CHAPTER 107

HYDRODYNAMIC FORCES ON BOTTOM-SEATED HEMISPHERE IN WAVES AND CURRENTS

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Abstract

The characteristics of hydrodynamic forces acting on a bottom-seated hemisphere in waves only, in currents only and in waves & currents are presented from the viewpoint of the design of an artificial reef. A large number of regular wave experiments were conducted in order to find the empirical formulas of the hydrodynamic coefficients for the bottom-seated hemisphere. It is shown that the proposed empirical formulas are sufficiently accurate for predicting the hydrodynamic forces. The effects of the normalized water depth and hemisphere spacing relative to its diameter are also elucidated experimentally.

Introduction

In order to provide a continued supply of marine products, the Japanese fishing industry has been shifting from fish catching to fish farming. Various artificial reefs have been used to attract fish by producing coherent eddies with upward flow as well as by providing hiding places for fish. Most of these reefs have rectangular shapes and caused tearing of fishing nets. In order to reduce entanglement of fishing nets, the authors have proposed bottom-seated hemispherical reefs. For the design of such a reef against waves and currents, the hydrodynamic forces acting on the reef need to be predicted.

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Hydraulic Experiments

Regular wave experiments were conducted in a wave flume which was 65 m long, 1.0 m wide and 1.6 m high. The flume was capable of generating steady currents in both directions relative to the direction of wave propagation. As a first attempt, plastic hemispheres without any opening were used in the experiments. The experimental results presented herein may hence be of interest to researchers working on the design of quarry stone armor units. The three-dimensional hydrodynamic force acting on an individual hemisphere was measured using a three-component waterproof load cell with a water pressure adjustment. Fluid velocities at the location of the hemisphere were measured using a two-component electromagnetic current meter. Free surface oscillations above the hemisphere are measured using a capacitance wave gage. For limited tests, flow visualization was performed to examine the flow pattern and eddy formation using a laser and a high speed camera, as shown in Photo.1. A large number of tests were performed for a single hemisphere and three hemispheres in a row.

Fig.1 shows the experimental setup for three hemispheres, where X=horizontal coordinate taken to be positive in the direction of wave propagation with X=0 at the center of the middle hemisphere; Z=vertical coordinate taken to be positive upward with Z=0 at the still water level (SWL); h=water depth below SWL; H=height of incident regular waves whose period is denoted by T; D=diameter of the hemisphere; and Lf=distance between the centers of two adjacent hemispheres. The experiments were conducted for the following conditions;
1) h=40, 60 and 80 cm
2) H=3.0, 6.0 and 9.0 cm
3) T=1.2, 1.6, 2.0, 2.4 and 2.8 sec
4) D=15.0 and 20.0 cm
5) Lf/D=1.0, 1.5, 2.0, 2.5 and 3.0
6) U=±5.0, ±10.0, ±20.0, ±30.0 and ±40.0 cm/sec
where U=depth-averaged steady current velocity which is negative for opposing currents.

Photo.1 An example of flow visualization in currents only
Data Analysis

The measured horizontal and vertical hydrodynamic forces acting on the hemisphere are assumed to be expressed in the form proposed by Morison (Morison et al. 1950).

\[ F_x = \frac{1}{2} \rho C_D \left( \pi D^2 \right) \left( u + U \right) + \rho C_M \left( \pi D^3 \right) \frac{du}{dt} \]  \hspace{1cm} (1)

\[ F_{ZM} = \frac{1}{2} \rho C_L \left( \pi D^2 \right) (u_m + U)^2 \]  \hspace{1cm} (2)

where \( F_x \)=horizontal force ; \( \rho \)=fluid density ; \( u \)=oscillatory horizontal fluid velocity ; \( U \)=steady fluid velocity ; \( C_D \)=drag coefficient ; \( C_M \)=inertia coefficient ; and \( C_L \)=lift coefficient .The velocities \( u \) and \( U \) are at the location of the hemisphere. The constant values of \( C_D \) and \( C_M \) for each test were determined using the method of least squares(Reid 1957). The constant value of \( C_L \) was estimated such that the maximum vertical force \( F_{ZM} \) could be predicted by Eq.(2) accurately. This was because Eq.(2) with constant \( C_L \) did not reproduce the entire variation of \( F_{ZM} \) with respect to time \( t \) very well.

