplitude vs the primary wave amplitude ζ_1 with $\zeta_2/\zeta_1 = 0.2$. and $\Delta\omega = 0.772$, which corresponds to series C in Kostense's experiments. Figures. 4 and 5 are, respectively, the variation of the bound wave amplitude and the reflected free wave (out-going) amplitude vs the wave group frequency $\Delta\omega$ with $\zeta_2/\zeta_1 = 0.2$ ($\zeta_1 = 0.055$m and $\zeta_2 = 0.011$m), which corresponds to series A in Kostense's experiments ($\gamma = 0.88$). The procedure of the calculation to simulate the experiments is as follows. For a given set of primary waves, the model is run until transients are no longer present in the solution domain. The surface elevation time series are then split into three components, viz. the incoming bound wave, the reflected free wave and an incoming free wave. The incoming free wave is negligible since it is not generated and the seaward boundary condition allows the reflected free wave to propagate out of the model area. The amplitudes of the incoming bound wave and the reflected free wave are determined by harmonic analysis as done in Kostense's experiments. Although more points are required for a more conclusive comparison to be made, Fig. 2 seems to show the bound wave amplitude increasing approximately quadratically, as expected. In fact, apart from one point the position of which seems slightly anomalous, results are very similar to those of Roelvink, as we would expect. The comparison with experiment is quite good. In Fig. 3 we perform a similar kind of comparison to that of Roelvink, by using two different values of bottom friction coefficient, $f_w = 0.05$ and 0.01. A similar trend is found with regard to the dependence of the amplitude of the outgoing free wave on the average amplitude of the primary waves. The comparison is not as good as for the incoming bound wave. This is not surprising since the reflected free wave will only emerge after the processes of short wave breaking and run-up have taken place and we only use linear theory to describe highly nonlinear processes in these regions. Nevertheless, the agreement is good in parts. The agreement in Fig. 4 is reasonably good (dependence of the bound wave on the group frequency), although it appears to deteriorate for lower values of the wave group frequency. The agreement in Fig. 5 is less than satisfactory at present. The experimental results clearly show a peak in the response at around 0.6 rad/s. More analysis needs to be done here if the interference patterns of Roelvink (1993) are to be reproduced.

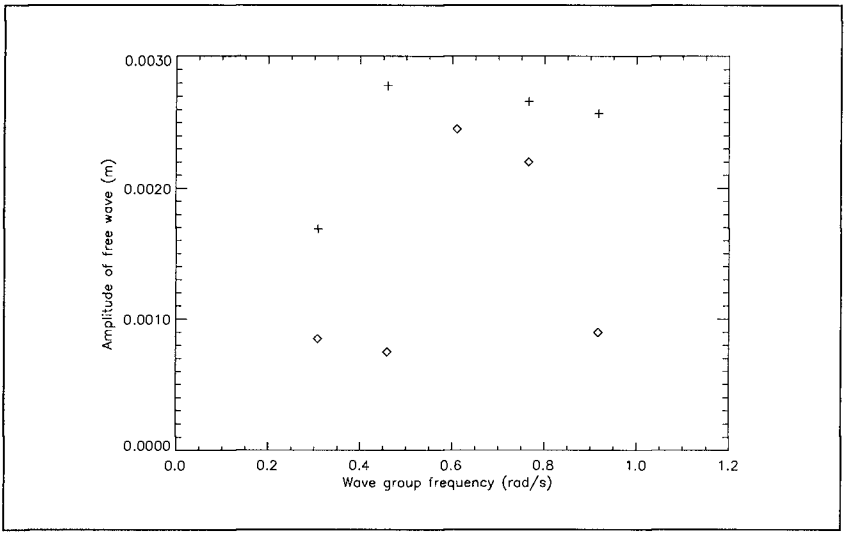

Figure 5. Amplitude of free waves vs group frequency. Diamonds = experiments (Kostense, 1984). Crosses = model.

Conclusions

A nonlinear surf beat model based on the same approach as that of Roelvink (1993) has been presented. It uses a novel (for this application) numerical scheme for its solution, which is highly efficient, especially for the case when long waves break. The comparisons done so far indicate that it performs reasonably well, but further validation is required, especially for random waves. The advantage of a nonlinear model over a linear surf beat model is that we do not have to identify beforehand the mechanism by which the free waves are generated, as is the case for linear models (eg, Symonds et al, 1982). Certain parameters are still present in the model however (unavoidably so), some of which must be calibrated for different types of applications.

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