CHAPTER 136

The Role of Rollers in Surf Zone Currents

William R. Dally¹, M. ASCE, and Daniel A. Osiecki²

Abstract

Employing a recently developed model for the creation and evolution of the aerated region of breaking waves, the relative importance of the roller in driving and mixing cross-shore and longshore currents is explored. Modeling results using linear wave theory confirm that in the mean balances of mass, momentum, and energy, the roller plays a role comparable to (and sometimes greater than) the underlying organized wave motion. It also appears that the roller is responsible for the landward shift of the peak cross-shore and longshore current observed in laboratory and field measurements, and is as important as the net convective acceleration in cross-shore mixing of the longshore current.

Introduction and Background

In the investigation and modeling of nearshore circulation, it has long been suspected that a significant role is played by the aerated region of the breaking wave (see e.g. Svendsen, 1984). However, a lack of understanding and modeling capabilities of the "roller" itself has thus far stymied attempts to clarify its importance in the mean mass, momentum, and energy balances of the surf zone.

¹Associate Professor, Ocean Engineering Program, Florida Institute of Technology, 150 W. University Blvd., Melbourne, FL 32901, U.S.A

²Graduate Research Assistant, Ocean Engineering Program, Florida Institute of Technology, 150 W. University Blvd., Melbourne, FL 32901, U.S.A

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A model that describes the growth, evolution, and decay of the breaking wave roller has recently been developed by Brown (1993) and Dally and Brown (1995). This model, annotated below, calculates the time-averaged cross-sectional area of the roller (essentially its mass flux) as a function of position in the surf zone. The roller model has a single fitting coefficient, which has been calibrated by including roller terms in the equations for the mean mass and momentum balances, and then tuning comparisons to data for cross-shore currents from a single wave channel experiment. With the calibration coefficient held fixed, additional comparisons to set-up and undertow data from other experiments reported in the literature are very favorable. Of particular note is a greatly improved ability to model the cross-shore structure of set-up and currents in the transition region of the outer surf zone.

The purpose of the present investigation is, by employing the calibrated and verified roller model, to examine in detail the role played by the rollers in driving set-up and cross-shore and longshore currents. Of specific interest is the magnitude of the roller mass and momentum fluxes relative to the familiar expressions for Stokes Drift and Radiation Stress components, as derived from linear wave theory.

Overview of the Roller Model

Based on a depth-integrated, period-averaged energy balance, Dally and Brown (1995) propose a governing equation for the creation and evolution of the aerated region of a breaking wave that is normally incident to the beach. Generalizing this model for the obliquely incident situation, but assuming longshore uniformity, yields

\[ \frac{\partial}{\partial x} \left( F_w \cos \alpha \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho_r c^2 \cos^2 \alpha \frac{A}{T} \right) = -\rho_r g \beta_p \frac{A}{T} \]  

(1)

where \( x \) is the cross-shore coordinate (directed onshore), \( F_w \) is the period-averaged energy flux associated with the organized wave motion, \( \alpha \) is the local wave angle relative to shore-normal, \( \rho_r \) is the mass density of the roller (including air), \( A \) is the cross-sectional area of the roller (including air), \( T \) is the wave period, \( g \) is gravity, and \( \beta_p \) is a dissipation coefficient related to the angle of inclination of the roller as it rides the face of the wave. \( \beta_p \) is the primary calibration factor for the model, for which a value of 0.1 has been established by Brown (1993).

In Dally and Brown (1995) it is noted that the dependent variable in Eq. (1) is essentially the period-averaged mass flux in the roller \( \rho_r A/T \). This obligingly
circumvents the need to specify/model the mass density of the aerated roller $\rho_r(x)$. Consequently, the volume flux of water in the roller in the direction of wave propagation, $Q_r$, and the momentum flux in the roller, $M$, are given by

$$Q_r = \frac{\rho_r A}{\rho T} \quad (2)$$

$$M = \rho_r c \frac{A}{T} = \rho Q_r c \quad (3)$$

Snell's Law is used to provide the local wave angle, and the boundary condition that $A=0$ at the breaker line is adopted. Using linear wave theory and the wave height decay model of Dally, Dean, and Dalrymple (1985) to compute $F$ and $c$, Figures 1 and 2 present results for the evolution of the volume and momentum fluxes. These quantities are nondimensionalized by their maximum values, which are found well landward of the point of incipient breaking, indicating the end of the transition region.