Drag, Inertia and Lift Coefficients

The efforts for developing the empirical relationships for \( C_D \), \( C_M \) and \( C_L \) were separated into;
- single hemisphere in waves only
- single hemisphere in currents only
- single hemisphere in waves & currents
- middle hemisphere among three hemispheres in waves only
- middle hemisphere among three hemispheres in currents only
For these tests, the inertia force was generally dominant in Eq.(1) and the horizontal force was normally greater than the vertical force. In addition, it was judged that there was little scale effect on hemispherical models because the influence of Reynolds number $Re=umD/\nu$ on the wave force coefficients were negligible under adopted experimental conditions, refer to Figs.2 and 3.

Fig.4 shows the relationship between the wave force coefficients and Keulegan-Carpenter number ($K.C.=umT/D$) for the single hemisphere in the water depth of 40 cm, where $um$ is the maximum value of $u$. As shown in Fig.4, $C_M$ in waves only is on the order of 1.35 and $C_D$ in waves only varies with $K.C.$ in a manner similar to a sphere (Jenkins and Inman 1976).

Eqs.(3), (4) and (5) are the empirical formulas of $C_D$, $C_M$ and $C_L$ in waves only respectively. They are statistically derived by means of the method of least squares.

\[
C_D = 6.79 \cdot (K.C.)^{-0.89} \quad \text{for } U=0 \quad (3)
\]
\[
C_M = 1.35 \quad \text{for } U=0 \quad (4)
\]
\[
C_L = 3.3 \cdot (K.C.)^{-0.98} \quad \text{for } U=0 \quad (5)
\]

Eqs.(6) and (7) are the empirical formulas of the drag and lift coefficients in currents only for the single hemisphere in the depth of 40 cm, respectively. The effect of Reynolds number $Re=UD/\nu$ on these coefficients seems to be also negligible for the case of currents only as shown in Fig.5.

\[
C_D = 0.48 \quad \text{for } um=0 \quad (6)
\]
\[
C_L = 0.8 \quad \text{for } um=0 \quad (7)
\]
Fig. 3 Inertia coefficient $C_M$ versus $Re = \frac{u_mD}{\nu}$ in waves only.

Fig. 4 Drag coefficient $C_D$, inertia coefficient $C_M$ versus $K.C.$ in waves only.

Fig. 5 Drag coefficient $C_D$ versus $Re = \frac{UD}{\nu}$ in currents only.
Fig. 6 shows the relationship between $C_D$ in waves & currents and the modified Keulegan-Carpenter number $K=(|U|+u_m)/D$ for the single hemisphere in fair currents for which $U>0$ (Iwagaki et al., 1983). The water depth is 40 cm. The parameter $\alpha$ shown in the figure is defined as $\alpha=|U|/u_m$ and two solid lines in this figure indicate the empirical formulas of $C_D$ in waves only [Eq.(3)] and in currents only [Eq.(6)], respectively. $C_D$ is slightly affected by $K$ and $\alpha$ and also approximately approaches 0.48 for large $\alpha$, corresponding to the value of $C_D$ for currents only.

On the basis of these characteristics shown in Fig. 6, the following empirical formula of $C_D$ for the single hemisphere in waves & currents with $U>0$ have been proposed;

$$C_D = 6.97 \cdot e^{-\beta} \cdot K^{-0.89} + 0.48(1 - e^{-\beta}) \text{ for } U=0$$

with \[ \beta = 0.019\alpha^2 + 0.99\alpha - 0.07 \]

When $\alpha$ is equal to zero and $K=K.C.$, Eq.(8) yields almost the same value of $C_D$ in waves only. On the other hand, this formula corresponds to $C_D$ in currents only when $\alpha$ tends to infinity. The broken lines in Fig.6 indicate Eq.(8) for $\alpha$ in the range of 0.25 to 2.5. In order to evaluate the accuracy of Eq.(8), Fig.7 compares $C_D$ calculated by Eq.(8) with $C_D$ obtained from the experiments. It is realized that the former agrees fairly with the latter, regardless of $\alpha$.

Fig.8, for the case of water depth 40 cm, shows the relationship between the inertia coefficient in waves & currents and $K$ for the single hemisphere in fair currents. It is obvious that $C_M$ increases with increasing $K$ in the region $C_M>1.35$, whereas $C_M$ decreases with increasing $K$ in the region $C_M<1.35$. Furthermore, $C_M$ decreases with increasing $\alpha$ if $K$ is assumed to be constant.

Considering the above characteristics, the following empirical formula of $C_M$ as a function of $\alpha$ and $K$ is derived from the method of least squares.

$$C_M = 1.35 \cdot \exp \left[ 0.084(1.18 - \alpha)K \right] \text{ for } U>0$$

Eq.(9) is equivalent to $C_M$ in waves only if $\alpha$ is equal to 1.18. Broken lines in Fig.8 indicate the above formula for $\alpha$ in the range of 0.25 to 3.5. It is found that Eq.(9) coincides fairly with the experimental values.