**Two-Layer Governing Equations for Cross/Longshore Currents**

Laboratory measurements of cross-shore flows (e.g. Nadaoka and Kondoh, 1982), and field measurements of longshore flows (e.g. Rodriguez, et al., 1994) clearly indicate that strong vertical gradients in mean discharge exist at the wave trough level. The cross-shore discharge switches from onshore-directed flow above the trough level (i.e. the Stokes Drift and roller mass flux), to offshore-directed below the trough level (i.e. the undertow) as required due to the presence of the shoreline. Although the longshore discharge does not change directions, it decays from a nearly depth-uniform current below the trough (see Visser, 1991) to zero at the wave crest level. That is, it appears that the most salient vertical structure in the mean horizontal discharge can be represented by splitting the flow in the vicinity of the wave trough level.

A complete set of governing equations for a two-layer (2D-H) current field, which includes terms associated with the roller, has been developed by Dally (1994). For the situation of longshore uniformity and steady currents, the period-averaged, depth-averaged conservation of mass reduces to

$$\frac{\partial}{\partial x} [U(h+\eta)] + \frac{\partial Q_{wx}}{\partial x} + \frac{\partial Q_{tx}}{\partial x} = 0 \quad (4)$$

in which $U$ is the depth-averaged cross-shore current, $h$ is
Figure 1 - Dimensionless volume ($Q_r$) and momentum fluxes ($M$) in the roller from numerical solution of Eq.(1), for a breaker angle of $20^\circ$.

Figure 2 - Same as Figure 1. Breaker angle of $0^\circ$. 
the still water depth, $\bar{\eta}$ is the mean water elevation, and $Q_{wx}$ and $Q_{rx}$ are the mean volume fluxes associated with the organized wave motion and the roller, respectively, projected in the $x$ direction. The period-averaged, depth-averaged momentum equations for the $x$ and $y$ directions are

$$\lambda \frac{\partial}{\partial x} [\rho U^2(\bar{h}+\bar{\eta})] + \frac{\partial S_{xx}}{\partial x} + \frac{\partial M_{xx}}{\partial x} + \rho g (\bar{h}+\bar{\eta}) \frac{\partial \bar{\eta}}{\partial x} = - \bar{\tau}_{bx}$$

and

$$\lambda \frac{\partial}{\partial y} [\rho UV(\bar{h}+\bar{\eta})] + \frac{\partial S_{xy}}{\partial x} + \frac{\partial M_{xy}}{\partial x} = - \bar{\tau}_{by}$$

where $V$ is the depth-averaged longshore current, $S_{xx}$, $S_{xy}$, $M_{xx}$, and $M_{xy}$ are the Radiation Stress and roller momentum flux components, and $\bar{\tau}_b$ is the mean bed stress. In this formulation the first terms in Eqs. (5) and (6) are parameterizations of the net convective acceleration associated with the vertical structure of the currents, in which $\lambda$ is an empirical coefficient. Although the importance of the net convective acceleration to the cross-shore mixing of the longshore current has been explored by Svendsen and Putrevu (1994), this mechanism is not yet well-resolved. However, for the immediate purpose of examining the influence of the roller on the currents, the parameterized form is adequate.

Adopting the quadratic bed stress model:

$$\bar{\tau}_{bx} = \rho \frac{f}{8} u_b (u_b^2 + v_b^2)^{1/2}$$

$$\bar{\tau}_{by} = \rho \frac{f}{8} v_b (u_b^2 + v_b^2)^{1/2}$$

in which $u_b$ and $v_b$ are the total instantaneous velocities at the bed, and the Darcy-Weisbach friction factor $f$ is related to Manning's $n$ by (see Smith, et al., 1993)

$$f = \frac{8 g n^2}{(\bar{h}+\bar{\eta})^{1/3}}$$

and a value of 0.019 sm$^{-1/3}$ is used herein.

Results and Discussion

Neglecting set-up and currents for the moment, and once again using linear wave theory and the breaker model of Dally, et. al (1985) to drive Eq.(1), the volume flux, energy flux, and momentum flux terms of Eqs.(4), (5), and
(6) can be calculated, and their relative magnitudes examined. In Figures 3 and 4, nondimensional quantities defined as

\[ Q'_x = \frac{Q_r}{Q_v} \]

\[ F'_i = \frac{1}{2} \rho Q_r c^2 \cos^2 \alpha \]

\[ M'_{xx} = \frac{\rho Q_r c \cos^2 \alpha}{S_{xx}} \]

\[ M'_{xy} = \frac{\rho Q_r c \sin \alpha \cos \alpha}{S_{xy}} \]

are plotted versus relative still-water depth for a planar beach of 1/30 slope, with $H_b/h_b = 0.8$, and breaker angles of 20° and 0°. It appears that once the roller is fully developed, $Q_r$ and $M_{xx}$ are nominally 1½ times greater than $Q_v$ and $S_{xx}$ computed from linear wave theory, whereas $M_{xx}$ and $F_r$ are respectively 1 and ¾ times the magnitude of $S_{xx}$ and $F_v$.

To include set-up and currents, the complete system of five equations (energy, Snell's Law, mass, x-momentum, and y-momentum) and five unknowns ($Q_r$, $\alpha$, U, V, and $\eta$) are to be solved numerically. Results for wave decay, set-up and undertow are shown in Figure 5 for 0° angle of incidence. Note that the maximum set-down and maximum cross-shore current are shifted landward of the breakpoint, in qualitative agreement with observations (e.g. Bowen, Inman, and Simmons, 1968). Dally and Brown (1995) present a comparison of similar model results to set-up and undertow laboratory data from the literature, in which it was discovered that quantitative agreement could be achieved if Stream Function wave theory was used to specify the quantities associated with the organized motion.

In contrast to Figure 5, Figure 6 displays results of the model if all roller terms are neglected (the problem reduces to four equations and four unknowns, with wave heights specified by the Dally, et al. breaker model). It is clear that the roller terms are responsible for the transition region.

Figures 7 and 8 are generated for a breaker angle of 20°, and a landward shift in the peak longshore current, due to the roller, is also evident. The convective acceleration is responsible for the longshore current found seaward of the breaker line, and the value for the coefficient $\lambda$ has been chosen as 0.2 to produce a
Figure 3 - Evolution of dimensionless volume, energy, and momentum flux components, defined in Eq.(9), for a breaker angle of 20°.

Figure 4 - Same as Figure 3. Breaker angle of 0°.
Figure 5 - Dimensionless wave height, set-up, and cross-shore current generated by numerical solution to Eqs. (1), (4), (5), and (6), for a breaker angle of $0^\circ$.

Figure 6 - Same as Figure 5, but roller terms neglected.
1.5

1

0.5

0

-0.5

-1

-1.5

H_b / L_o = 0.02
\alpha_b = 20^\circ
\lambda = 0.2

Figure 7 - Dimensionless wave height, set-up, cross-shore current, and longshore current generated by numerical solution to Eqs. (1), (4), (5), and (6), for a breaker angle of 20°.

1.5

1

0.5

0

-0.5

-1

-1.5

H_b / L_o = 0.02
\alpha_b = 20^\circ
\lambda = 0.2

Figure 8 - Same as Figure 7, but roller terms neglected.
reasonable tail. Inside the surf zone, it appears that the roller is as important as the convective acceleration in determining the cross-shore structure of the longshore current. In comparing Figure 5 with Figure 7, altering the breaker angle does not change the cross-shore structure of the dimensionless set-up or undertow.

Conclusions

By utilizing the very simple roller model developed by Dally and Brown (1995), the long-suspected role played by the aerated region in driving and mixing surf zone currents is confirmed. Because Dally and Brown (1995) found the net convective acceleration to be negligible in the cross-shore momentum balance, it can be concluded that the roller is almost solely responsible for the observed landward shift in set-up and peak cross-shore current.

The roller also appears to contribute significantly to the landward shift in the peak longshore current, as well as to cross-shore mixing in the surf zone. In light of the findings of Svendsen and Putrevu (1994), it is reasonable to conclude that the momentum flux associated with the roller is significantly more important to the horizontal structure of the currents than the turbulent mixing (Reynolds Stress) associated with the wake left behind.

The modeling results also indicate that the roller could be of importance comparable to the net convective acceleration associated with the vertical structure of the currents (Svendsen and Putrevu, 1994); however, a more precise assessment awaits closer study of the vertical structure of the currents. Svendsen and Putrevu (1994) assume it is vertical structure in the longshore current that leads to mixing, where comparison of the laboratory measurements of Nadaoka and Kondoh (1982) for undertow to those of Visser (1991) for the longshore current indicate that it is actually the undertow that is more depth-dependent.

Finally, it is stressed that all of the modeling done herein utilizes linear wave theory to represent the organized wave motion. In light of the findings of Dally and Brown (1995) for set-up and undertow driven by normally incident waves, it is anticipated that Stream Function, or another suitable nonlinear wave theory, will be required to achieve satisfactory comparisons of the model to longshore currents observed in the laboratory and field.
References


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