The relationship between $C_M$ calculated by Eq.(9) and $C_M$ obtained from the experimental results is also shown in Fig.9. The reason why the accuracy of Eq.(9) is lower as the parameter $\alpha$ becomes larger is that the drag force is more predominant than the inertia force and it is hard to estimate $C_M$ accurately for large $\alpha$. 
Fig. 6 Drag coefficient $C_D$ versus $K$ in waves & currents (fair current)

Fig. 7 Drag coefficient $C_D(\text{EXP})$ versus Drag coefficient $C_D(\text{CAL})$
Fig. 8 Inertia coefficient $C_M$ versus $K$ in waves & currents (fair current)

Fig. 9 Inertia coefficient $C_M$(EXP) versus inertia coefficient $C_M$(CAL)
Fig. 10 shows the relationship between $C_L$ in waves & currents and $K$ for the single hemisphere in the water depth of 40 cm. The variations of $C_L$ with respect to these dimensionless parameters were similar to those associated with $C_D$. The following formula in fair currents is introduced.

$$C_L = 3.3 \cdot e^{-0.5\alpha} \cdot K^{-0.98} + 0.8(1 - e^{-0.5\alpha})^{0.16} \text{ for } U>0 \quad (10)$$

The relationships between the hydrodynamic coefficients and $K$ for the single hemisphere in adverse currents for which $U<0$ and $\alpha$ are shown in Figs. 11 and 12. Moreover, the following empirical formulas for adverse currents are derived using the same procedure.

$$C_D = 6.97 \cdot e^{-\beta} \cdot K^{-0.89} + 0.48(1 - e^{-\beta}) \quad \text{for } U<0 \quad (11)$$

with $\beta = -0.08\alpha^2 + 1.05\alpha - 0.1$

$$C_M = 1.35 \cdot \exp \left[ 0.063(1.34 - \alpha)K \right] \quad \text{for } U<0 \quad (12)$$

$$C_L = 3.3 \cdot e^{-0.6\alpha} \cdot K^{-0.98} + 0.8(1 - e^{-0.6\alpha})^{0.43} \quad \text{for } U<0 \quad (13)$$

Those formulas should be reevaluated because free surface oscillations became larger and more irregular as waves propagated over a long distance in adverse currents.

Fig. 10 Lift coefficient $C_L$ versus $K$ in waves & currents (fair current)
Fig. 11 Drag coefficient $C_D$ versus $K$ in waves & current (adverse current)

Fig. 12 Inertia coefficient $C_M$ versus $K$ in waves & currents (adverse current)
Applicability of Empirical Formulas

The above experimental results were for the water depth of 40cm. It is very important to study the effect of the variation of water depth on \( C_D \) and \( C_M \) in waves & currents. Figs.13 and 14 show the effects of the water depth normalized by the diameter \( D \) on \( C_D \) and \( C_M \), respectively. These coefficients are almost constant, regardless of the dimensionless parameter \( h/D \). In other words, \( C_D \) and \( C_M \) in waves & currents are not practically affected by the effect of the water depth in the range \( h/D=2.0 \sim 4.0 \) tested here.

Fig.15 shows the relationship between the dimensionless \( C_{MG}/C_M \) and \( L_f/D \), where \( C_{MG} \) is the inertia coefficient of the middle hemisphere among three hemispheres and \( L_f \) is defined in Fig.1. \( C_{MG}/C_M \) approximately approaches 1 for large \( L_f/D \). The empirical formulas on the inertia coefficient proposed in this paper are valid for the range \( L_f/D > 2.0 \). The effect of the adjacent hemispheres on the drag coefficient was also negligible for \( L_f/D > 2.0 \).

Conclusions

The experimental results in this paper are summarized as follows;
(1)The empirical formulas of \( C_D \) and \( C_M \) in waves & currents are derived using the modified Keulegan-Carpenter number \( K \) and the parameter \( \alpha \) indicating the current strength relative to the wave velocity. These formulas are accurate enough to estimate the hydrodynamic forces.
(2)These hydrodynamic coefficients in waves & currents are not affected by the normalized water depth \( h/D \) in the range \( h/D = 2.0 \sim 4.0 \).
(3)The empirical formulas proposed in this study are valid for the range \( L_f/D > 2.0 \) for the conditions shown in Fig.1.

Fig.13 Drag coefficient \( C_D \) versus \( h/D \)  Fig.14 Inertia coefficient \( C_M \) versus \( h/D \)
Fig. 15 $CMG/CM$ versus $L_f/D$

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**References**


